Communication and Concurrency: CCS

R. Milner, “A Calculus of Communicating Systems”, 1980

cours SSDE – Master 1

Why calculi?

- Prove properties on programs and languages
- Principle: tiny syntax, small semantics, to be handled on paper or mechanically
- Prove properties on the principles of a language or a programming paradigm

- Examples: lambda calculus, sigma calculus, …

Static semantics : examples

- Checks non-syntactic constraints
- compiler front-end :
  - declaration and utilisation of variables,
  - typing, scoping, …
- or back-ends : optimisers
- defines legal programs :
  - static typing => no execution error ?
  - Java byte-code verifier

What can we do/know about a program without executing it?

Dynamic semantics

- Gives a meaning to the program (a semantic value)
- Describes the behaviour of a (legal) program
- Defines a language interpreter

  e -> e’

  let i=3 in 2*i   -> 2*3 -> 6

Objective = prove properties on Program execution (optimizations, determinacy, subject reduction, …)

The different semantic families

- Denotational semantics
  - mathematical model, high level, abstract
- Axiomatic semantics
  - provides the language with a theory for proving properties / assertions of programs
- Operational semantics
  - expresses the evaluation of a program
  - used to build evaluators, simulators.

What about concurrency and communication?

- Different timing (synchronous/asynchronous …)
- Different programming models (what is the unit of concurrency? What is sufficient to characterize an execution?...?)
- Interaction between communication/ concurrency/shared memory!

Through CCS, this course is a simple study of synchronous communications

SEMANTICS

Operational Semantics

- Describes the computation
- Generally uses states and configuration of an abstract machine:
  - Stack, memory state, registers, heap...
- Abstract machine transformation steps
- Several different operational semantics

Natural Semantics : big steps (Kahn 1986)

- Defines the results of evaluation.
- Direct relation from programs to results

env |- prog => result

  - env: binds variables to values
  - result: value given by the execution of prog

Reduction Semantics : small steps

describes **each elementary step** of the evaluation

- **rewriting relation** : reduction of program terms
- **stepwise reduction**: <prog, s> -> <prog’, s ’>
  - infinitely, or until reaching a **normal form**.

Deduction Rules

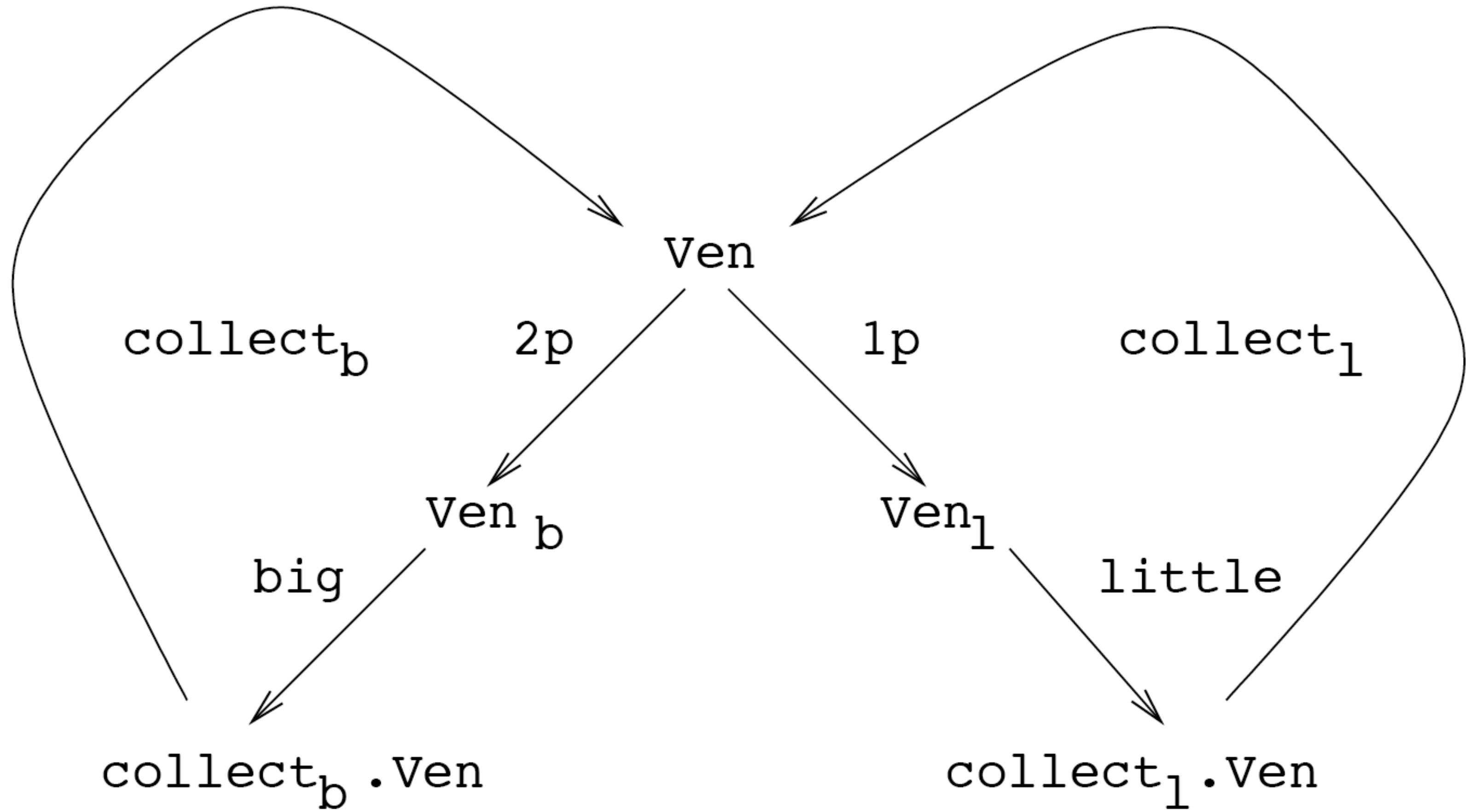
$$\frac{P \rightarrow Q \quad P}{Q}$$

$$\frac{P}{P \vee Q} \qquad \frac{Q}{P \vee Q}$$

Labelled Transition Systems (LTS)

- Basic model for representing reactive, concurrent, parallel, communicating systems.
- Definition:
  - < S, s0, L, T>
  - S = set of states
  - $S0 \in S$ = an initial state
  - L = set of labels (events, communication actions, etc)
  - $T \subseteq S \times L \times S$ = set of transitions
  - Notation: $s1 \xrightarrow{a} s2 = (s1, a, s2) \in T$

An example



Exercise:
What are the possible traces (output sequences) of Ven?

CCS – SYNTAX AND SEMANTICS

CCS syntax

- Channel names: a, b, c , . . .
- Co-names: $\bar{a}, \bar{b}, \bar{c}, \ldots$
- Silent action: τ
- Actions: $\mu ::= a \mid \bar{a} \mid \tau$
- Processes:

$$
\begin{array}{lcll}
P, Q & ::= & 0 & \text{inaction} \\
 & \mid & \mu.P & \text{prefix} \\
 & \mid & P \mid Q & \text{parallel} \\
 & \mid & P + Q & \text{(external) choice} \\
 & \mid & (\nu a)P & \text{restriction} \\
 & \mid & \mathrm{rec}_K P & \text{process } P \text{ with definition } K = P \\
 & \mid & K & \text{(defined) process name}
\end{array}
$$

A tiny example

$$rec_{C1}(Tick.C1)$$

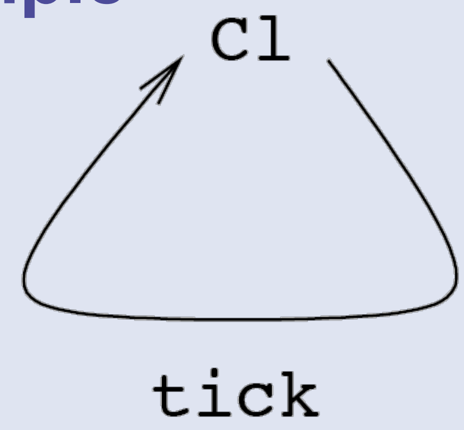


Figure: The transition graph for C1

Labelled graph

- vertices: process expressions
- labelled edges: transitions
- Each derivable transition of a vertex is depicted
- Abstract from the derivations of transitions

Exercise:
What are the possible traces (output sequences) of C1?

CCS : behavioural semantics (1)
Operators and rules

- Action prefix:

$$\frac{}{\mu.P \xrightarrow{\mu} P}$$

- Communication:

$$\frac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

- Parallelism

$$\frac{P \xrightarrow{\mu} P'}{P|Q \xrightarrow{\mu} P'|Q} \qquad \frac{Q \xrightarrow{\mu} Q'}{P|Q \xrightarrow{\mu} P|Q'}$$

CCS : behavioural semantics (2)
Operators and rules

- Non-deterministic choice

$$\frac{Q \xrightarrow{\mu} Q'}{P+Q \xrightarrow{\mu} Q'} \qquad \frac{P \xrightarrow{\mu} P'}{P+Q \xrightarrow{\mu} P'}$$

- Scope restriction

$$\frac{P \xrightarrow{\mu} P' \qquad \mu \neq a, \overline{a}}{(\nu a)P \xrightarrow{\mu} (\nu a)P'}$$

- Recursive definition

$$\frac{P[\mathrm{rec}_K P / K] \xrightarrow{\mu} P'}{\mathrm{rec}_K P \xrightarrow{\mu} P'}$$

Derivations
(construction of each transition step)

$$
\dfrac{\dfrac{\dfrac{}{a.P \xrightarrow{\ a\ } P}\ \text{Prefix}}{a.P \mid Q \xrightarrow{\ a\ } P \mid Q}\ \text{Par-L} \qquad \dfrac{}{\overline{a}.R \xrightarrow{\ \overline{a}\ } R}\ \text{Prefix}}{(a.P \mid Q) \mid \overline{a}.R \xrightarrow{\ \tau\ } (P \mid Q) \mid R}\ \text{Par-2}
$$

Par-2(Par_L(Prefix), Prefix)

One amongst 3 possible derivations of $(a.P \mid Q) \mid a.R$

Exercise: what are the other possible derivations?

More general recursion

- To have a recursion over several variables we can use:

let rec K1=P1

  and K2=P2

  and ….

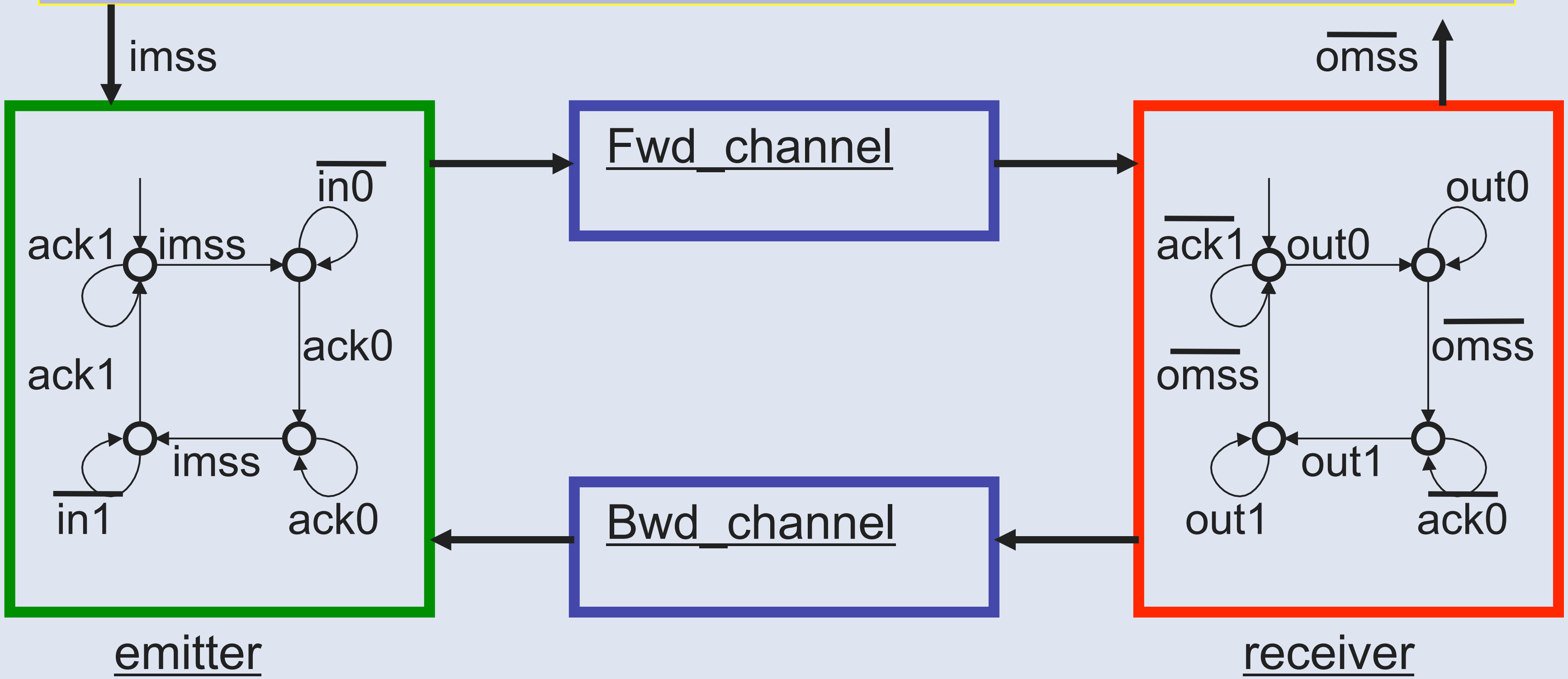
in Pn

for example

let rec A=a.B and B=b.A in A+B

$\xrightarrow{a}$ let rec A=a.B and B=b.A in B

Exercise: Alternated Bit Protocol



Write emitter in CCS (use let rec)

Hypotheses: channels can loose messages

Requirement:

the protocol ensures no loss of messages

Example: Alternated Bit Protocol (2)

- **emitter =**

  let rec em0 = ack1 . em0 + imss . em1

  and em1 = $\overline{\text{in0}}$. em1 + ack0 . em2

  and em2 = ack0 . em2 + imss . em3

  and em3 = $\overline{\text{in1}}$. em3 + ack1 . em0

  in em0

- **ABP =**

  emitter | Fwd_channel | Bwd_channel | receiver

Note this shows how to build a CCS term from a LTS, we have seen the other direction

Example: Alternated Bit Protocol (3)

Channels that loose and duplicate messages (in0 and in1) but preserve their order ?

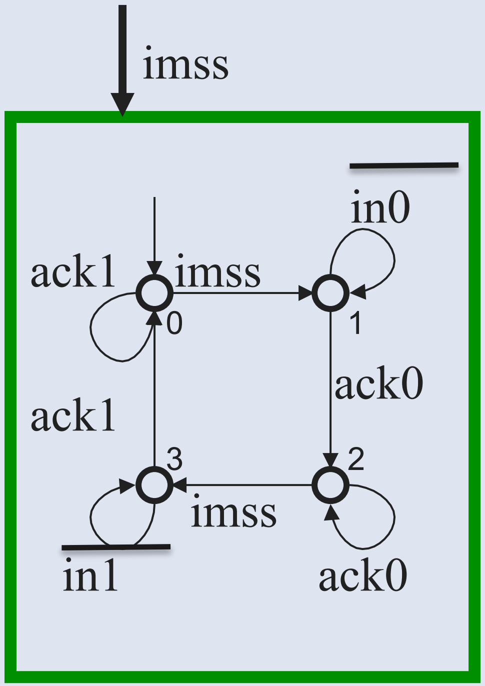
- Exercise :
  0) Draw the LTS describing the perfect channel (no loss – no duplication)
  1) Draw an LTS describing the loosy channel behaviour
  2) Write the same description in CCS

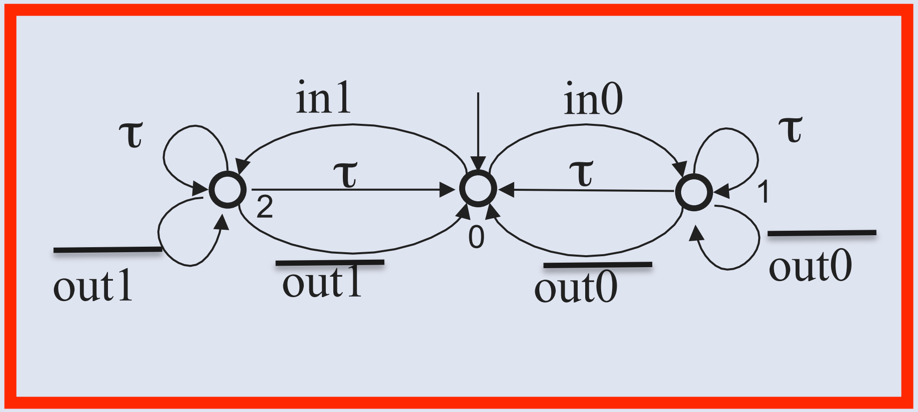
Exercise (4): synchronized product

Compute the synchronized product of the LTS representing the ABP emitter with the (forward) Channel:

new {in0, in1} in

(Emitter | Channel)

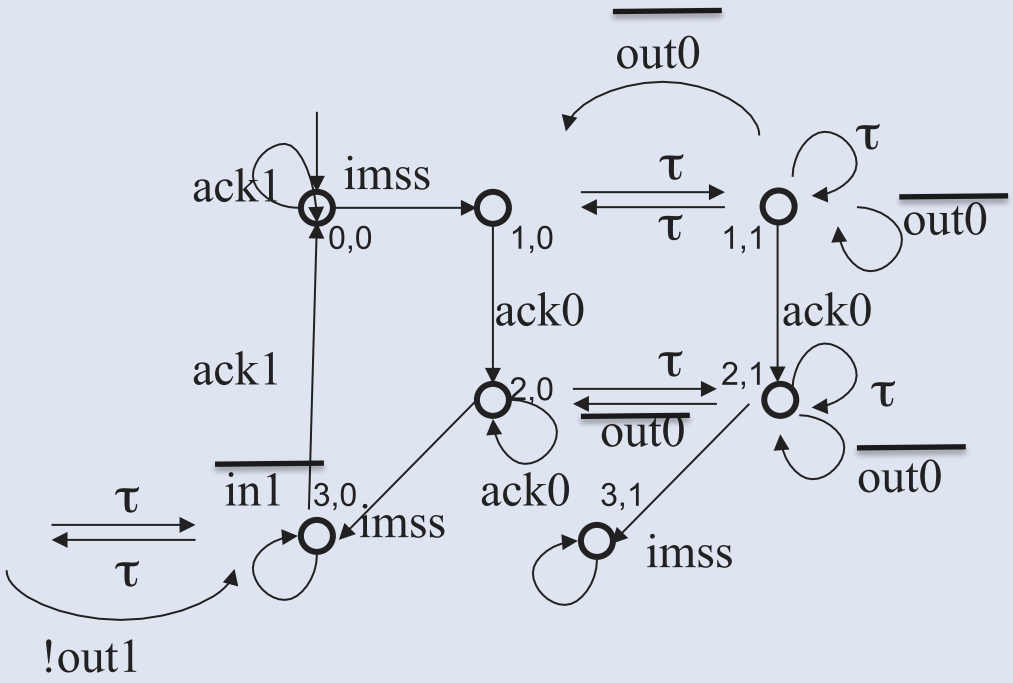




Exercise: synchronized product

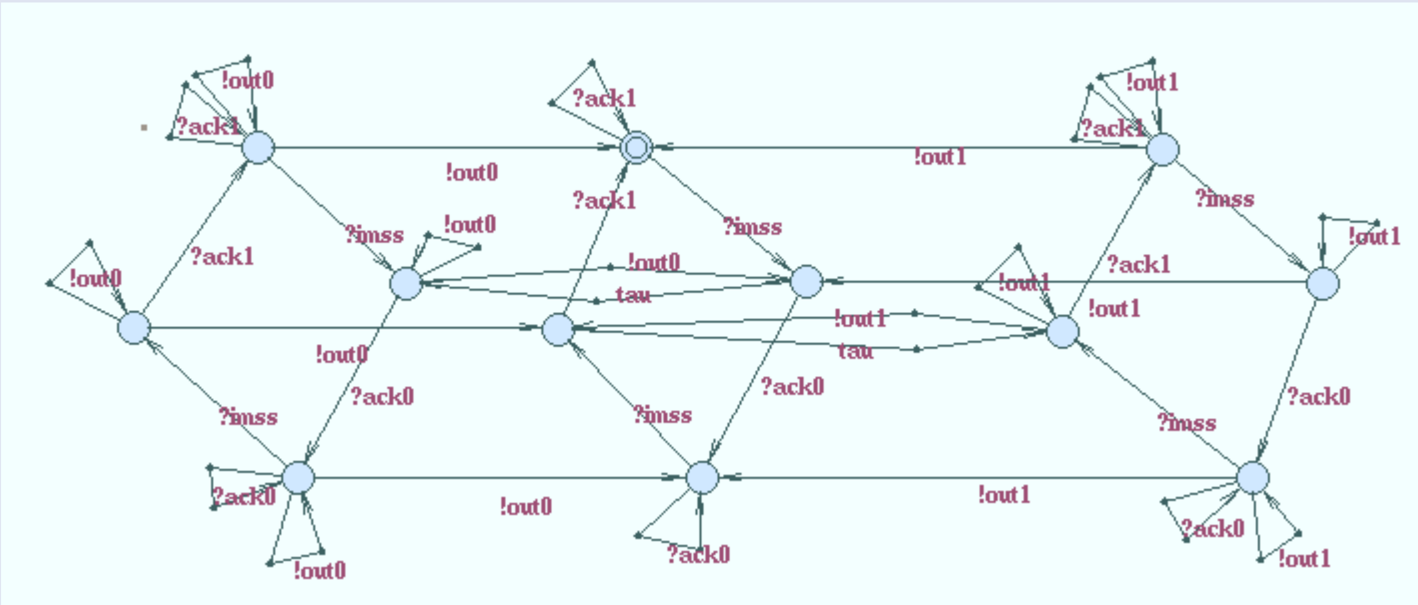
Correction ? partially…

local {in0, in1} in
(Emitter || Channel)



Exercise: synchronized product

Correction ? Tool generated LTS…



EQUIVALENCES

Why an equivalence relation?

- Identify similar processes
- Idea: 2 equivalent processes should behave the same - or more or less the same
- What does “behave the same” mean?
- Strict structural equality is not sufficient (optimisation / alternative implementation / …)
- What is an equivalence relation?
  - symmetrical:
  - transitive:
  - reflexive:

Behavioural Equivalences

- Intuition:
  - Same possible sequences of observable actions
  - Finite / infinite sequences
  - Various refinements of the concept of observation
- Definition: Trace Equivalence

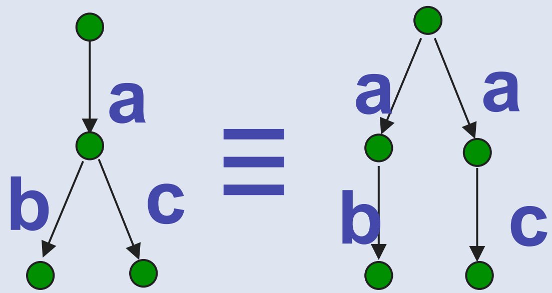
  For a LTS (S, s0, L, T) its Trace language ***T*** is the set of finite sequences {(t = t_1, …, t_n such that $\exists s_0,\ldots,s_n \in S^{n+1,}$ and $(s_{n-1},t_n,s_n) \in T$}

  Two LTSs are Trace equivalent iff their Trace languages are equal.

  Corresponding Ordering: Trace inclusion

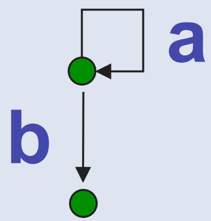
Trace Languages, Examples

- Those 2 systems are trace equivalent:



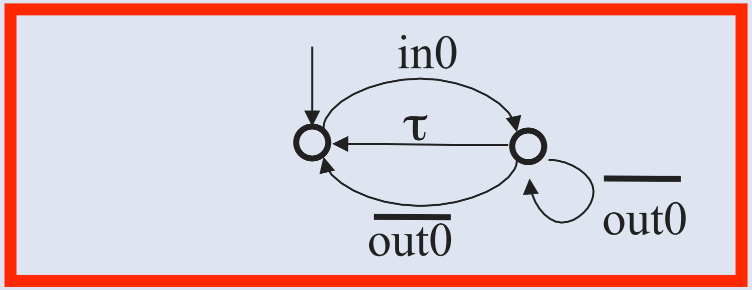
$\boldsymbol{T} = \{(), (a), (a,b), (a,c)\}$

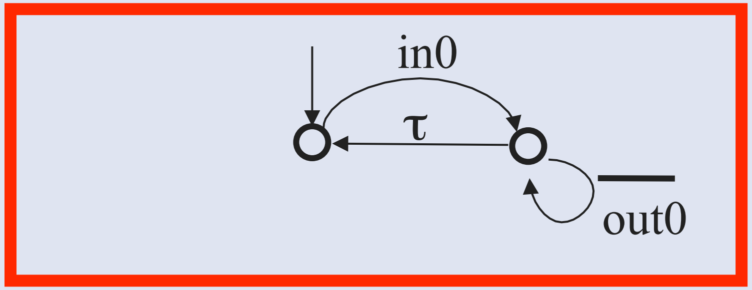
- A trace language can be an infinite set:



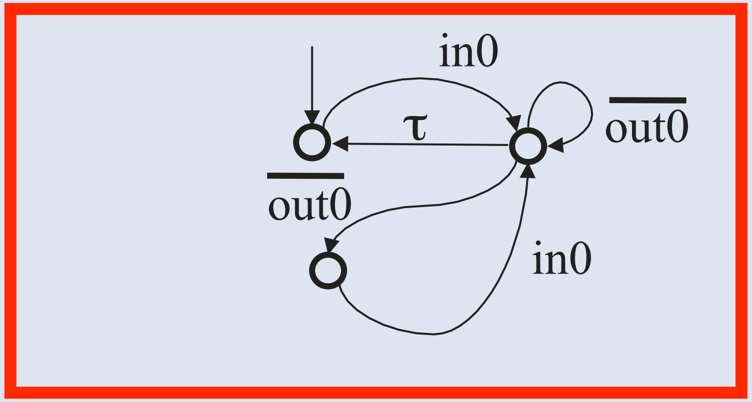
$\boldsymbol{T} = \{(), (a), (a,a), (a,\ldots,a),\ldots$
$\quad (a,b), (a,a,b), (a,a,\ldots,a,b),$
$\ldots\}$

Exercice: Trace equivalence





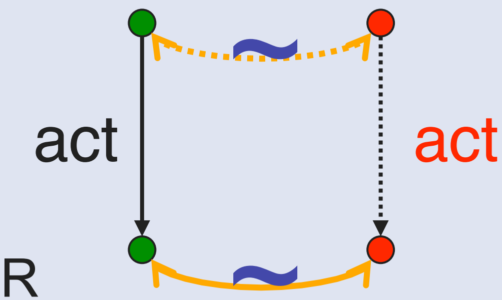
Are those 3 LTSs trace-equivalent?



Bisimulation

- **Behavioural Equivalence**
  - non distinguishable states by observation:

    two states are equivalent if for all possible transitions labelled by the same action, there exist equivalent resulting states.



- **Bisimulations**

  R ⊆ SxS is a simulation iff
  - $\forall (p,q) \in R$,

    $p \xrightarrow{l} p' \in T \Rightarrow \exists\, q'.\ q \xrightarrow{l} q' \in T$ and $(p',q') \in R$
  - R is a **bisimulation** if the same condition hold with q too:

    $\forall (p,q) \in R$,

    $q \xrightarrow{l} q' \in T \Rightarrow \exists\, q'.\ q \xrightarrow{l} q' \in T$ and $(p',q') \in R$

- **~ is the coarsest bisimulation:**

  p~q if there exists a bisimulation R such that p R q

  2 LTS are bisimilar iff their initial states are in ~

  -> all their reachable states are in ~

Transitivity

- If **R**, **S** are bisimulations, then so is their composition

RS = {(P, P') | ∃ Q. P **R** Q and Q **S** P'}

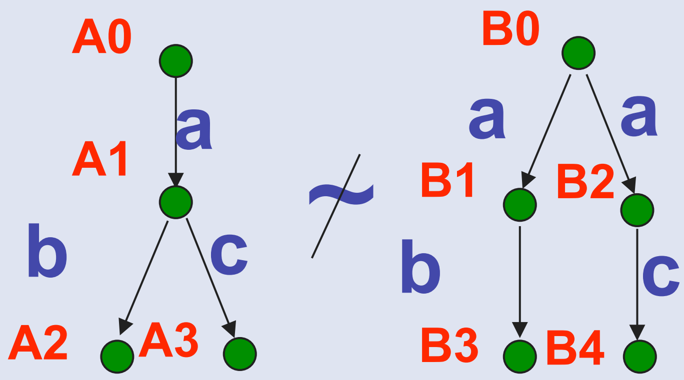
- In particular, $\sim\sim \subseteq \sim$, i.e., bisimilarity is transitive
- $\sim$ is an equivalence relation

Exercise:

Explain why **RS** is a bisimulation

Bisimulation Properties

- More precise than trace equivalence :



No state in B is equivalent to A1 - **Check**

- Preserves deadlock properties.
- Can be built by adding elements in the equivalence relation
- Coinductive definition (biggest set verifying …)

Bisimulation Properties (2)

- Congruence laws:

  P1~P2 => a.P1 ~ a.P2 (∀ P1,P2,a)

  P1~P2, Q1~Q2 => P1+Q1 ~ P2+Q2

  P1~P2, Q1~Q2 => P1|Q1 ~ P2|Q2

  Etc…

- ~ is a congruence for all CCS operators :

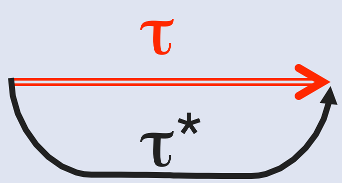
  for any CCS context C[.], C[P] ~ C[Q] <=> P~Q

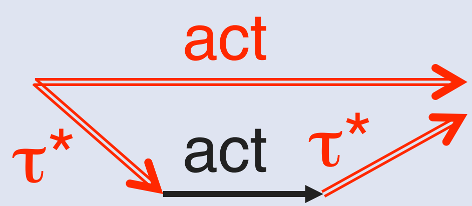
  Basis for compositional proof methods

- Maximal trace is not a congruence

Weak bisimulation(1)

- Weak bisimulation
  - Let us hide some actions (tau transitions)
  - We define a new reduction $\stackrel{\mu}{\Rightarrow}$ that allows for arbitrary many internal actions, more preciisely:





Weak bisimulation (2)

A weak bisimulation is a relation R *such that*

$P\ \mathbf{R}\ Q \Rightarrow \forall \mu, P, P'\ (P \xrightarrow{\mu} P' \Rightarrow \exists Q'.\ Q \overset{\mu}{\Rightarrow} Q' \text{ and } P'\ \mathbf{R}\ Q')$
and conversely

- Note the dissymetry between the use of $\xrightarrow{\mu}$ on the left and of $\overset{\mu}{\Rightarrow}$ on the right
- Two processes are *weakly bisimilar* (notation $P \approx Q$) *if* ***there exists a weak bisimulation R such that P R Q.***

Coffee machine Exercise

- $\mathrm{rec}_K \mathit{coin}.(\mathit{coffee}.\overline{\mathit{ccup}}.K + \mathit{tea}.\overline{\mathit{tcup}}.K)$
- $\mathit{coin}.\mathrm{rec}_K(\mathit{coffee}.\overline{\mathit{ccup}}.\mathit{coin}.K + \mathit{tea}.\overline{\mathit{tcup}}.\mathit{coin}.K)$
- $\mathrm{rec}_K(\mathit{coin}.\mathit{coffee}.\overline{\mathit{ccup}}.K + \mathit{coin}.\mathit{tea}.\overline{\mathit{tcup}}.K)$
- Question: which of these machines can we safely consider equivalent?
- Note that these machines have all the same traces.

ADDITIONAL NOTATIONS AND CONSTRUCTS

Alternative Notations (if you read books or papers or for other courses)

- $\stackrel{\text{def}}{=}$

$$rec_{C1}(Tick.C1) \Leftrightarrow \texttt{Cl} \stackrel{\text{def}}{=} \texttt{tick.Cl}$$

- Input/output: a=?a ; $\overline{a}$ = !a
- | or ||

Extension: Parameterized actions

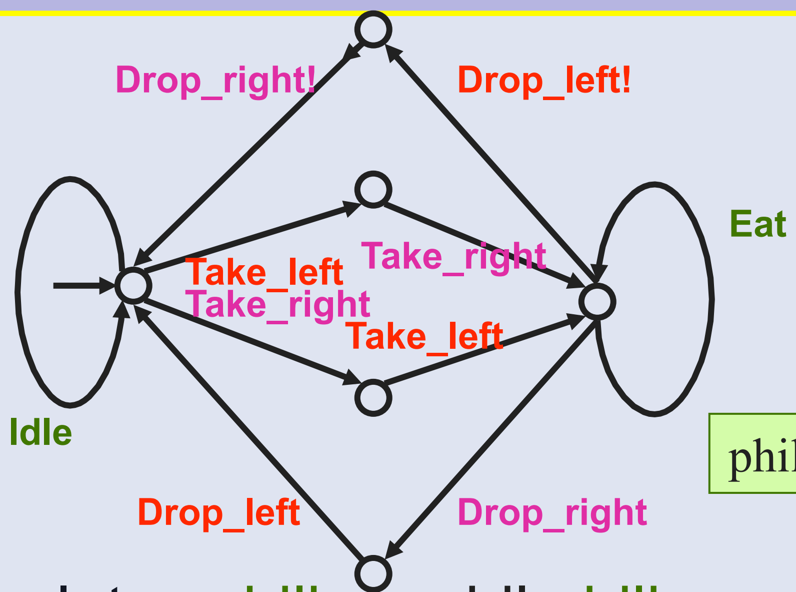
- input of data at port a, $a(x).P$
- $a(x)$ binds free occurrences of x in P .
- Port a represents $\{a(v):v\in D\}$ where D is a family of data values
- Output of data at port a, $\overline{a}(e).P$ where e is a data expression.
- Transition Rules depend on extra machinery for expression evaluation: Val(e) is the data value in D to which e evaluates
- R (in) $a(x).P \xrightarrow{a(v)} P\{v/x\}$ if $v\in D$ where $\{v/x\}$ is substitution
- R (out) $\overline{a}(e).P \xrightarrow{\overline{a}(v)} P$ if $Val(e) = v$
- Example $Reg_i = \overline{read}(i).Reg_i + write(x).Reg_x$

CONCLUSION

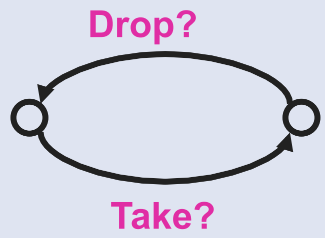
- A synchronous communication language
- A (complex but) efficient notion of equivalence on processes
- What is missing?
  - Channel communication (like in pi-calculus):
    a channel name can be communicated over another channel
    -> much more complex
  - No data or computation

EXERCISES

Guided exercise: dining philosophers



philosopher



fork

let rec idling = idle.idling + take_left.take_right.eating + take_right.take_left.eating

and eating = eat.eating + drop_left.drop_right.idling + drop_right.drop_left.idling

in idling

Consider 2 philos and 2 forks

Deadlock or not ?
Mutual exclusion ?

(trivial) example: Milner's Scheduler

- Processes iteratively start and finish executing tasks (one task per process)
- Task starts are cyclically ordered

cycler = $\overline{\alpha}$.start.(β.0 | end.cycler)

scheduler_3 = new $\alpha1$, $\alpha2$, $\alpha3$ in

([$\alpha1$/ α, $\alpha2$/β, start1/start, end1/end] cycler

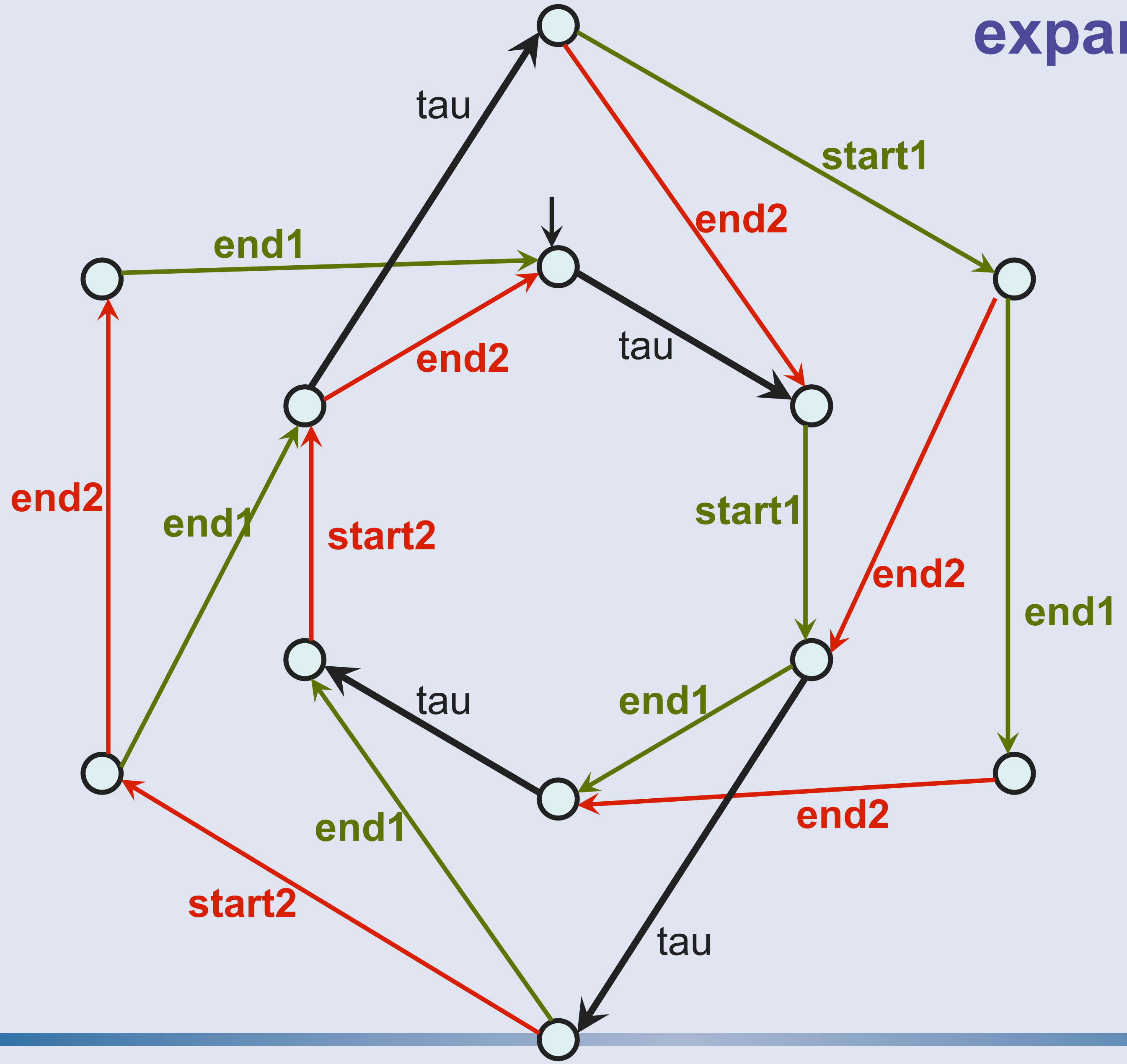
| [$\alpha2$/ α, $\alpha3$/β, start2/start, end2/end] cycler

| [$\alpha3$/ α, $\alpha1$/β, start3/start, end3/end] cycler

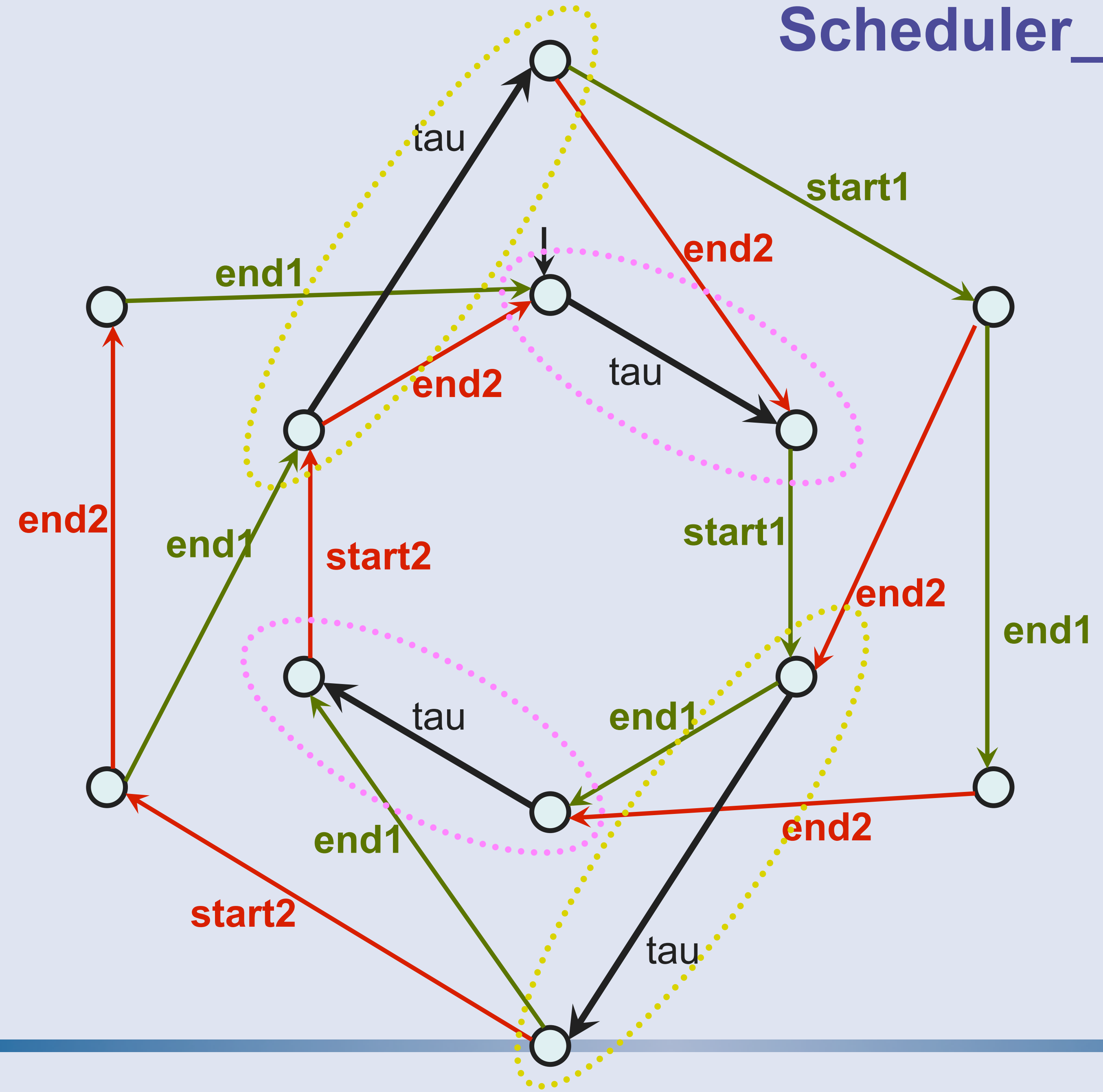
| $\alpha1$.0)

properties?

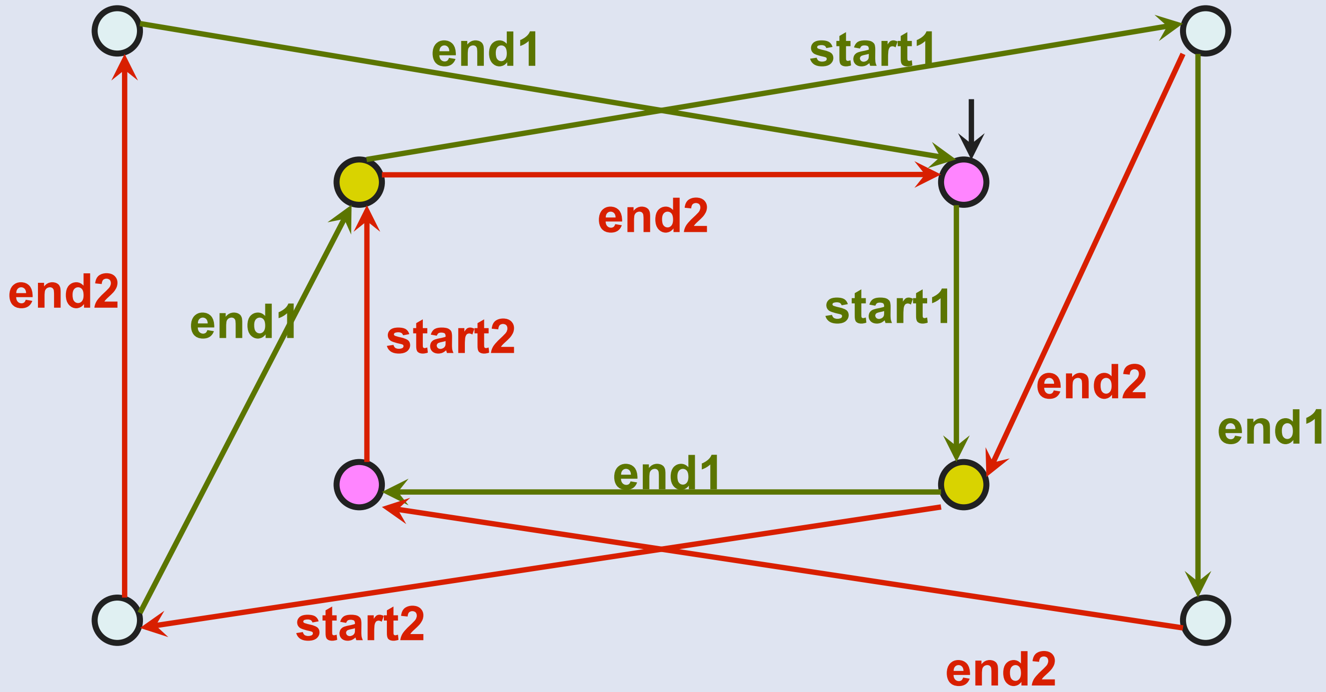
Scheduler_2 expanded



Scheduler_2 reducing

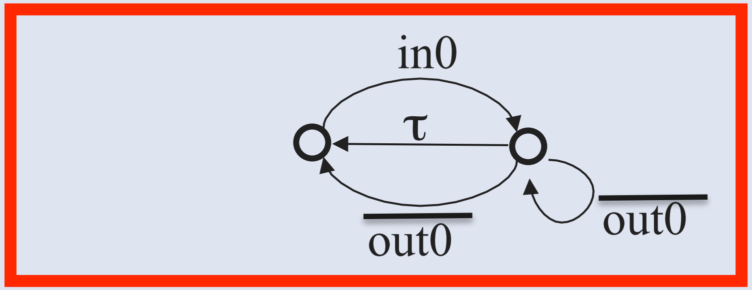


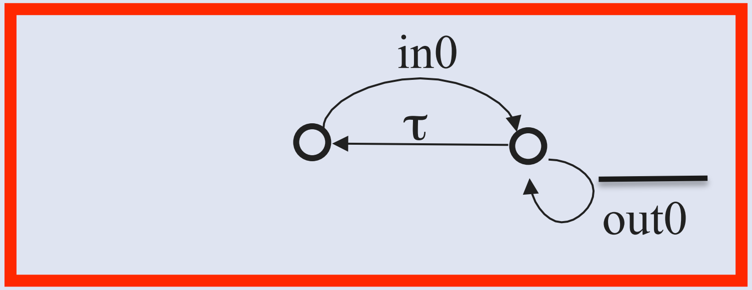
Scheduler_2 reduced

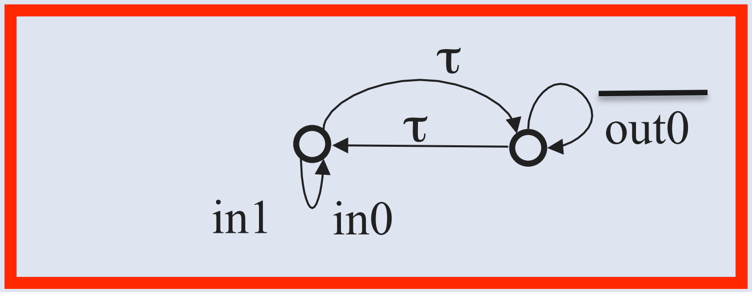


is this LTS bisimilar to the first one?

Exercise: Bisimulations







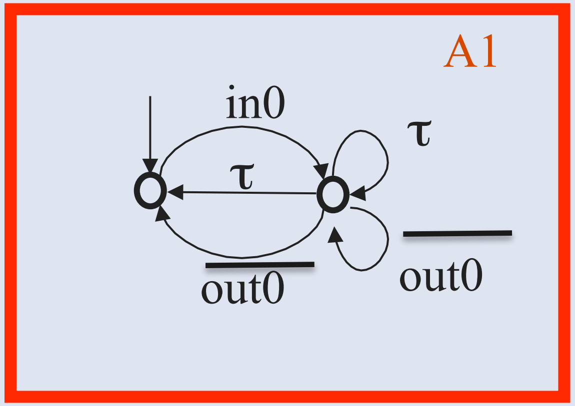
Are those 3 LTSs equivalent by:

- Strong bisimulation?

- Weak bisimulation ?

In each case, give a proof.

Exercise: Bisimulation



- Exercice :
  1) Compute the strong minimal automaton for A1.
  2) Compute the weak minimal automaton for A1.

Exercise

- Compare the construct $\stackrel{\text{def}}{=}$ and rec_K :
  1. Let us start by a simple pair of processes

$$\begin{aligned} A &\stackrel{def}{=} \bar{a}.A + b.B \\ B &\stackrel{def}{=} a.A \end{aligned}$$

  2. Suppose rec can accept several variables:
     rec (K=P,L=Q) express the same term
  3. Is it possible to express the same thing with a single variable K? Here are some possible hints:
     - Define a recursive process All that contains A and B and can trigger each of them by the reception of a message on channel cA or cB
     - (we suppose cA and cB cannot be used elsewhere)
     - What kind of equivalence between the two expressions do you have?

Additional exercise

- Why is maximal trace not a congruence? give an example. (small hint – use the example of the course)

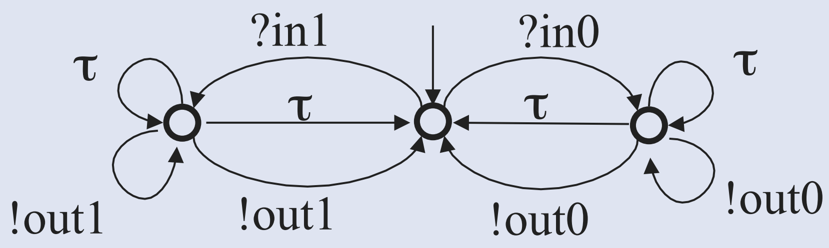
CORRECTION

Exercice: Alternated Bit Protocol

Correction (1):

Channels that loose and duplicate messages (in0 and in1) but preserve their order ?

1) Draw an automaton describing the loosy channel behaviour



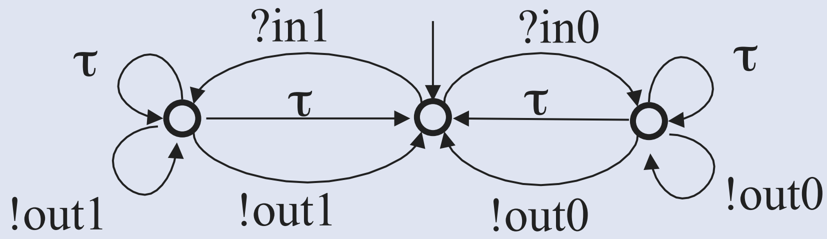
- It is a symmetric system, receiving ?in0 and ?in1 messages, then delivering 0 , 1 or more times the corresponding !out0 or !out1 message.
- On each side (bit 0 or 1), the initial state has a single transition for the reception.
- In the next state, it can either : return silently to the initial state (= lose the message), deliver the message and return to the initial state (exactly one delivery), or deliver the message and stay in the same state (thus enabling duplication).

Exercice: Alternated Bit Protocol

Correction (2):

Channels that loose and duplicate messages (in0 and in1) but preserve their order ?

2) Write it in CCS



- **Lousy channel** =

let rec {ch0 = ?in0 :ch1 + ?in1:ch2

and ch1 = τ :ch1 + τ :ch0 + !out0 :ch1 + !out0 :ch0

and ch2 = τ :ch2 + τ :ch0 + !out0 :ch2 + !out0 :ch0

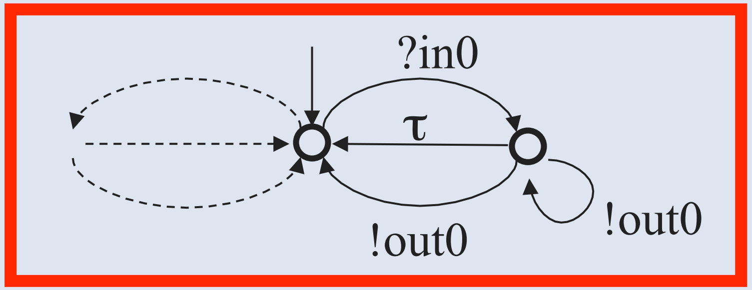
}

in ch0

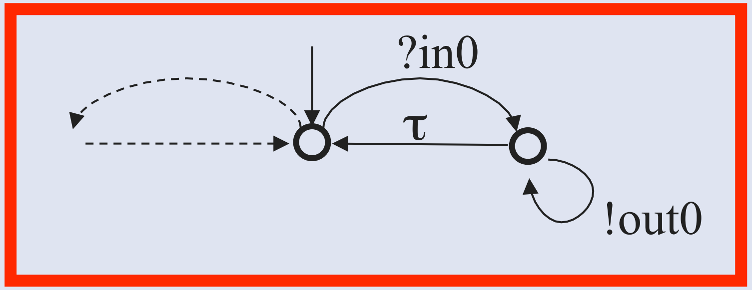
Exercice: Alternated Bit Protocol

Correction (3):

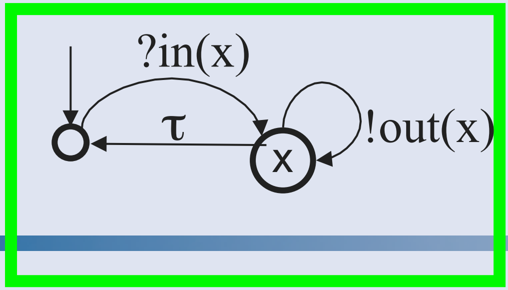
Channels that loose and duplicate messages (in0 and in1) but preserve their order ?



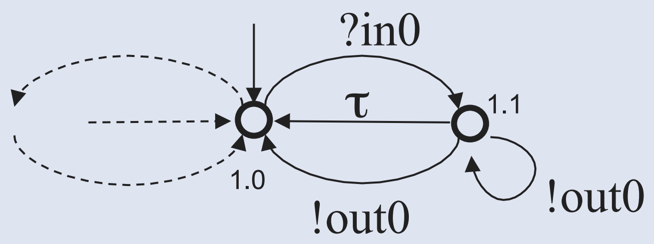
Other Solutions:

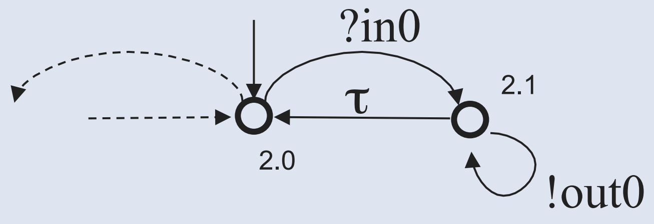


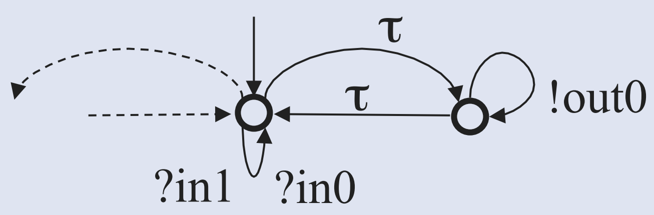
More generally, parameterized model :



Exercice 2 : Bisimulations







Are those 3 LTSs equivalent by:

- Strong bisimulation?

NO ! Need find non equivalent states. E.g. counter example for $1 \neq 2$:

States 1.0 and 1.1 are different because 1.0 can do ?in0 and 1.1 cannot.

Then 1.1 and 2.1 are different because 1.1 can do !out0 -> 1.0, while no 2.1 !out0 transitions can go to a state equivalent to 1.0.

- Weak bisimulation ?

YES. Exhibit a partition of equivalent states:

1={1.0,2.0}, 2={1.1, 2.1}

Check all possible (τ*aτ*) transitions:

1 - !in0 -> 2, … , 2 - !out0.τ* -> 1

Remark: this transition set defines the minimal representant modulo weak bisimulation…