Timed Automata
vs
Bounded Time Petri nets

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Motivations

In terms of timed language acceptance:

$TPN = TA$ [Bérard et al., 2005][Bouyer et.al 2006]

In terms of weak timed bisimulation:

$TPN < TA$ [Cassez/Roux, 2004]

$TPN = TA^{-}$ [Bérard et al., 2005]

? : TPN^{+} such that

$TPN^{+} \equiv TA$ [BB/FP/FV, FORMATS 2006]

Timed transition systems (TTS)

A structure $\langle Q, q^0, \Sigma^\epsilon, \rightarrow \rangle$ where:

- Q is a set of *states*
- $q^0 \in Q$ is the *initial state*
- $\Sigma^\epsilon = \Sigma \cup \{\epsilon\}$ is a finite set of *actions* ($\epsilon \notin \Sigma$)
- $\rightarrow \subseteq Q \times (\Sigma \cup \{\epsilon\} \cup \mathbf{R}^+) \times Q$ is the *transition* relation.

Product of TTS:

$$\frac{q_1 \xrightarrow{a}_1 q_1'}{q_1 \mid\mid q_2 \xrightarrow{a} q_1' \mid\mid q_2} \quad (a \in \Sigma_1^\epsilon \backslash \Sigma_2) \qquad \frac{q_2 \xrightarrow{a}_2 q_2'}{q_1 \mid\mid q_2 \xrightarrow{a} q_1 \mid\mid q_2'} \quad (a \in \Sigma_2^\epsilon \backslash \Sigma_1)$$

$$\frac{q_1 \xrightarrow{a}_1 q_1' \qquad q_2 \xrightarrow{a}_2 q_2'}{q_1 \mid\mid q_2 \xrightarrow{a} q_1' \mid\mid q_2'} \quad (a \neq \epsilon)$$

Timed bisimilarity of TTS

Strong: $\langle Q_1, q_1^0, \Sigma_1^\epsilon, \rightarrow_1 \rangle$ strong timed bisimilar $\langle Q_2, q_2^0, \Sigma_2^\epsilon, \rightarrow_2 \rangle$ iff

- $q_1^0 \sim q_2^0$
- whenever $q_1 \sim q_2$ and $a \in \Sigma_1^\epsilon \cup \Sigma_2^\epsilon \cup \mathbf{R}^+$:

  (1) $q_1 \xrightarrow{a}_1 q_1' \Rightarrow (\exists q_2')(q_2 \xrightarrow{a}_2 q_2' \wedge q_1' \sim q_2')$

  (2) $q_2 \xrightarrow{a}_2 q_2' \Rightarrow (\exists q_1')(q_1 \xrightarrow{a}_1 q_1' \wedge q_1' \sim q_2')$

Weak: replace $\xrightarrow{a}$ by $\stackrel{d}{\Longrightarrow}$, with:

$$\frac{q \xrightarrow{a} q'}{q \stackrel{a}{\Longrightarrow} q'} \qquad \frac{q \stackrel{d}{\Longrightarrow} q' \quad q' \stackrel{d'}{\Longrightarrow} q''}{q \stackrel{d+d'}{\Longrightarrow} q''}$$

$$\frac{q \stackrel{a}{\Longrightarrow} q' \quad q' \xrightarrow{\epsilon} q''}{q \stackrel{a}{\Longrightarrow} q''} \qquad \frac{q \xrightarrow{\epsilon} q' \quad q' \stackrel{a}{\Longrightarrow} q''}{q \stackrel{a}{\Longrightarrow} q''}$$

Timed Automata (TA)

A tuple $\langle Q, q^0, X, \Sigma^\epsilon, T\rangle$ in which:

- Q is a finite set of *locations*
- $q^0 \in Q$ is the *initial location*
- X is a finite set of clocks
- $\Sigma^\epsilon = \Sigma \cup \{\epsilon\}$ is a finite set of actions ($\epsilon \notin \Sigma$)
- $T \subseteq Q \times (\mathcal{C}(X) \times \Sigma^\epsilon \times 2^X) \times Q$ is a finite set of transitions

Semantics:

The TTS $\langle S, s^0, \Sigma^\epsilon, \rightarrow\rangle$ where $S = Q \times (\mathbf{R}^+)^{|X|}$, $s^0 = (q^0, \overline{0})$, $\rightarrow$:

- $(q, v) \xrightarrow{a} (q', v')$ iff $(\exists(q, g, a, R, q') \in T)(g(v) \wedge v' = v[R := 0])$
- $(q, v) \xrightarrow{d} (q, v + d)$ iff $d \in \mathbf{R}^+$

Timed Automata ...

Product of TA: compositional

Progress requirements: Invariants, urgency, deadlines, ...

Priorities: do not add expressiveness

if t_1, with guard g_1, has lower priority than t_2, with guard g_2

then replace g_1 by $g_1 \wedge \neg g_2$

Time Petri nets with priorities ($PrTPN$)

$\langle P, T, \mathbf{Pre}, \mathbf{Post}, m^0, Is, \succ, \mathbf{\Sigma}^\epsilon, L \rangle$ in which:

- $\langle P, T, \mathbf{Pre}, \mathbf{Post}, m^0 \rangle$ is a Petri net
- $Is : T \rightarrow \mathbf{I}^+$ is the *Static Interval* function,
- $\succ \subseteq T \times T$ is the Priority relation

  s irreflexive, asymmetric and transitive,
- $\mathbf{\Sigma}^\epsilon = \mathbf{\Sigma} \cup \{\epsilon\}$ is a finite set of *Actions* ($\epsilon \notin \Sigma$)
- $L : T \rightarrow \mathbf{\Sigma}^\epsilon$ is the *Labeling* function.

Semantics of $PrTPN$

The TTS $\langle S, (m^0, Is^0), \Sigma, \rightarrow \rangle$ in which:

- $Is^0 = Is$ restricted to the transitions enabled at m^0
- the states of S are pairs (m, I) with $m : P \rightarrow \mathbf{N}^+$ and $I : T \rightarrow \mathbf{I}^+$
- discrete transitions: $(m, I) \xrightarrow{L(t)} (m', I')$ iff $t \in T$ and
  1. $m \geq \mathbf{Pre}(t)$
  2. $0 \in I(t)$
  3. $(\forall k \in T)(m \geq \mathbf{Pre}(k) \wedge 0 \in I(k) \Rightarrow \neg(k \succ t))$
  4. $m' = m - \mathbf{Pre}(t) + \mathbf{Post}(t)$
  5. $(\forall k \in T)(m' \geq \mathbf{Pre}(k) \Rightarrow$
     $I'(k) =$ **if** $k \neq t \ \wedge \ m - \mathbf{Pre}(t) \geq \mathbf{Pre}(k)$ **then** $I(k)$ **else** $Is(k))$
- continuous transitions: $(m, I) \xrightarrow{d} (m, I')$ iff

  $(\forall k \in T)(m \geq \mathbf{Pre}(k) \Rightarrow d \leq \uparrow I(k) \wedge I'(k) = I(k) \ \dot{-} \ d)$

Product of $PrTPN$

Of transitions:

Given a PN $\langle P, T, \mathbf{Pre}, \mathbf{Post}, m^0 \rangle$ and $E \subseteq T$
the *product* of the transitions in E is t such that:

$$\mathbf{Pre}(t)(p) = \sum_{k \in E} \mathbf{Pre}(k)(p) \quad \text{and} \quad \mathbf{Post}(t)(p) = \sum_{k \in E} \mathbf{Post}(k)(p)$$

Of Petri nets:

$\langle P_1, T_1, \mathbf{Pre}_1, \mathbf{Post}_1, m_1^0, \mathbf{\Sigma}_1^\epsilon, L_1 \rangle \mid\mid \langle P_2, T_2, \mathbf{Pre}_2, \mathbf{Post}_2, m_2^0, \mathbf{\Sigma}_2^\epsilon, L_2 \rangle$ built as follows:

- Start with $N = N_1 \cup N_2$, after removing transitions labeled on $\mathbf{\Sigma}_1 \cap \mathbf{\Sigma}_2$,
- For each $(t_1, t_2) \in T_1 \times T_2$ such that $L_1(t_1) = L_2(t_2) \neq \epsilon$,

  add a transition defined as the product of t_1 and t_2, inheriting their label.

Product of $PrTPN$ ...

Of $PrTPN$:

- $Is(t_1 \times t_2) = Is_1(t_1) \cap Is_2(t_2)$,
- $\succ =$ the transitive closure of R, assuming it asymmetric, where

  $R = \{(x, y) \in T \times T | (\exists (t, t') \in \succ_1 \cup \succ_2)(x \in S(t) \wedge y \in S(t')\}$, where

  $S(t) = \{t\}$ if t is not synchronized

  $S(t) =$ the set of product transitions involving t otherwise

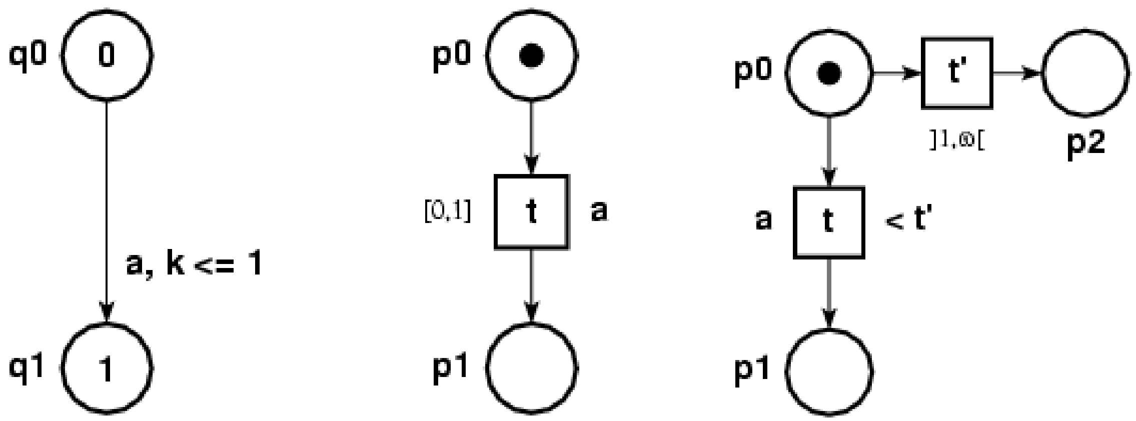
Theorem:

If the transitions synchronized have interval $[0, \infty[$

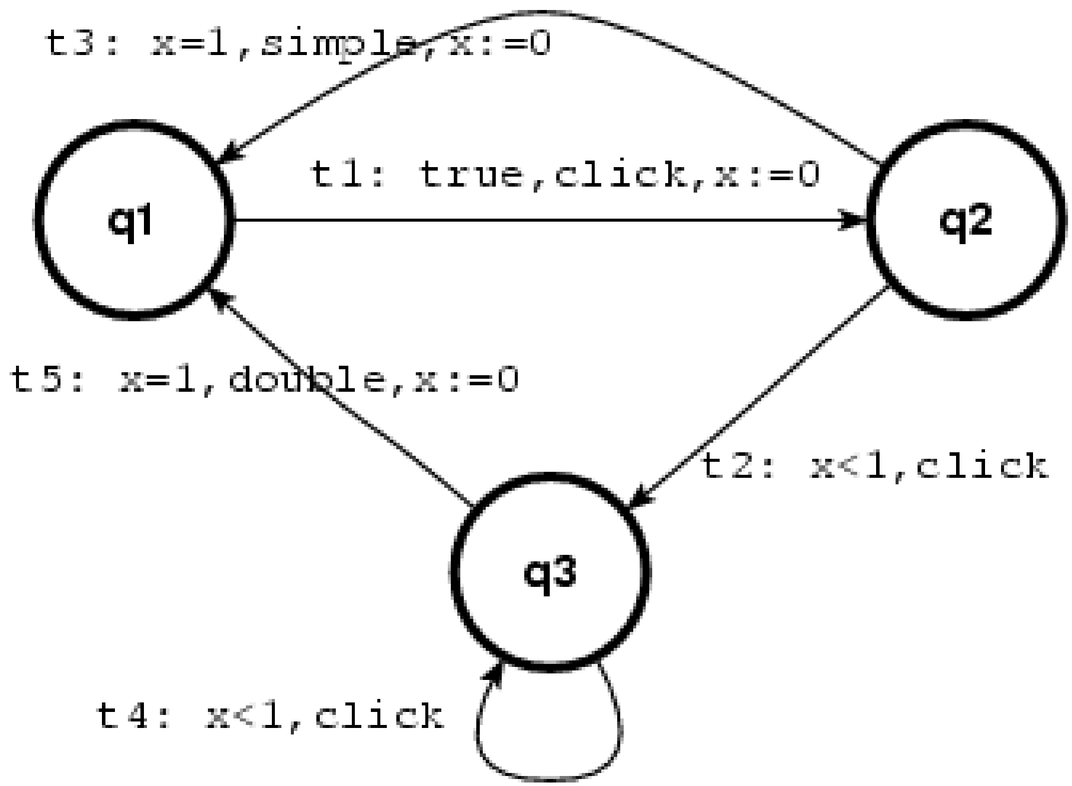
And $\succ$ does not relate non-sync transitions in different components

Then $||$ is compositional

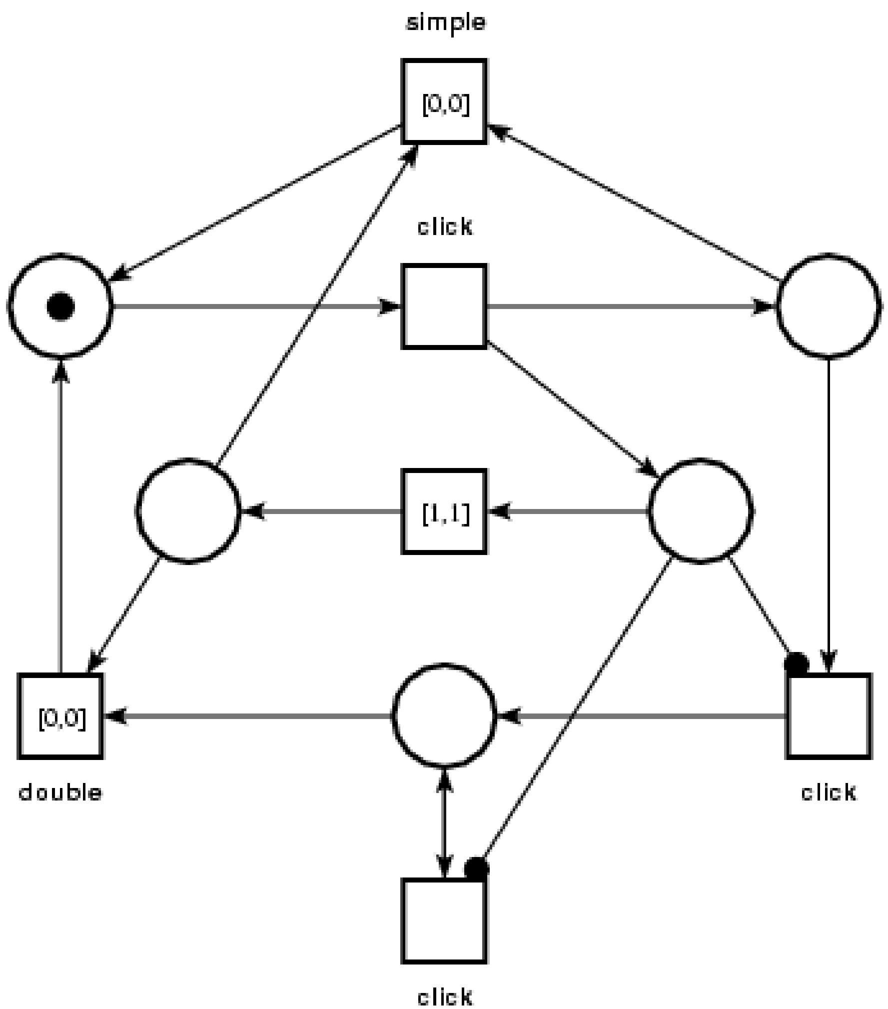
Priorities add expressiveness to TPN



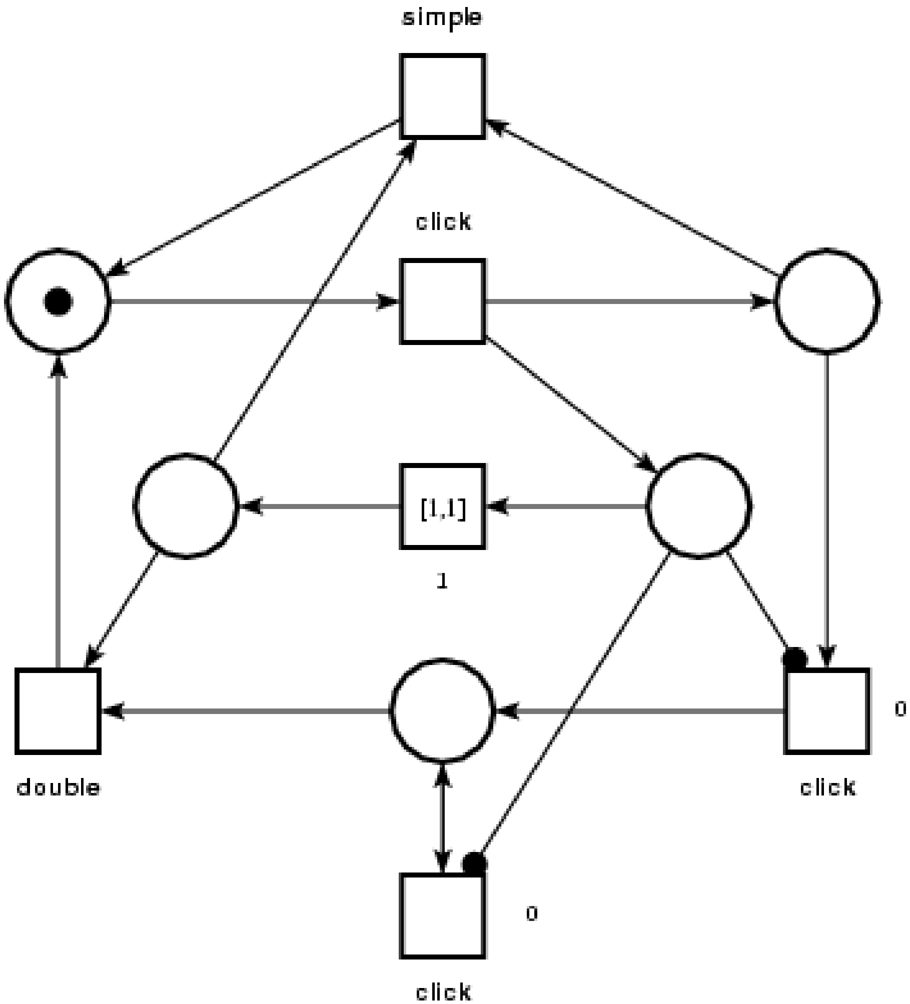
Double click TA example



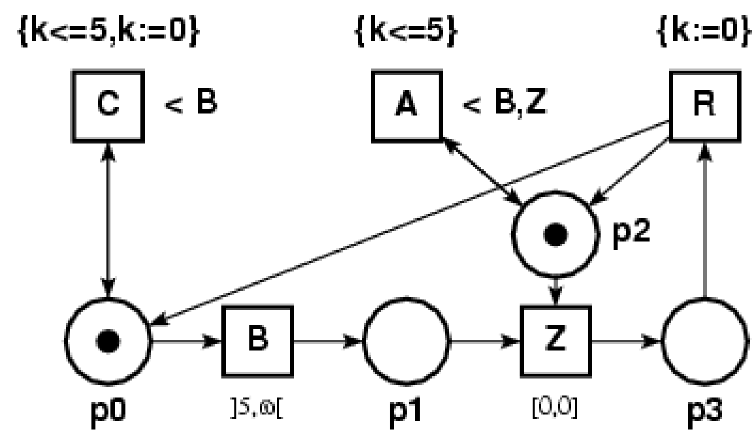
Approximated double click in TPN

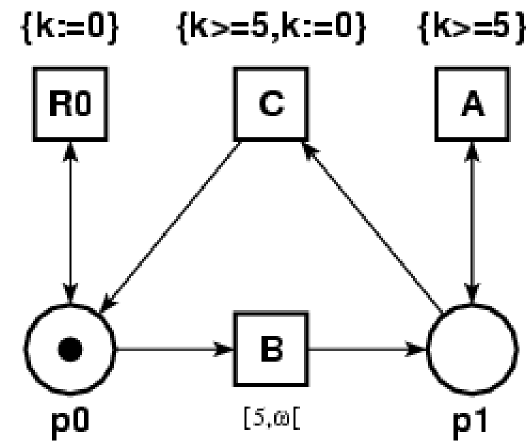


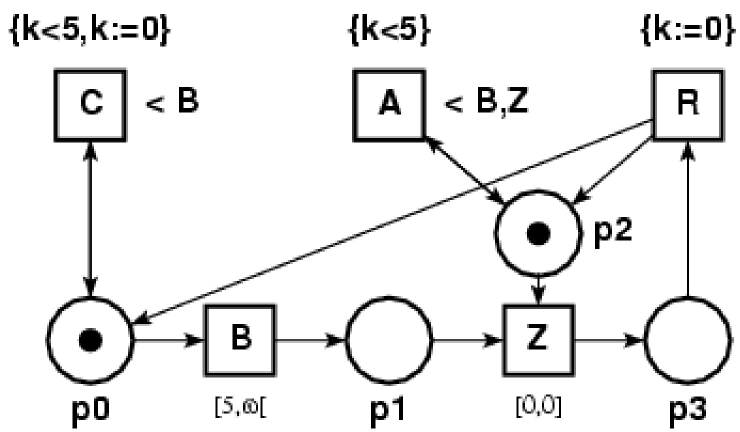
Double click in $PrTPN$

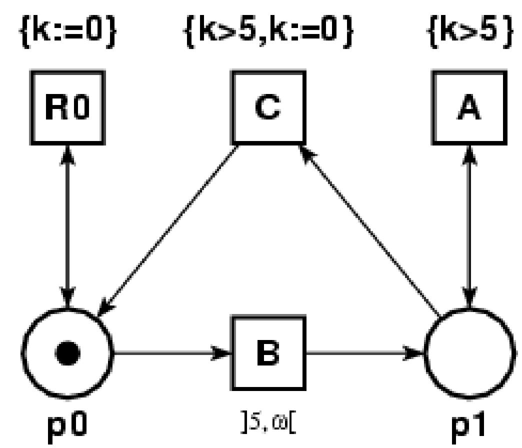


Encoding TA guards

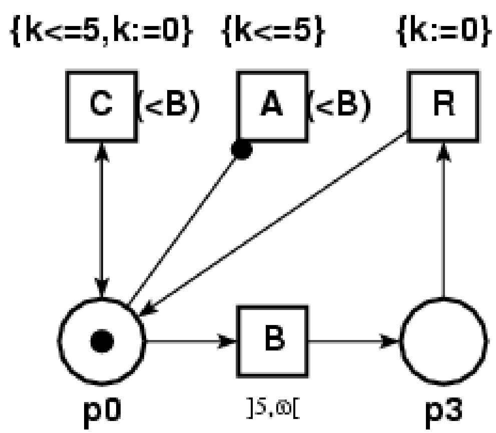


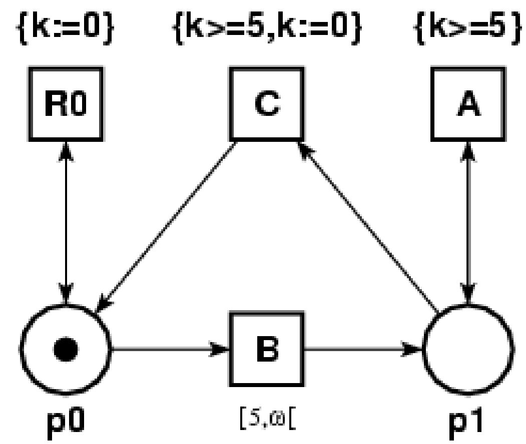


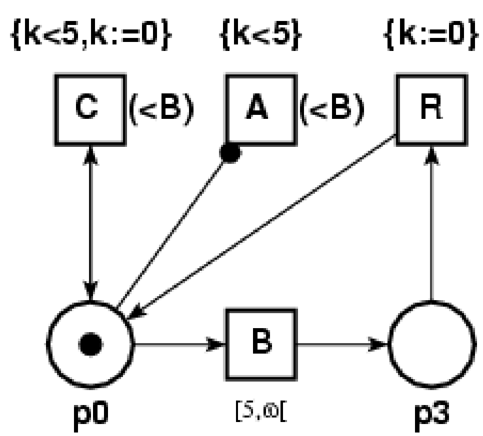


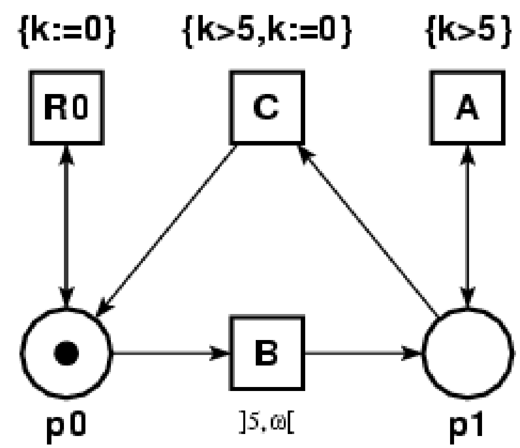


Same, using read arcs

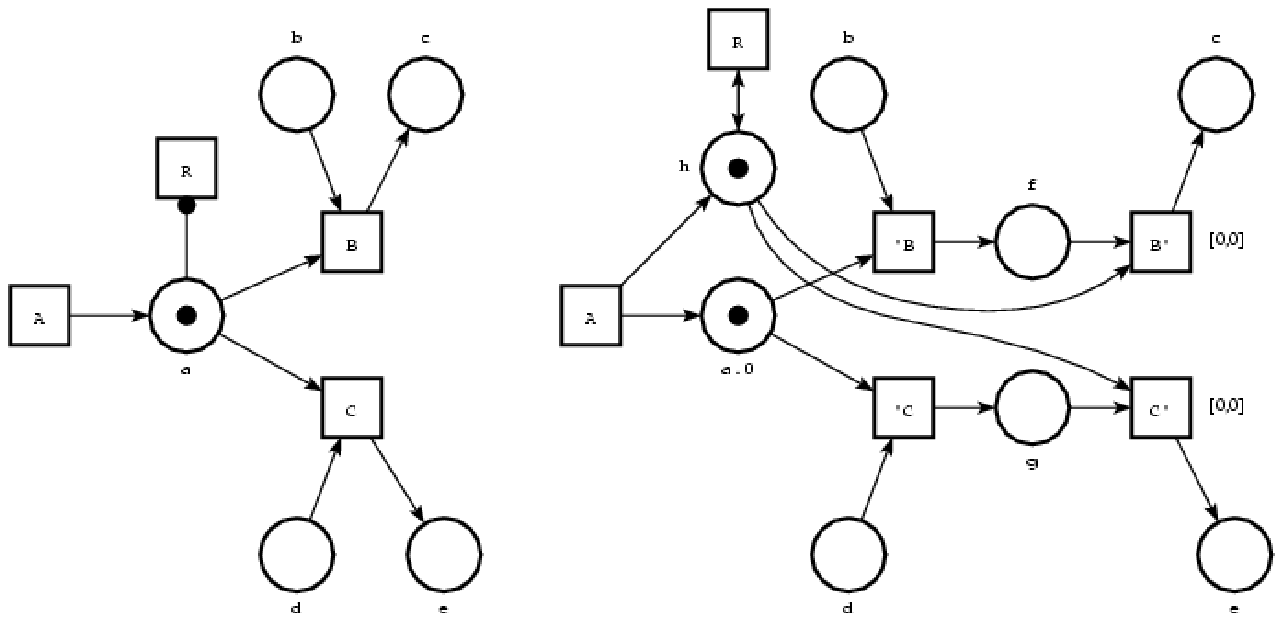








Removing read arcs



Theorem

Let k be the time elapsed since the initial marking was established

- the transitions whose label includes a condition on k
  are firable exactly at the times at which that condition holds,
- all transitions whose label includes $k := 0$ restore the
  initial state of the net,
- at any time some transition whose label includes $k := 0$ is firable.

Encoding a clock

For each $k \in X$, let N_k be the net built as follows:

1. Assume $\{c_1, \ldots, c_n\}$ is the set of nets encoding the guards involving k. Let $K = (c_1 \backslash F \parallel \ \ldots \ \parallel c_n \backslash F) \backslash H$, with F and H as follows:
   - F relabels any transition whose label includes $k := 0$ with ρ (ρ new),
   - H relabels any t obtained from a product of transitions by the union of their labels in nets c_i.
2. Starting with $N_k = K$, add to N_k, for each $E \subseteq G_k$ with $card(E) > 1$, a transition labeled E defined as the product of all transitions of K with their label intersecting E.

Net N_k has as transitions:

- Those internal to the component nets (unlabelled),
- For each nonempty $E \subseteq G_k$, a transition labeled E,
- For each (possibly empty) $E \subseteq G_k$, a transition labeled $E \cup \{k := 0\}$.

Theorem

Let k be the time elapsed since the initial marking was established

- the transitions whose label includes a condition are firable exactly at the times at which that condition holds,
- all transitions whose label includes $k := 0$ restore the initial state of the net,
- at any time some transition whose label includes $k := 0$ is firable.

Encoding a TA

$\mathcal{A} = \langle Q, q^0, X, \Sigma^\epsilon, T \rangle$

1. Let N_A be the net built as follows:
   - For each $q \in Q$ add a new place to N_A, and mark the place encoding q^0,
   - For each transition $q \xrightarrow{t} q'$ of $\mathcal{A}$, add to N_A a transition between the places encoding q and q', labeled E_t
2. Next, let N_K be the net built as follows:
   - Start with $N_K = ||_{k \in X}\ N_k$ where N_k encodes clock k,
   - Then, for each E_t, add to N_K a transition labeled E_t defined as the product of all transitions of N_K with their label belonging to E_t,
   - Remove all labeled transitions of N_K whose label is not in any E_t,
3. Finally, let $\mathcal{N} = N_A \mid\mid N_K$ and relabel each labeled transition by the action from Σ^ϵ belonging to its label.

Theorem

Any TA can be encoded into a $PrTPN$ without right-open intervals, preserving weak timed bisimilarity, or:

$TA \lesssim_{\mathcal{W}} PrTPN$ with unbounded or right-closed intervals.

TA **Invariants**

TA with invariants:

$\langle Q, q^0, X, \Sigma^\epsilon, T, I\rangle$ where

- $\langle Q, q^0, X, \Sigma^\epsilon\rangle$ is a TA
- $I : Q \rightarrow \mathcal{C}(X)$ maps clock constraints with locations

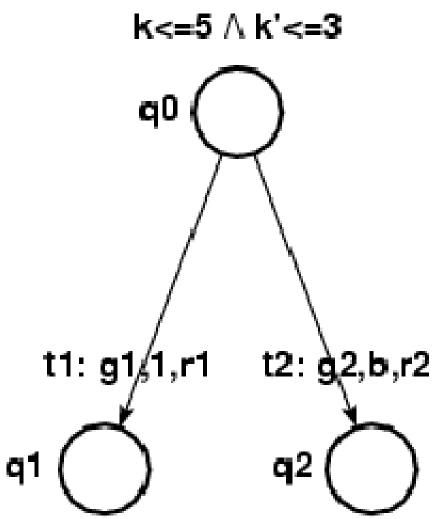
  invariants typically built from $\{\leq, <, \wedge\}$

Semantics

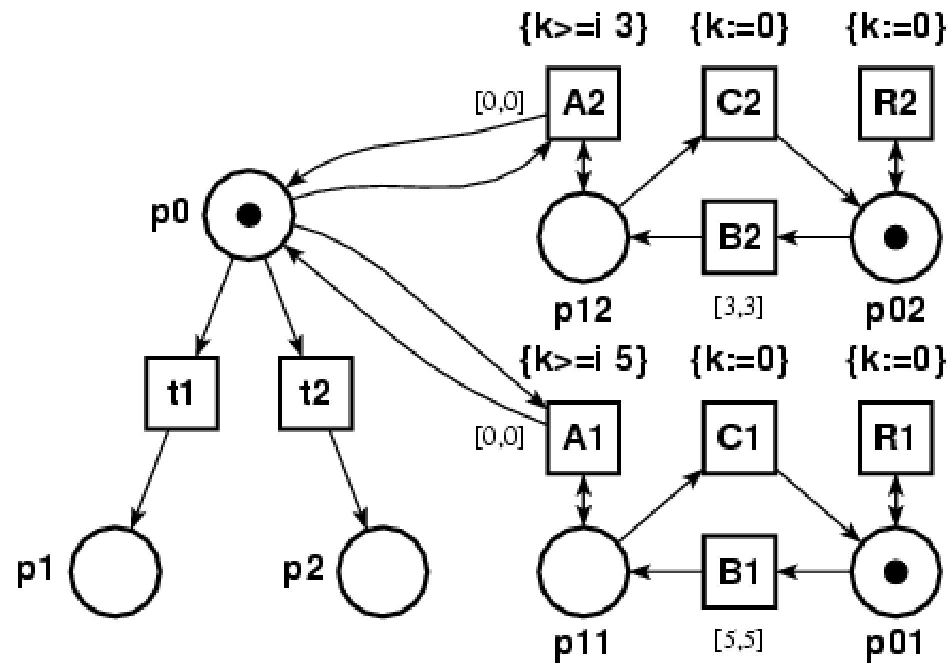
- $(q, v) \xrightarrow{d} (q, v + d)$ iff $d \in \mathbf{R}^+ \wedge (\forall d')(0 \leq d' \leq d \Rightarrow I(q))$

Encoding Invariants built from $\{\leq, \wedge\}$

Adapting the translation of [Bérard et al., 2005]:



$\Rightarrow$



Theorems

So:

(Th 1) $TA+\{\leq,\wedge\} \lesssim_{\mathcal{W}} PrTPN$ with unbounded or right-closed intervals

Adapting the encoding of Cassez/Roux of TPN into TA:

(Th 2) $PrTPN \lesssim_{\mathcal{W}} TA + \{\leq,<,\wedge\}$.

Equivalence results:

(Th 3) $TA+\{\leq,\wedge\} \approx_{\mathcal{W}} PrTPN$ with right-closed or unbounded intervals

(Cor 1) TA with guards built from $\{\geq,>,\wedge\} \approx_{\mathcal{W}} TPN$ with unbounded intervals

(Cor 2) $TA+\{\leq,\wedge\}$ with guards built from $\{\geq,>,\wedge\} \approx_{\mathcal{W}} TPN$ with right-closed or unb
intervals.

Summary of comparisons

	TA guards	invariants		TPN intervals	$\succ$
Th 1	$\leq\ <\ \geq\ >\ \wedge$	$\leq\ \wedge$	$\lesssim_{\mathcal{W}}$	$(]a$—$[a),(b]$—$\infty[)$	Y
Th 2	$\leq\ <\ \geq\ >\ \wedge$	$\leq\ <\ \wedge$	$\gtrsim_{\mathcal{W}}$	$(]a$—$[a),(b]$—$b[$—$\infty[)$	Y
Th 3	$\leq\ <\ \geq\ >\ \wedge$	$\leq\ \wedge$	$\approx_{\mathcal{W}}$	$(]a\vert[a),(b]\vert\infty[)$	Y
CR	$\geq\ >\ \wedge$	$\leq\ <\ \wedge$	$\gtrsim_{\mathcal{W}}$	$(]a$—$[a),(b]$—$b[$—$\infty[)$	N
Cor 1	$\geq\ >\ \wedge$	$\leq\ \wedge$	$\approx_{\mathcal{W}}$	$(]a\vert[a),(b]\vert\infty[)$	N
Cor 2	$\geq\ >\ \wedge$	$\emptyset$	$\approx_{\mathcal{W}}$	$(]a\vert[a),\infty[$	N

Conclusion

New equivalence results % TA

Better understanding of differences

Allow sharing of analysis methods for TA and $PrTPN$

Note: Priorities make TPN compositional

State space abstractions for $PrTPN$

Improved encodings