Window Flow Control in FIFO Networks with Cross–Traffic

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Stochastic Networks - Leiden

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Outline

I Window Flow Control

II Maximum Throughput

III Bounds on the Maximum Throughput

IV Examples

V Conclusion

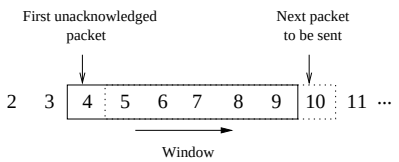
I - Window Flow Control

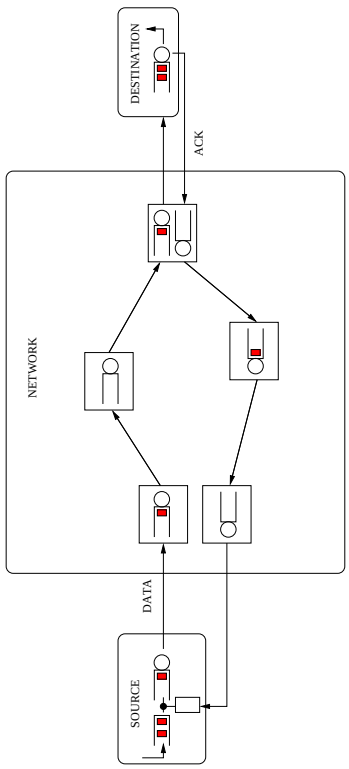
End-to-end control of packet transmission

"TCP over the Internet"

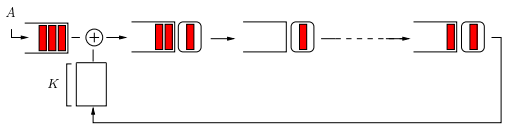
Error control: Each packet received by the destination is acknowledged;

Flow control: The number of unacknowledged packets in transit in the network is limited to a maximal value K called the *window*.





Model of Window Flow Control



This system is (max,+)–linear.

Let λ be the *intensity* of the arrival process A. There exists a constant μ, such that

- If $\lambda < \mu$, the system is stable;
- If $\lambda > \mu$, the system is unstable.

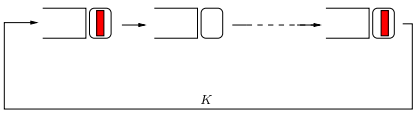
In this sense, μ is the *maximum throughput* of the connection.

Case of Deterministic Service Times

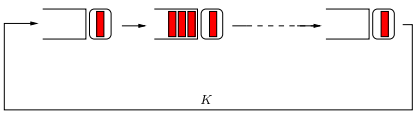
Denote by $\sigma_i = \mu_i^{-1}$ the (deterministic) service time at station i. Then,

$$\mu = \min\left(\frac{K}{\sum_i \sigma_i}, \min_i \mu_i\right).$$

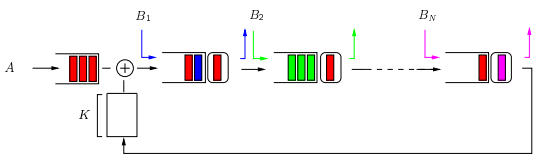
Case 1: The *window* limits the throughput.



Case 2: The *network* limits the throughput.



Model with Cross–Traffic



The cross–flows $B_1, \ldots, B_N$ are *stationary* and *ergodic*, and *independent* of A.

Denoting by $\rho_1, \ldots, \rho_N$ their traffic intensity, we assume that

$$\forall i = 1, \ldots, N, \quad \rho_i < 1.$$

Questions:

- Existence of a maximum throughput?
- Saturation rule?
- Evaluation of this maximum throughput?

Stochastic Framework

Controlled flow:

Let $(\Omega_A, \mathbb{P}_A)$ be a probability space, equipped with a *stationary* and *ergodic* flow $\{\theta_A(t)\}$.

$$A \text{ is } \{\theta_A(t)\}\text{-compatible.}$$

Cross–flows:

Let $(\Omega_B, \mathbb{P}_B)$ be a probability space, equipped with a *stationary* and *ergodic* flow $\{\theta_B(t)\}$.

$$B_1, \ldots, B_N \text{ are } \{\theta_B(t)\}\text{-compatible.}$$

Probability space:

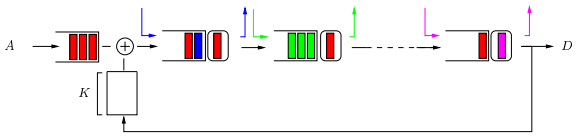
Let $(\Omega, \mathbb{P})$ be the product space:

$$\Omega = \Omega_A \times \Omega_B \quad \text{and} \quad \mathbb{P} = \mathbb{P}_A \mathbb{P}_B.$$

We define a flow $\{\theta(t)\}_{t\in\mathbb{R}}$ on this space by

$$\forall t \in \mathbb{R}, \quad \theta(t) = \theta_A(t) \times \theta_B(t).$$

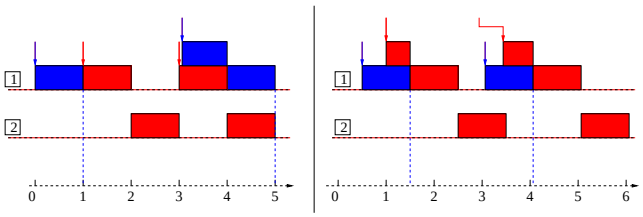
II - Maximum Throughput



Lemma 1 (Monotonicity) *Let A and A' be two arrival processes with the same (finite) number of points. Then,*

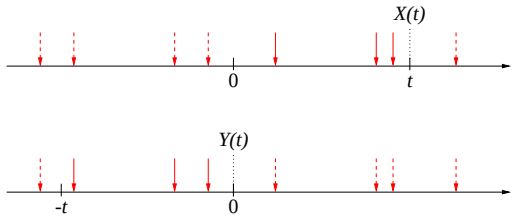
$$A \leq A' \quad \Rightarrow \quad D \leq D'.$$

Remark: This monotonicity does not hold with respect to the cross–flows.



Stationary Regime

For any $t \geq 0$, define $X(t)$ and $Y(t)$ as follows:



By monotonicity, we can define

$$Z = \lim_{t \to +\infty} Y(t).$$

Lemma 2 (Stationarity) *The process $\{X(t)\}$ tends to Z for the weak convergence.*

Lemma 3 (Ergodicity) *The random variable Z is a.s. finite or a.s. infinite.*

Maximum Throughput

Theorem 1 *There exists a constant $\bar{\lambda}$, such that for any stationary arrival point process A of intensity λ,*

- *If $\lambda < \bar{\lambda}$, the system is stable;*

$$X(t) \Rightarrow Z \quad \text{when } t \to +\infty$$

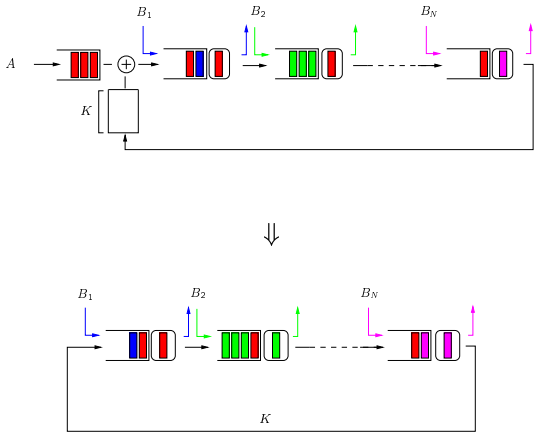
- *If $\lambda > \bar{\lambda}$, the system is unstable.*

$$X(t) \Rightarrow \infty \quad \text{when } t \to +\infty$$

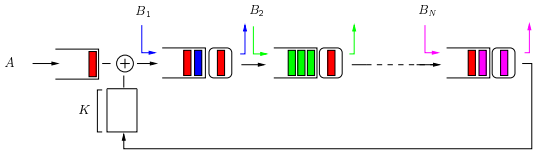
Remark: In particular, the stability region does *not* depend on the statistical characteristics of the controlled flow (except for the critical case).

Saturation Rule

Theorem 2 *The maximum throughput $\bar{\lambda}$ is obtained by saturating the input queue with an infinite number of customers.*



III - Bounds on the Maximum Throughput



Upper Bounds:

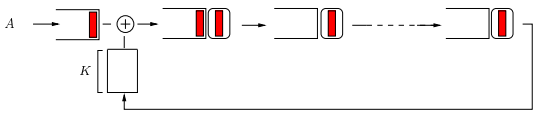
$$\bar{\lambda} \leq \mu \quad \text{and} \quad \bar{\lambda} \leq \min_i \mu_i(1-\rho_i).$$

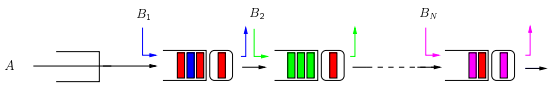
Lower Bound:

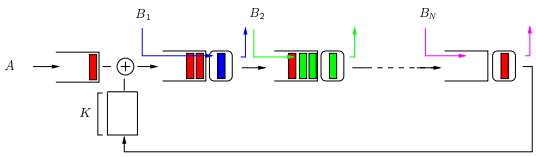
$$\bar{\lambda} \geq \mu\, \mathbb{P}(W(0)=0).$$

If the cross–flows are *mutually independent*,

$$\bar{\lambda} \geq \mu \prod_i (1-\rho_i).$$



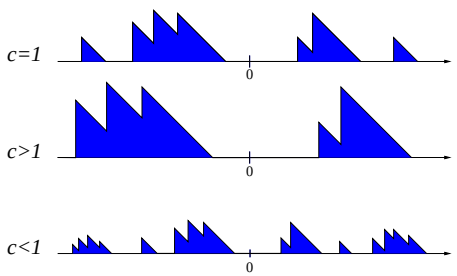




Tightness of the Bounds

We have

$$\underbrace{\mu \prod_i (1-\rho_i)}_{\lambda_{\min}} \le \bar{\lambda} \le \underbrace{\min\left\{\mu, \min_i \mu_i(1-\rho_i)\right\}}_{\lambda_{\max}}.$$



Theorem 3 (Time–Space Scaling) *For any $c > 0$, let $\bar{\lambda}(c)$ be the maximum throughput obtained when the cross–flows are scaled by c. Then,*

$$\lim_{c\to+\infty} \bar{\lambda}(c) = \lambda_{\min} \quad \text{and} \quad \lim_{c\to 0} \bar{\lambda}(c) = \lambda_{\max}.$$

Numerical Example

Consider the case of a controlled connection which goes through $N = 10$ routers.

The window is $K = 10$, so that

$$\mu = \min_i \mu_i \stackrel{\triangle}{=} 1.$$

If the traffic intensities of the exogenous flows are the same ($\rho_1 = \ldots = \rho_N = \rho$), we get

$$\lambda_{\max} = \min\left\{\mu, \min_i \mu_i(1-\rho_i)\right\} = 1-\rho,$$

and

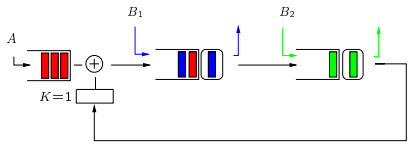
$$\lambda_{\min} = \mu \prod_i (1-\rho_i) = (1-\rho)^{10}.$$

ρ	0.1	0.2	0.5	0.8
$\lambda_{\max}$	0.9	0.8	0.5	0.2
$\lambda_{\min}$	0.3	0.1	10^{-3}	10^{-7}

IV - Examples

Consider the simplest non–trivial case:

$$N = 2, \quad K = 1.$$



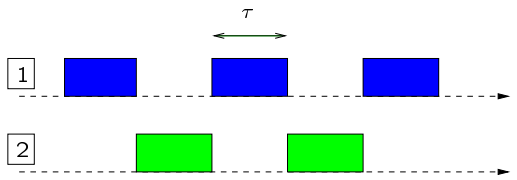
$$\mu_1 = \mu_2 = 1 \quad \text{and} \quad \mu = \frac{1}{2}$$

Ex 1: B_1, B_2 periodic, $B_2 = B_1 + \tau$;

Ex 2: B_1, B_2 periodic, mutually independent;

Ex 3: B_1 periodic, $B_2 = \emptyset$.

Example 1



$$\lambda_{\min} = 0 \quad \text{and} \quad \lambda_{\max} = \min(\mu, \frac{1}{2}) = \frac{1}{2}$$

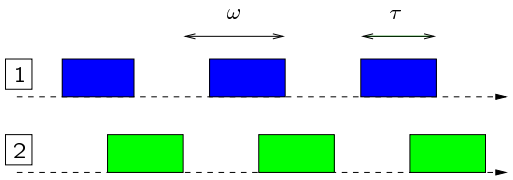
For any $\tau \ge 1$, we have

$$\bar{\lambda} = \frac{1}{2\tau}.$$

In particular,

$$\lim_{\tau\to+\infty} \bar{\lambda} = 0.$$

Example 2



$$\lambda_{\min} = \mu \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \quad \text{and} \quad \lambda_{\max} = \frac{1}{2}$$

Here the maximum throughput is a random variable, which depends on the *phase* ω.

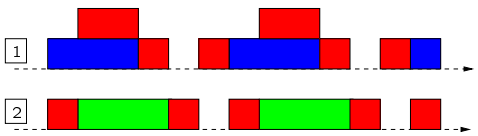
If $\tau = 2n - 1$, where n is a positive integer, there is a *spectrum* of n possible values for $\bar{\lambda}$, namely

$$\bar{\lambda} \in \{1, \ldots, n\} \times \frac{1}{2\tau}.$$

Remark: In this non–ergodic case, $\lambda_{\min}$ does not provide a lower bound for the maximum throughput but for its *expectation*.

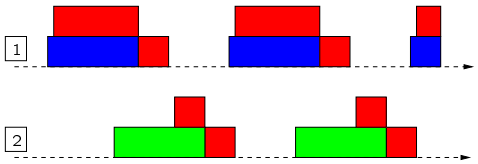
Example 2: Case $\tau = 3$

Cross–flows "in phase":



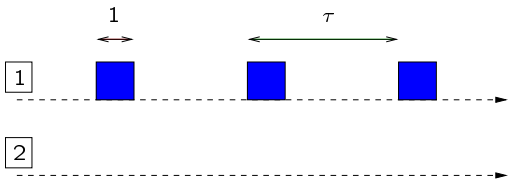
$$\bar{\lambda} = \frac{1}{\tau}.$$

Cross–flows "out of phase":



$$\bar{\lambda} = \frac{1}{2\tau}.$$

Example 3



The traffic intensity of the cross–flow is:

$$\rho = \frac{1}{\tau}.$$

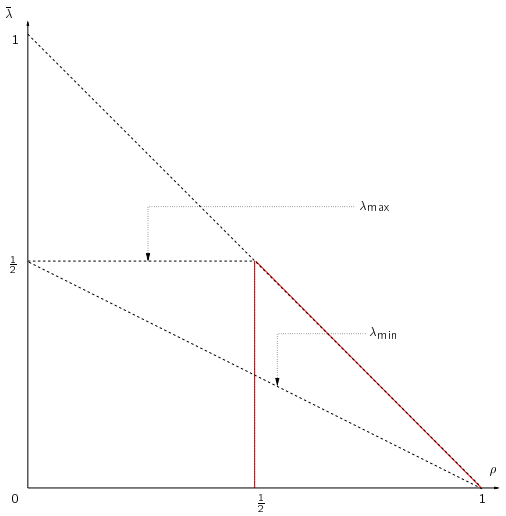
We have

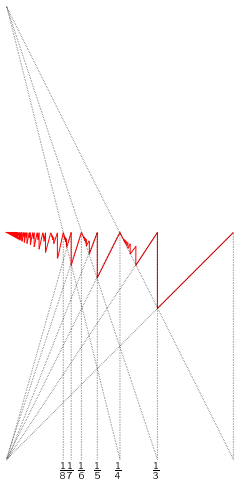
$$\lambda_{\min} = \frac{1}{2}(1 - \rho)$$

and

$$\lambda_{\max} = \min(\frac{1}{2}, 1 - \rho).$$

Example 3: Stability Region





V - Conclusion

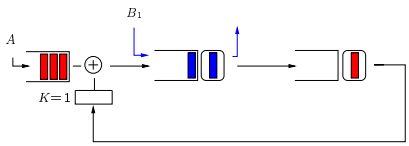
The efficiency of the window flow control is very *sensitive* to the statistical characteristics of the cross–flows:

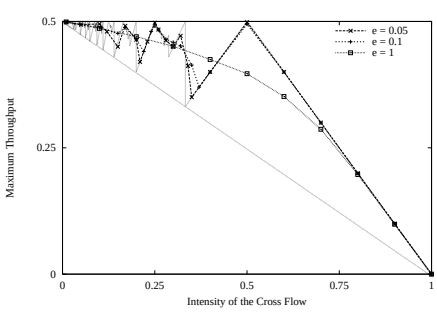
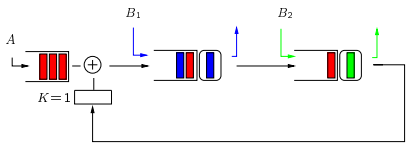
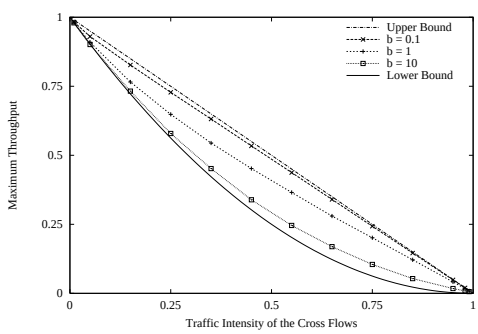
- Periodic flows *vs* random flows;
- Bursty flows *vs* fluid flows.

This is due to the fact that the model is not *monotone* with respect to the cross–flows, as soon as $N > 1$.

These results can be easily extended to more general models (e.g. presence of propagation delays, multicast connection,...).

Effect of Randomization

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