

The Rodin Platform for Incremental Modelling in Event-B

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Outline

- Complex Systems Modelling in Event-B
- Interplay of proof and modelling
- Worked example: Access to secure building
- Tool Animation
- Conclusion

Design of complex systems

- Need for **rigorous** modelling
- The main **purpose** of modelling is reasoning
 - to **improve** our understanding of the system
 - to **clarify** assumptions about the system
 - to **increase** quality of system model
- Need for **refinement**
 - **Too many details** to address final system directly

Reasoning about complex systems

- Requires **rigorous** reasoning
 - **Formal** notation
 - **Formal** reasoning
- This is offered by **formal proof**
 - **Why** prove?
 - To incrementally **improve** a model
 - To take advantage of **failing proofs**
 - **What** to prove?
 - **Proof obligations** associated with each model

List of core Event-B proof obligations

- **Feasibility** of events
- **Invariant** preservation by events
- **Refinement** of events
- **Introduction** of new events
- **Convergence** of events
- **Enabledness** of events

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Example of an Event-B machine

invariants

$inv1 : auth \in User \leftrightarrow Room$

$inv2 : in \in User \mapsto Room$

$inv3 : in \subseteq auth$

event *enter*

any

u, r

when

$grd1 : u \notin \text{dom}(in)$

$grd2 : u \mapsto r \in auth$

then

$act1 : in := in \cup \{u \mapsto r\}$

end

parameters

guards

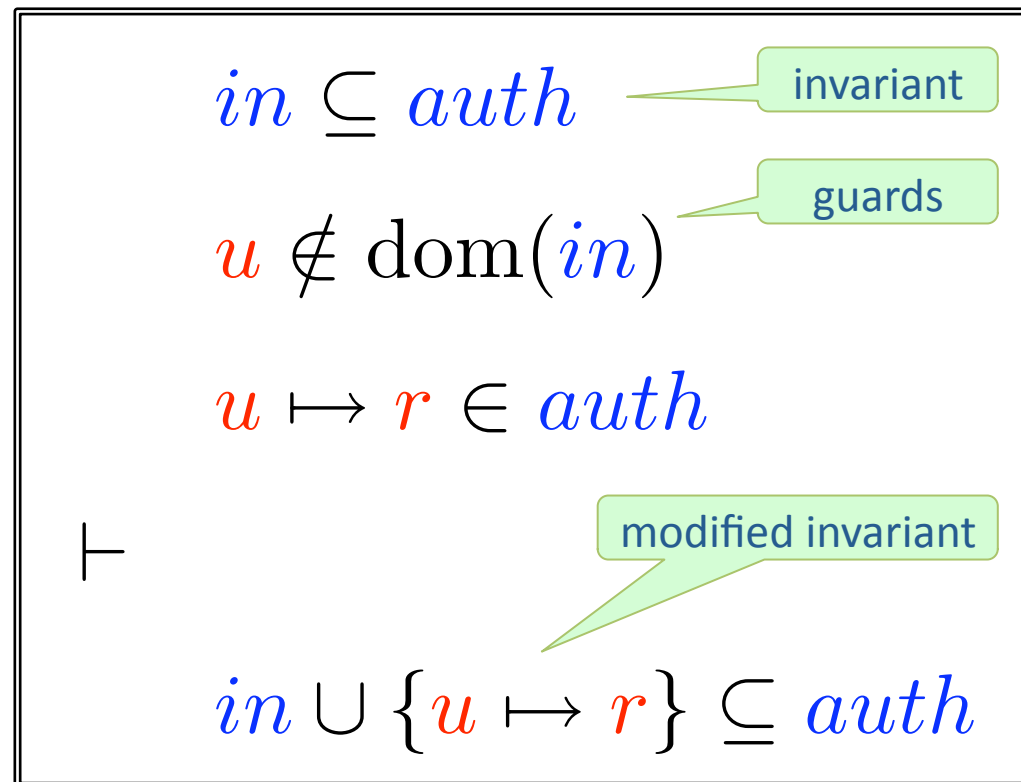
actions

Invariant properties:

- $inv1$: A user is **authorised** to be in certain rooms
- $inv2$: A user can be **at most in one** room
- $inv3$: A user can only be **in rooms** where he is **authorised** to be

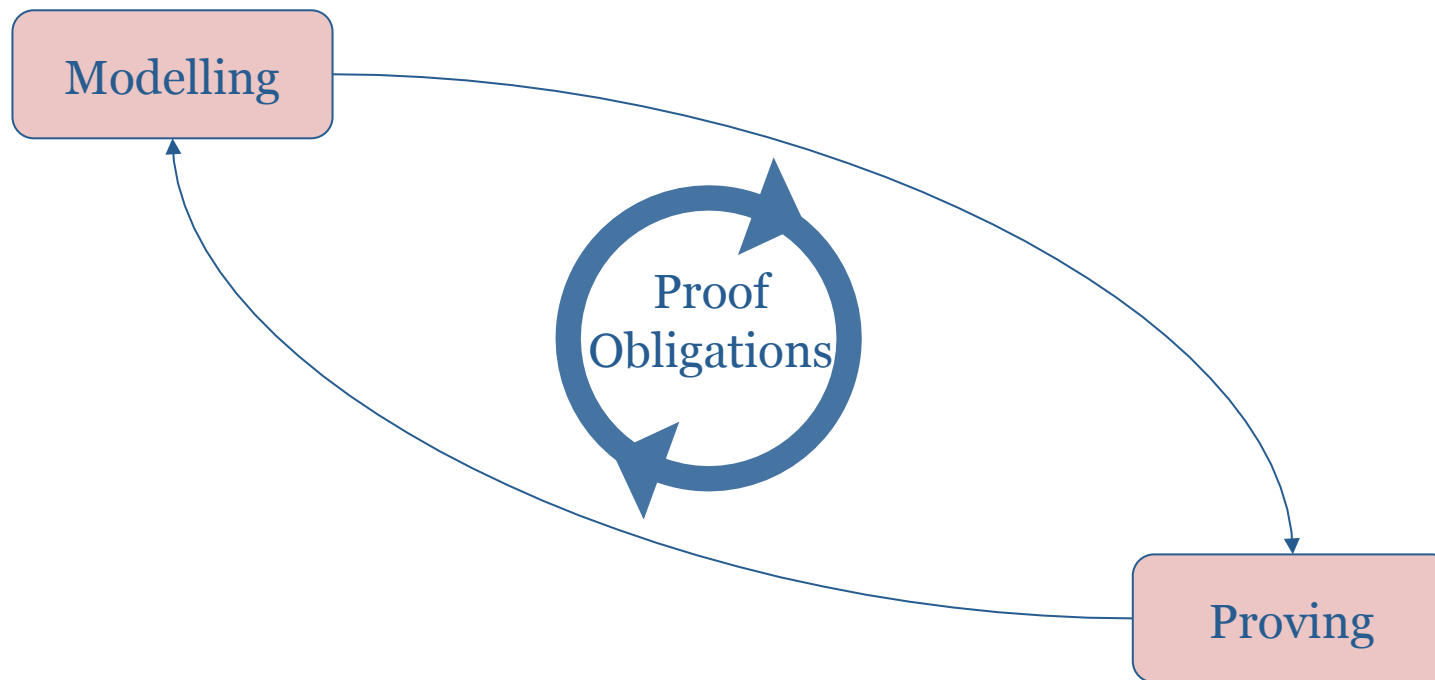
Preservation of $in \subseteq auth$ by event $enter$

- Proof obligation:



Use of proof obligations for modelling

- Modelling is an **incremental** activity



- Proof obligations are **automatically** generated

Creating a model incrementally

- What does this **mean**?
- We do **not** demonstrate the actual tool “Rodin”
 - But focus on **essential features**
 - **Removed** everything that could distract
- Aim:
 - To **illustrate** method and tool
 - **Not to** get distracted by features of the Rodin tool
- By way of an **example**
 - Access to a **secure building**

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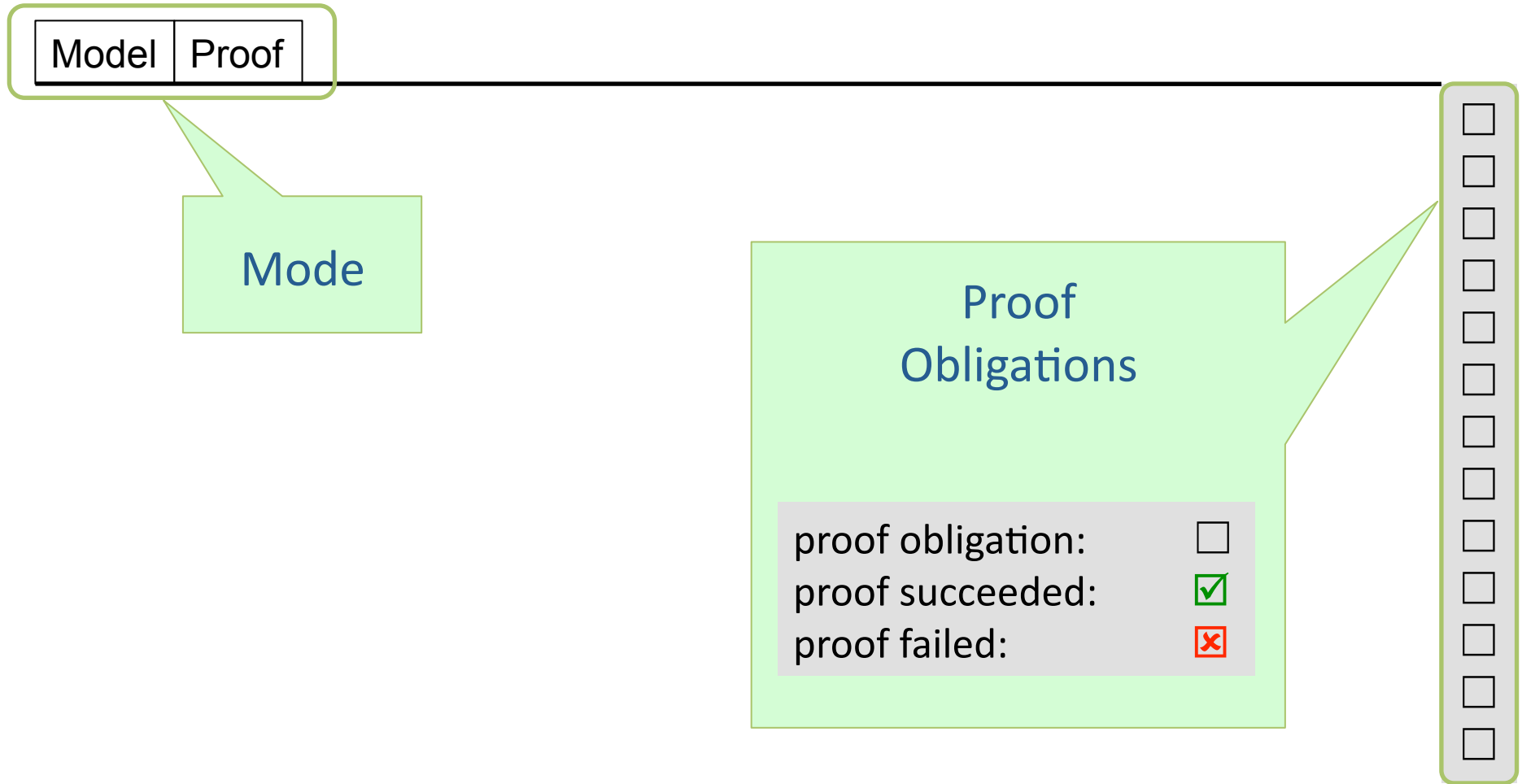
Description of the secure access model

- **Abstract** model
 - *Users, Rooms*
 - Entering/leaving rooms; adding/removing authorisations
- **Refined** model
 - *Tokens*
 - Data-refinement of abstract model
- The model itself is **not of importance**
 - But **the way we create** it is important

Focus on three modelling scenarios

- **Adding** events
 1. Reasoning about **guards** and invariants
- **Refining** an event
 2. Reasoning about **parameters**
 3. Reasoning about **guards** and invariants

Layout of the modelling canvas



Colouring conventions for formulas

Model	Proof
-------	-------

constants are blue-green

parameters are red

$utok(t) \mapsto rtok(t) \in auth$

variables are blue

Creation of the **abstract model**

Model	Proof
-------	-------

Summary:

- Variables: *in*, *auth*
- Events:
 - *enter* – enter building
 - *leave* – leave building
 - *addAuth* – add authorisation
 - *remAuth* – remove authorisation

Location and authorisation of users

Model	Proof
-------	-------

invariants

$inv1 : auth \in User \leftrightarrow Room$

$inv2 : in \in User \leftrightarrow Room$

$inv3 : in \subseteq auth$

Location and authorisation of users

Model	Proof
-------	-------

invariants

$inv1 : auth \in User \leftrightarrow Room$

$inv2 : in \in User \leftrightarrow Room$

$inv3 : in \subseteq auth$

initialisation

$act1 : in := \emptyset$

$act2 : auth := \emptyset$

Location and authorisation of users

Model	Proof
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$inv1 : auth \in User \leftrightarrow Room$

$inv2 : in \in User \leftrightarrow Room$

$inv3 : in \subseteq auth$

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Location and authorisation of users

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$inv3 : in \subseteq auth$

initialisation

$act1 : in := \emptyset$

$act2 : auth := \emptyset$



Entering a room

Model	Proof
-------	-------

```
event enter
  any
     $u, r$ 
  when
     $grd1 : u \notin \text{dom}(in)$ 
     $grd2 : u \mapsto r \in auth$ 
  then
     $act1 : in := in \cup \{u \mapsto r\}$ 
  end
```



Entering a room

Model	Proof
-------	-------

```
event enter
  any
     $u, r$ 
  when
     $grd1 : u \notin \text{dom}(in)$ 
     $grd2 : u \mapsto r \in auth$ 
  then
     $act1 : in := in \cup \{u \mapsto r\}$ 
  end
```

Proof obligation:

Preservation of
invariant $inv3$



Entering a room

Model	Proof
-------	-------

Preservation of invariant $inv3$

$$in \subseteq auth$$

$$u \notin \text{dom}(in)$$

$$u \mapsto r \in auth$$

⊢

$$in \cup \{u \mapsto r\} \subseteq auth$$

-
-
-
-
-

Proof obligation:
Preservation of
invariant $inv3$

Entering a room

Model	Proof	Preservation of invariant $inv3$
-------	-------	----------------------------------

$$in \subseteq auth$$

$$u \notin \text{dom}(in)$$

$$u \mapsto r \in auth$$

⊢

$$in \cup \{u \mapsto r\} \subseteq auth$$



Proof obligation:

Preservation of
invariant $inv3$

Entering a room

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  end
```



Entering a room

Model	Proof
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```
event enter
  any
     $u, r$ 
  when
     $grd1 : u \notin \text{dom}(in)$ 
     $grd2 : u \mapsto r \in auth$ 
  then
     $act1 : in := in \cup \{u \mapsto r\}$ 
  end
```



Leaving a room

Model	Proof
-------	-------

```
event leave
  any
    u
  when
    grd1 : u ∈ dom(in)
  then
    act1 : in := {u} ◁ in
  end
```



Leaving a room

Model	Proof
-------	-------

```
event leave
  any
    u
  when
    grd1 : u ∈ dom(in)
  then
    act1 : in := {u} ◁ in
  end
```



Leaving a room

Model	Proof
-------	-------

```
event leave
  any
    u
  when
    grd1 : u ∈ dom(in)
  then
    act1 : in := {u} ◁ in
  end
```



Adding an authorisation

Model	Proof
-------	-------

```
event addAuth
  any
    u, r
  when
    grd1 : u ∈ User
    grd2 : r ∈ Room
  then
    act1 : auth := auth ∪ {u ↦ r}
end
```



Adding an authorisation

Model	Proof
-------	-------

```
event addAuth
  any
    u, r
  when
    grd1 : u ∈ User
    grd2 : r ∈ Room
  then
    act1 : auth := auth ∪ {u ↦ r}
end
```



Adding an authorisation

Model	Proof
-------	-------

```
event addAuth
  any
    u, r
  when
    grd1 : u ∈ User
    grd2 : r ∈ Room
  then
    act1 : auth := auth ∪ {u ↦ r}
end
```



Removing an authorisation

Model	Proof
-------	-------

event *remAuth*

any

u, r

when

grd1 : *u* ∈ *User*

grd2 : *r* ∈ *Room*

then

act1 : *auth* := *auth* \ {*u* ↦ *r*}

end



Removing an authorisation

Model	Proof
-------	-------

event *remAuth*

any

u, r

when

grd1 : *u* ∈ *User*

grd2 : *r* ∈ *Room*

then

act1 : *auth* := *auth* \ {*u* ↦ *r*}

end



Removing an authorisation

Model	Proof
-------	-------

```
event remAuth
  any
    u, r
  when
    grd1 : u ∈ User
    grd2 : r ∈ Room
  then
    act1 : auth := auth \ {u ↦ r}
  end
```

Proof obligation:

Preservation of
invariant *inv3*



Removing an authorisation

Model	Proof	Preservation of invariant <i>inv3</i>
-------	-------	---------------------------------------

$$in \subseteq auth$$

$$u \in User$$

$$r \in Room$$

⊢

$$in \subseteq auth \setminus \{u \mapsto r\}$$

Possible remedies:

- Modify action:
 →Remove user *u* from building
- Modify guard:
 →Require user *u* not in building
- Modify guard:
 →Require user *u* not in room *r*

$$u \mapsto r \notin in$$



Removing an authorisation

Model	Proof
-------	-------

event *remAuth*

any

u, r

when

grd1 : *u* ∈ *User*

grd2 : *r* ∈ *Room*

then

act1 : *auth* := *auth* \ {*u* ↦ *r*}

end



Removing an authorisation

Model	Proof
-------	-------

event *remAuth*

any

u, r

when

grd1 : *u* ∈ *User*

grd2 : $u \mapsto r \notin in$

then

act1 : $auth := auth \setminus \{u \mapsto r\}$

end



Removing an authorisation

Model	Proof	Preservation of invariant <i>inv3</i>
-------	-------	---------------------------------------

$$in \subseteq auth$$

$$u \in User$$

$$u \mapsto r \notin in$$

⊢

$$in \subseteq auth \setminus \{u \mapsto r\}$$



Removing an authorisation

Model

Proof

Preservation of invariant $inv3$

$$in \subseteq auth$$

$$u \in User$$

$$u \mapsto r \notin in$$

⊢

$$in \subseteq auth \setminus \{u \mapsto r\}$$



Creation of the **refined model**

Model	Proof
-------	-------

Summary:

- Variables: *tok*, *auth*
- Events:
 - *enter* – enter building
 - other events not shown

Replacing Users and Rooms by Tokens

Model	Proof
-------	-------

axioms

$axm1 : utok \in Token \rightarrow User$

$axm2 : rtok \in Token \rightarrow Room$

Abstract model of a **record type** with two fields
utok and *rtok*

Replacing Users and Rooms by Tokens

Model	Proof
-------	-------

invariants

$inv4 : \forall u, r \cdot$

$u \mapsto r \in in$

\Leftrightarrow

$\exists t \cdot t \in tok \wedge u = utok(t) \wedge r = rtok(t)$

Using some **set-theoretic notation** of Event-B

$inv4$ can be stated more concisely:

$in = utok^{-1} ; (tok \triangleleft rtok)$

Replacing Users and Rooms by Tokens

Model	Proof
-------	-------

invariants

$$inv4 : in = utok^{-1} ; (tok \triangleleft rtok)$$

Replacing Users and Rooms by Tokens

Model	Proof
-------	-------

invariants

$$inv4 : in = utok^{-1} ; (tok \triangleleft rtok)$$

initialisation

$$act1 : tok := \emptyset$$

$$act2 : auth := \emptyset$$

Replacing Users and Rooms by Tokens

Model	Proof
-------	-------

invariants

$$inv4 : in = utok^{-1} ; (tok \triangleleft rtok)$$

initialisation

$$act1 : tok := \emptyset$$

$$act2 : auth := \emptyset$$



Replacing Users and Rooms by Tokens

Model	Proof
-------	-------

invariants

$$inv4 : in = utok^{-1} ; (tok \triangleleft rtok)$$

initialisation

$$act1 : tok := \emptyset$$

$$act2 : auth := \emptyset$$



Entering a room (refined model)

Model

Proof

```

event enter
  any
    t
  when
    grd1 : t  $\notin$  tok
    grd2 : utok(t)  $\mapsto$  rtok(t)  $\in$  auth
  then
    act1 : tok := tok  $\cup$  {t}
  end
  
```

```

abstract event enter
  any
    u, r
  when
    grd1 : u  $\notin$  dom(in)
    grd2 : u  $\mapsto$  r  $\in$  auth
  then
    act1 : in := in  $\cup$  {u  $\mapsto$  r}
  end
  
```



Entering a room (refined model)

Model

Proof

```

event enter
  any
    t
  when
    grd1 : t  $\notin$  tok
    grd2 : utok(t)  $\mapsto$  rtok(t)  $\in$  auth
  then
    act1 : tok := tok  $\cup$  {t}
  end
  
```

```

abstract event enter
  any
    u, r
  when
    grd1 : u  $\notin$  dom(in)
    grd2 : u  $\mapsto$  r  $\in$  auth
  then
    act1 : in := in  $\cup$  {u  $\mapsto$  r}
  end
  
```



Entering a room (refined model)

Model

Proof

```

event enter
  any
    t
  when
    grd1 : t ∉ tok
    grd2 : utok(t) ⇨ rtok(t) ∈ auth
  then
    act1 : tok := tok ∪ {t}
  end
  
```

abstract event *enter*

any

u, r

when

grd1 : *u* ∉ dom(*in*)

Proof obligation:

Strengthening of
 guard *grd2*



r}

Entering a room (refined model)

Model	Proof	Strengthening of guard <i>grd2</i>
-------	-------	------------------------------------

$$t \notin tok$$

$$utok(t) \mapsto rtok(t) \in auth$$

⊢

$$u \mapsto r \in auth$$

Must relate t to u and r

Choose witnesses for u and r :

$$u = utok(t)$$

$$r = rtok(t)$$



Entering a room (refined model)

Model

Proof

```

event enter
  any
    t
  when
    grd1 : t ∉ tok
    grd2 : utok(t) ↦ rtok(t) ∈ auth
  then
    act1 : tok := tok ∪ {t}
  end
  
```

```

abstract event enter
  any
    u, r
  when
    grd1 : u ∉ dom(in)
    grd2 : u ↦ r ∈ auth
  then
    act1 : in := in ∪ {u ↦ r}
  end
  
```



Entering a room (refined model)

Model	Proof
-------	-------

event *enter*

any

t

when

grd1 : $t \notin tok$

grd2 : $utok(t) \mapsto rtok(t) \in auth$

with

$u \mid u = utok(t)$

$r \mid r = rtok(t)$

then

act1 : $tok := tok \cup \{t\}$

end

abstract event *enter*

any

u, r

when

grd1 : $u \notin \text{dom}(in)$

grd2 : $u \mapsto r \in auth$

then

act1 : $in := in \cup \{u \mapsto r\}$

end



Entering a room (refined model)

Model	Proof
-------	-------

Strengthening of guard *grd2*

$t \notin tok$

$utok(t) \mapsto rtok(t) \in auth$

⊢

$utok(t) \mapsto rtok(t) \in auth$



Entering a room (refined model)

Model	Proof	Strengthening of guard <i>grd2</i>
-------	-------	------------------------------------

$t \notin tok$

$utok(t) \mapsto rtok(t) \in auth$

⊢

$utok(t) \mapsto rtok(t) \in auth$



Proof obligation:
 Strengthening of
 guard *grd1*

Entering a room (refined model)

Model	Proof
-------	-------

```

event enter
  any
    t
  when
    grd1 : t  $\notin$  tok
    grd2 : utok(t)  $\mapsto$  rtok(t)  $\in$  auth
  with
    u | u = utok(t)
    r | r = rtok(t)
  then
    act1 : tok := tok  $\cup$  {t}
  end
  
```

abstract event *enter*

```

any
  u, r
when
  grd1 : u  $\notin$  dom(in)
  grd2 : u  $\mapsto$  r  $\in$  auth
then
  act : tok := tok  $\cup$  {t}
end
  
```



Proof obligation:
 Strengthening of
 guard *grd1*

Entering a room (refined model)

Model	Proof	Strengthening of guard <i>grd1</i>
-------	-------	------------------------------------

$$in = utok^{-1} ; (tok \triangleleft rtok)$$

$$t \notin tok$$

$$utok(t) \mapsto rtok(t) \in auth$$

⊢

$$utok(t) \notin \text{dom}(in)$$



Entering a room (refined model)

Model	Proof	Strengthening of guard <i>grd1</i>
-------	-------	------------------------------------

$$in = utok^{-1} ; (tok \triangleleft rtok)$$

$$t \notin tok$$

$$utok(t) \mapsto rtok(t) \in auth$$

⊢

$$utok(t) \notin \text{dom}(utok^{-1} ; (tok \triangleleft rtok))$$



Entering a room (refined model)

Model	Proof	Strengthening of guard <i>grd1</i>
-------	-------	------------------------------------

$$in = utok^{-1} ; (tok \triangleleft rtok)$$

$$t \notin tok$$

$$utok(t) \mapsto rtok(t) \in auth$$

⊢

$$utok(t) \notin \text{dom}((utok^{-1} \triangleright tok) ; rtok)$$

rtok is total

- ✓
- ✓
- ✗
- ✓

Entering a room (refined model)

Model	Proof
-------	-------

Strengthening of guard *grd1*

$$in = utok^{-1} ; (tok \triangleleft rtok)$$

$$t \notin tok$$

$$utok(t) \mapsto rtok(t) \in auth$$

⊢

$$utok(t) \notin \text{dom}(utok^{-1} \triangleright tok)$$



Entering a room (refined model)

Model	Proof	Strengthening of guard <i>grd1</i>
-------	-------	------------------------------------

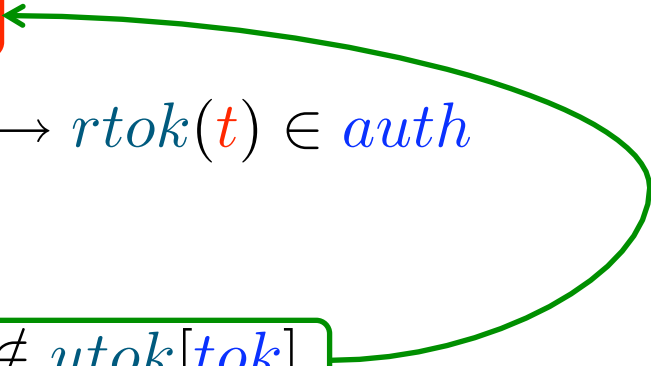
$$in = utok^{-1} ; (tok \triangleleft rtok)$$

$$t \notin tok$$

$$utok(t) \mapsto rtok(t) \in auth$$

⊢

$$utok(t) \notin utok[tok]$$



Guard
 $t \notin tok$
 is too weak.

$utok(t) \notin utok[tok]$
 is a better choice for
 guard *grd1*

- ✓
- ✓
- ✗
- ✓

Entering a room (refined model)

Model

Proof

```

event enter
  any
    t
  when
    grd1 : t ∉ tok
    grd2 : utok(t) ↦ rtok(t) ∈ auth
  with
    u | u = utok(t)
    r | r = rtok(t)
  then
    act1 : tok := tok ∪ {t}
  end
  
```

```

abstract event enter
  any
    u, r
  when
    grd1 : u ∉ dom(in)
    grd2 : u ↦ r ∈ auth
  then
    act1 : in := in ∪ {u ↦ r}
  end
  
```



Entering a room (refined model)

Model

Proof

event *enter*

any

t

when

grd1 : $utok(t) \notin utok[tok]$

grd2 : $utok(t) \mapsto rtok(t) \in auth$

with

u | $u = utok(t)$

r | $r = rtok(t)$

then

act1 : $tok := tok \cup \{t\}$

end

abstract event *enter*

any

u, r

when

grd1 : $u \notin \text{dom}(in)$

grd2 : $u \mapsto r \in auth$

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end



Entering a room (refined model)

Model

Proof

event *enter*

any

t

when

grd1 : $utok(t) \notin utok[tok]$

grd2 : $utok(t) \mapsto rtok(t) \in auth$

with

u | $u = utok(t)$

r | $r = rtok(t)$

then

act1 : $tok := tok \cup \{t\}$

end

abstract event *enter*

any

u, r

when

grd1 : $u \notin \text{dom}(in)$

grd2 : $u \mapsto r \in auth$

then

act1 : $in := in \cup \{u \mapsto r\}$

end



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- Tool Animation
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Conclusion

- **Close relationship** between modelling and proving
- Proof can give hints for **improvements** of model
- **Incremental modelling** based on heuristics
 - As opposed to **refinement** based on proof
- There was no time to show
 - **finding invariants** by inspection of proofs
 - modifying an abstract model to **improve a refinement**

Community

- Web:

- www.event-b.org
- wiki.event-b.org

- Credits

Jean-Raymond Abrial, Jens Bendisposto, Michael Butler, Dominique Cansell, Mathieu Clabaut, Kriangsak Damchoom, Fabian Fritz, Thai Son Hoang, Sonja Holl, Cliff Jones, Thierry Lecomte, Michael Leuschel, Farhad Mehta, Christophe Métayer, Renato Silva, Colin Snook, François Terrier, Laurent Voisin, . . .