Solving Distance CSP with Uncertainties

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Outline

- Problem statement (Distance CSP and uncertainties)
- Some applications
- Classic approach for solving Distance CSP
  - Some disadvantage
- A new methodology
  - Space Separation Algorithm and Feasibility Checker
  - Special Quantifier Elimination and Generalized Intervals
  - A new conditions for inner boxes
- Conclusions and future works

Problem Statement

- Solving CSP (Constraint Satisfaction Problem)
- What is a CSP?
  - Variables (set of decision objects)
  - Domains (set of values for variables)
  - Constraints (relations on the variables)
- Example

$P = (V, D, C)$

$V = \{x, y\}$ D = {{1,2}, {2,5}} C = {x = y}

- What means “to solve a CSP”? $x = 2, y = 2$

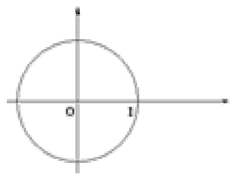
Problem Statement

- Distance CSP
  - Continuous domains
  - Specific constraints
  - Geometric representation

$$c : \sum_{k=1}^{n} (x_k - y_k)^2 = d_i^2$$

- Example

$P = (V, D, C)$
$V = \{x, y\}$
$D = \{[-10,10],[-10,10]\}$
$C = \{\ x^2 + y^2 = 1^2$



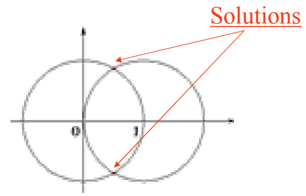
Problem Statement

- Distance CSP
  - Continuous domains
  - Specific constraints
  - Geometric representation

$$c:\sum_{k=1}^{n}(x_k-y_k)^2=d_i^2$$

- Example

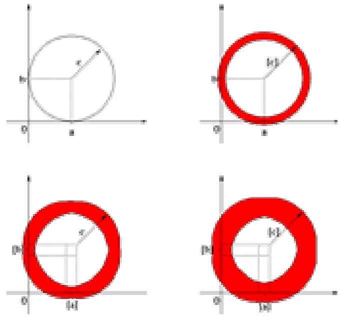
$$P=(V,D,C)$$
$$V=\{x,y\}$$
$$D=\{[-10,10],[-10,10]\}$$
$$C=\{\ x^2+y^2=1^2$$
$$(x-1)^2+y^2=1^2\}$$



Problem Statement

- Uncertainties?
  - Parameters
    - Interval values
  - Distance (easy) or Centre (difficult)
- Examples

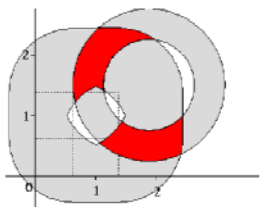
$$(x-a)^2+(y-b)^2=c^2$$
$$(x-a)^2+(y-b)^2=[c]^2$$
$$(x-[a])^2+(y-[b])^2=c^2$$
$$(x-[a])^2+(y-[b])^2=[c]^2$$



Problem Statement

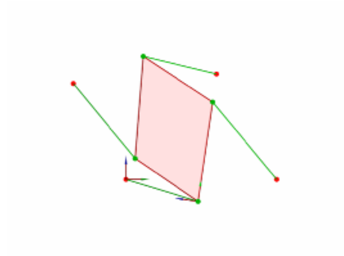
- Solving a Distance CSP
  - Find a description of solutions set as precise as possible

$$CSP\quad P=(V,D,C)$$
$$V=\{\vec{P}_1,\dots,\vec{P}_k\}$$
$$D=\{II_1^n,\dots,II_k^n\}$$
$$C=\{d(\vec{P}_i,\vec{P}_j)=d_{ij}\dots\}$$

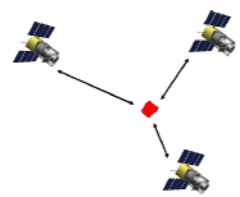


Applications

Robotics

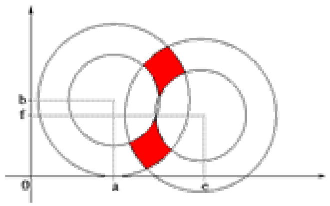


Positioning



A Classic Approach

- Applying a Branch and Prune Algorithm
  - Filtering phase
  - Bisection phase
  - Calculating a paving with a given precision
- Problems
  - A set of boxes without additional information
  - Inefficient computation bisecting inner boxes



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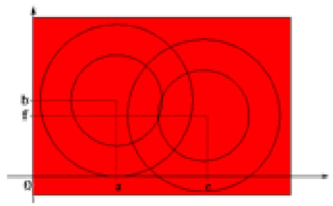

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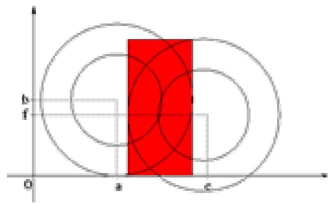

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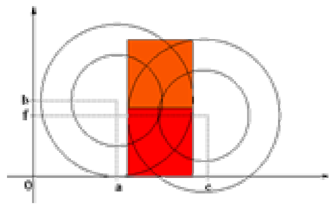

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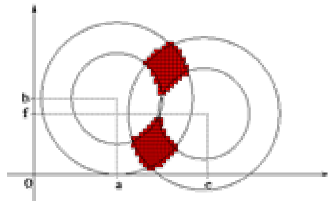

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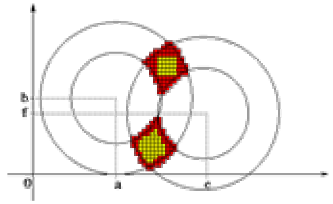

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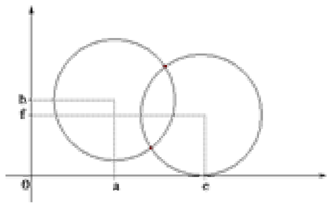
A New Methodology (CP 2005)

- Initial space separation (isolate)
  - Solving the problem without considering the uncertainties
  - Applying a Space Separation Algorithm
  - Solving a set of CSPs with uncertainties
- Feasibility Checker
  - Using a feasibility checker each time the filtering phase does not reduce the domains in order to detect inner boxes.



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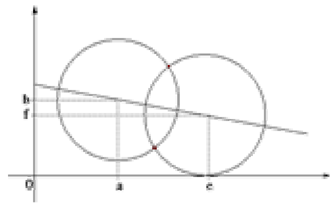

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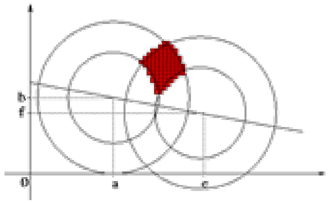

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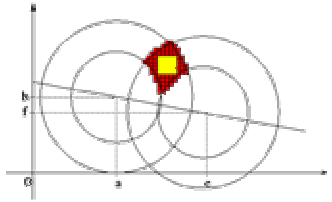

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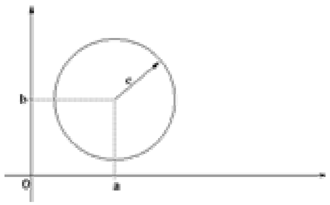
Feasibility Checker (FC)

$$(x-a)^2+(y-b)^2=c^2$$



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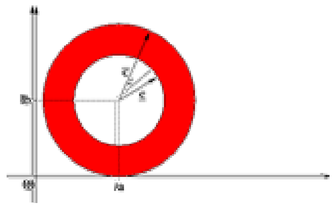
Feasibility Checker (FC)

$$(x-a)^2+(y-b)^2=[\underline{c},\overline{c}]^2$$



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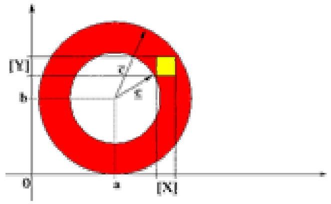
Feasibility Checker (FC)

$$(x-a)^2+(y-b)^2=[\underline{c},\overline{c}]^2$$

$$F(X,Y)=([X]-a)^2+([Y]-b)^2$$
$$F(X,Y)\subseteq[\underline{c},\overline{c}]^2$$

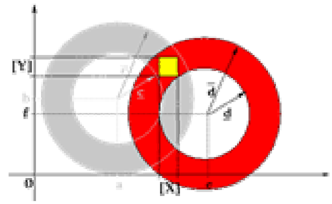
$$\forall x\in[X],\forall y\in[Y]\ \exists c\in[C]$$
$$((x-a)^2+(y-b)^2=c^2)$$



Feasibility Checker (FC)

$$(x-a)^2+(y-b)^2=[\underline{c},\overline{c}]^2$$

$$(x-e)^2+(y-f)^2=[\underline{d},\overline{d}]^2$$



Feasibility Checker (FC)

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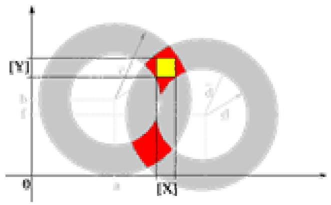


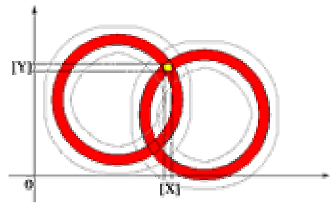
Feasibility Checker (FC)

$$(x-[a])^2+(y-[b])^2=[c]^2$$

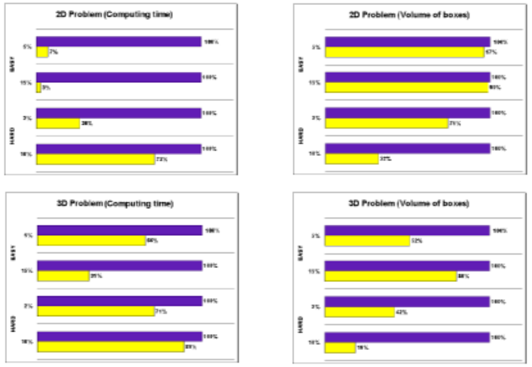
$$\downarrow$$

$$([X]-\dot{a})^2+([Y]-\dot{b})^2\subseteq[c]^2$$

(sufficient condition)



Preliminary Results



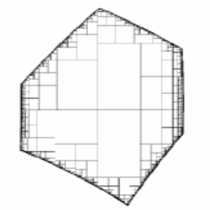
Inner Boxes Checkers (SAC 2006)

- A Specific Quantifier Elimination (QE)

- Generalized Interval Evaluation (GI)
  - Classic
    - $x+a:[1,3]+[4,8]=[5,11]$
    - $x+a:[1,3]+[4,5]=[5,8]$
  - Generalized
    - $x+a:[1,3]+[8,4]=[9,7]$
    - $x+a:[1,3]+[5,4]=[6,7]$

Inner Boxes Checkers

- Some Results



- Sufficient Conditions (necessary in some cases)

A New Inner Boxes Checker (?)

- Proposition

  $if(\vec{P}_x \cap \vec{P}_a = \phi) \Rightarrow GI$ is a necessary condition

  - Example

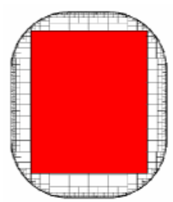
    $([x]-[a])^2+([y]-[b])^2 \subseteq [c]^2$

- New checker

  $Z_1=([x]-[a])^2$

  $Z_2=([y]-[b])^2$

  $Z_1+Z_2-[c] \subseteq [0,0]$



- Sufficient and Necessary condition

Conclusions

- A new approach for solving Distance CSP with uncertainties was proposed
  - Improving information in the solver results
  - Improving computing time by using a feasibility checker
- Different inner boxes checkers were analysed
  - Classic IA, Quantified Elimination, Generalized Interval
  - A new necessary a sufficient condition was found
- Future Works
  - Studying a Generalization of the new condition
  - Combining with a decomposition algorithm

Questions…

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