Fault-Tolerant Broadcasting in Radio Networks

Evangelos Kranakis, Danny Krizanc
Carleton University, Ottawa, Canada
Andrzej Pelc (speaker)
Université du Québec, Hull, Canada

Radio network:

Collection of transmitter-receiver devices (called *nodes*), located in a geographical region. Nodes send messages in synchronous time slots. Each node v has a given transmission range. All nodes located within this range can receive messages from v. However, a node situated in the range of two or more nodes that send messages simultaneously, cannot receive these messages and hears only noise.

Broadcasting:

One node of the network, called the *source*, has a piece of information which has to be transmitted to all other nodes. Remote nodes get the source message via intermediate nodes, in several hops. One of the most important performance parameters of a broadcasting scheme is the total time it uses to inform all nodes of the network.

Previous work: network modeled as an undirected graph in which nodes are adjacent if they are in the range of each other.

Alon, Bar-Noy, Linial, Peleg (1991): family of n-node networks of radius 2, for which any broadcast requires time $\Omega(\log^2 n)$

Gaber, Mansour (1995): broadcasting can be done in time $O(D+\log^5 n)$ for any n-node network of diameter D.

Sen, Huson (1996): communication graphs that arise from actual geometric locations of nodes in the plane. Scheduling optimal broadcasting is NP-hard even when restricted to such graphs; $O(n \log n)$ algorithm to schedule an optimal broadcast when nodes are on a line.

This paper:

fast broadcasting algorithms in radio networks subject to permanent node failures of unknown location.

Two scenarios:

1. nodes are situated at integer points of a line,

2. nodes are situated at grid points of a square or hexagonal mesh.

Every node v of the network has the same *range*. This is a segment of length $2R$ in case of the line, a square of side $2R$ in case of the square mesh and a regular hexagon of side R in case of the hexagonal mesh, in all cases with center at node v.

At most t nodes are faulty. Faults are permanent (i.e., the fault status of a node does not change during the broadcast), and faulty nodes do not send or receive any messages. The location of faults is unknown and it is assumed to be worst-case.

Graph G whose vertices are fault-free nodes, and adjacent vertices are those in each other's range.

The *domain* is the connected component of G containing the source. The *diameter* (denoted by D) is the maximum length of all shortest paths between the source and nodes of the domain.

We consider *nonadaptive* algorithms, in which all transmissions have to be scheduled in advance

and

adaptive algorithms, in which nodes have the ability to schedule future transmissions on the basis of their communication history. This enables them to perform a preprocessing phase during which nodes learn the fault status of some other nodes from obtained messages and even from noise.

Results:

For all the above scenarios we give broadcasting algorithms whose worst-case time has optimal order of magnitude, and we prove corresponding lower bounds.

Nonadaptive algorithms: $\Theta(D + t)$,

Adaptive algorithms: $\Theta(D + \log(\min(R, t)))$,

where D is the diameter.

The line.

Nodes of the radio network are situated at the points $0, 1, ..., n$, on a line. The range of every node v has the same radius R, i.e., it includes nodes $v, v+1, ..., v+R, v-1, ..., v-R$. Assume that the source is at node 0 and define the ith *segment* to be $\{(i-1)R+1, ..., iR\}$.

The nonadaptive case

Number nodes in the ith segment $a_{i1}, ..., a_{iR}$ in decreasing order.

Algorithm Line-NA

1. The source transmits in time 1.

2. Node a_{ij} transmits in time $i+j+c(R+1)$, for $c = 0, 1, 2, ...$

Lemma.

The algorithm Line-NA informs the domain in time at most $D + t + \lceil t/R \rceil$.

Lower bound based on:

Lemma.

Suppose that the source message is known to a set A of nodes, where $|A| \geq t$. Any nonadaptive broadcasting algorithm informing a node v outside of A must use time larger than t.

Theorem.

The worst-case minimum time to inform the domain on the line by a nonadaptive broadcasting algorithm is $D + \Theta(t)$. Algorithm Line-NA achieves this performance.

The adaptive case.

Algorithm Line-ADA

1. Preprocessing: find the smallest and the largest fault-free node in each segment. Time $O(\log t)$.

2.

Time $2D$.

Lemma.

If $t < R$, algorithm Line-ADA informs the domain in time $2D + O(\log t)$.

Lower bound based on:

Lemma.

Suppose that the source message is known to a set A of nodes, where $|A| \geq t$. Any adaptive broadcasting algorithm informing a node v outside of A must use time larger than $\lfloor \log t \rfloor$ in the worst case.

Theorem.

The worst-case minimum time to inform the domain on the line by an adaptive broadcasting algorithm is $\Theta(D + \log(min(R, t))$. Algorithm Line-ADA achieves this performance.

The mesh.

Nodes are situated at points with coordinates $(i, j) : 1 \leq i, j \leq n$. Every square $\{(i-1)R + 1, ..., iR\} \times \{(j-1)R+1, ..., jR\}$, for $1 \leq i, j < n/R$ is called a *cell*.

Nonadaptive:

Theorem.

The worst-case minimum time to inform the domain on the mesh by a nonadaptive broadcasting algorithm is $\Theta(D + t)$.

Adaptive: preprocessing + proper broadcast

The aim of preprocessing is to elect representatives among all fault-free nodes in each cell. These representatives conduct message passing in the second phase. We use the following procedure independent of the topology of the radio network.

Let A and B be two sets of nodes of a radio network, such that every pair of nodes in A and every pair of nodes in B are in each other's range. We describe the Procedure Elect Couple that finds fault-free nodes $a \in A$ and $b \in B$, such that a and b are in each other's range, if such nodes exist. The procedure works in time $O(\log(|A| + |B|))$.

Fix a binary partition of each of the sets A and B: Divide each of the sets into halves, these halves into halves again, etc., down to singletons.

Basic step of Procedure Elect Couple (iterated $\lceil \log(\max(|A|, |B|)) \rceil$ times):

Before this step a member set X of the binary partition of A and a member set Y of the binary partition of B are *promoted*. (They contain fault-free nodes in each other's range). After this step, one of the halves of X and one of the halves of Y (those containing fault-free nodes in each other's range) are promoted, the other halves are *defeated*. This basic step works in 8 time units as follows.

1. All fault-free nodes in X_1 transmit.

2. All fault-free nodes in Y_1 that heard something in time 1, transmit. (If transmissions occur, Y_1 has been promoted).

3. All fault-free nodes in Y_2 that heard something in time 1 but nothing in time 2, transmit. (If transmissions occur, Y_2 has been promoted).

4. All fault-free nodes in X_1 that heard something in time 2 or 3, transmit. (If transmissions occur, X_1 has been promoted).

5. All fault-free nodes in X_2 that heard nothing in time 4, transmit.

6. All fault-free nodes in Y_1 that heard something in time 5, transmit. (If transmissions occur, Y_1 has been promoted).

7. All fault-free nodes in Y_2 that heard something in time 5 but nothing in time 6, transmit. (If transmissions occur, Y_2 has been promoted).

8. All fault-free nodes in X_2 that heard something in time 6 or 7, transmit. (If transmissions occur, X_2 has been promoted).

Consider all pairs of neighboring cells. Partition these pairs into groups in such a way that distinct pairs in the same group do not contain neighboring cells. (36 groups are sufficient for the square mesh.) To each pair assign the number (0 to 35) of the group to which this pair belongs.

Algorithm Mesh-ADA

1. Run Procedure Elect Couple for all pairs of neighboring cells, in 36 distinct phases, pairs of cells from the same group in the same phase, in parallel. Partners elected for a pair of neighboring cells get the number 0 to 35 corresponding to this pair. (The same node may get more than one number.)

2.The source transmits the message.
After an elected node heard the source message for the first time, it transmits it at the first time unit $u \equiv i \bmod 36$, where i is a number assigned to this node, and stops.

Theorem.

The worst-case minimum time to inform the domain on the mesh by an adaptive broadcasting algorithm is $\Theta(D + \log(\min(R, t)))$. Algorithm Mesh-ADA achieves this performance.