Fixed Point Models and Congestion Pricing for TCP and Related Schemes

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Collaborators

- Frank Kelly, Richard Gibbens (Stats Lab)
  - preprint: *A note on resource pricing and congestion control for networks with delay and loss*, R J Gibbens and P B Key
- Derek McAuley, Paul Barham, Koenraad Laevens, Dave Stewart, (Microsoft Res Ltd)
  - submitted: *Differential QOS and Pricing in Networks: where flow-control meets game theory*, Peter Key & Derek McAuley, UKPEW1998

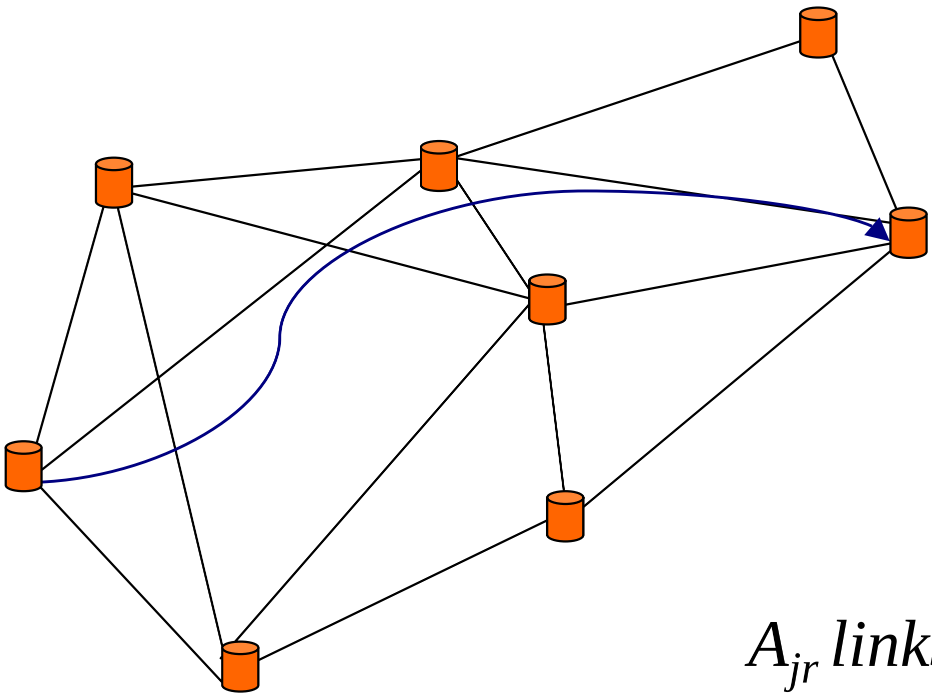
Outline

- A single resource system
- Implied cost type models
- Multiple Resources
- A distributed game!

Resource system ('network')

Resource $j \in \mathcal{J}$

Capacity C_j



User
/route
$r \in \mathcal{R}$

A_{jr} *links users to resources*

Simple models of TCP (CA)

- Single resource, lose pkts with prob. *p, x* is window in MSS

$$x_{t+1} = x_t + 1 - p x_t x_t / 2 \quad \Rightarrow x^* = \sqrt{2/p}$$

- Strictly periodic loss (Sawtooth)

$$x^* = \sqrt{3/2p}$$

- Random loss, ...

$$x^* = 1.309\sqrt{1/p}$$

Simple TCP model, multiple connections

n sources, single resource capacity C

$$\frac{dW_t}{dt} = 1 - \Pr\{\text{packet lost}\} \cdot W_t \cdot \frac{W_t}{2}$$

$$= 1 - \frac{\left(\sum_i W_t^i - C\right)^+}{\sum_i W_t^i} W_t \cdot \frac{W_t}{2}$$

So in steady state, with equal connections

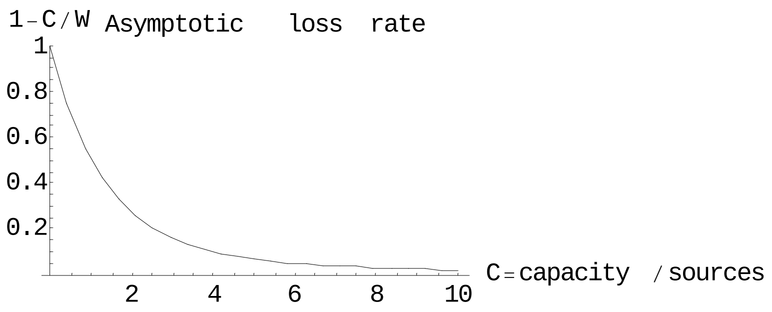
$$\Rightarrow \quad W = \frac{C + \sqrt{C^2 + 8n^2}}{2n}$$

Simple TCP model with feedback

Resource loses excess load, *n* number of sources,

nC capacity (=n x resource capacity*RTT/MSS),

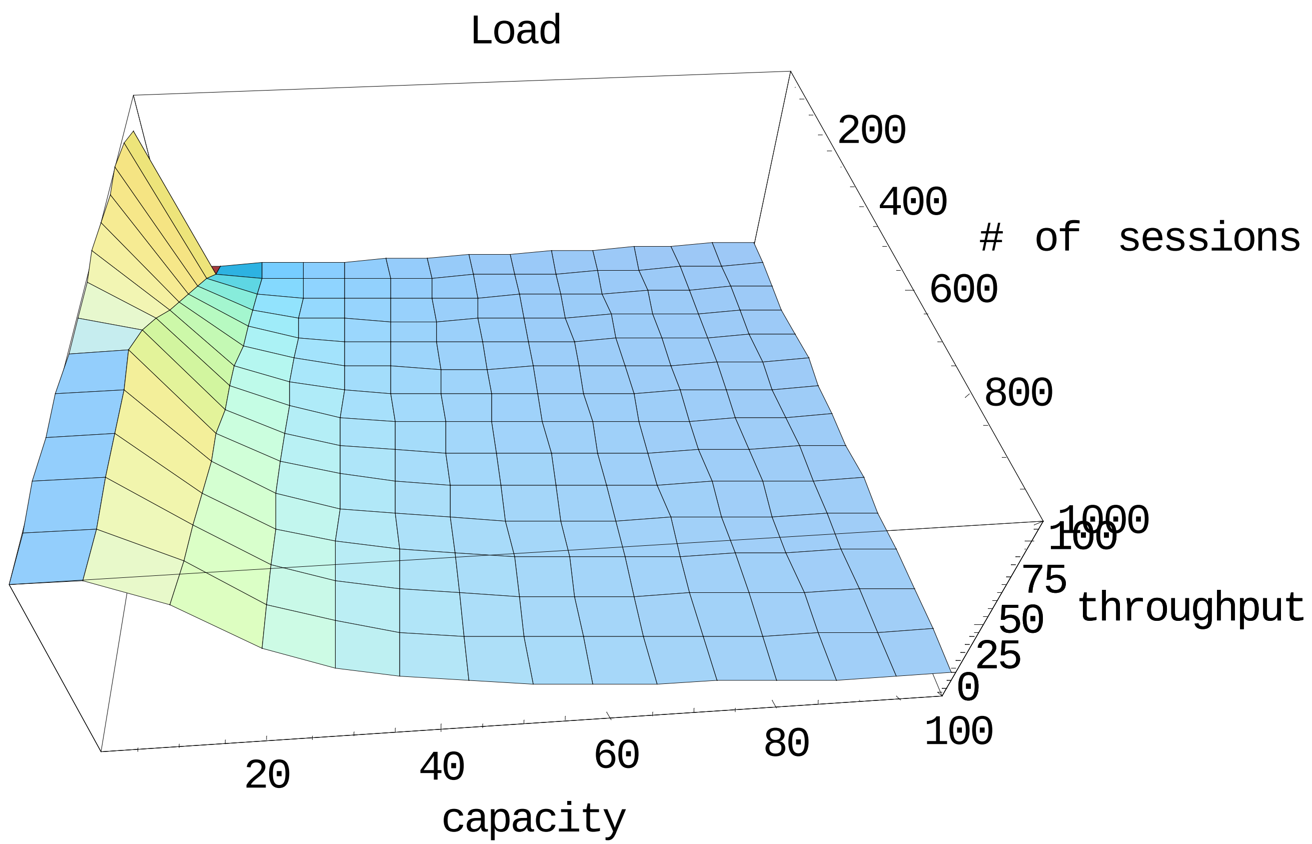
RTT=1ms, cap=1MB/s, C=244



window=

$$\frac{1}{2}\left(C + \sqrt{8 + C^2}\right)$$

Normalised Load



Stochastic Network Formulation

- Suppose user has utility U_r and offers at rate ν_r objective:

$$\text{Maximise} \quad \sum_r U_r(x_r) \qquad \text{over} \quad x_r \geq 0 \quad \text{s.t. } x_r \leq \nu_r$$

where

$$x_r = \nu_r \prod_j (1 - B_j)^{A_{jr}} \stackrel{def}{=} \nu_r (1 - L_r)$$

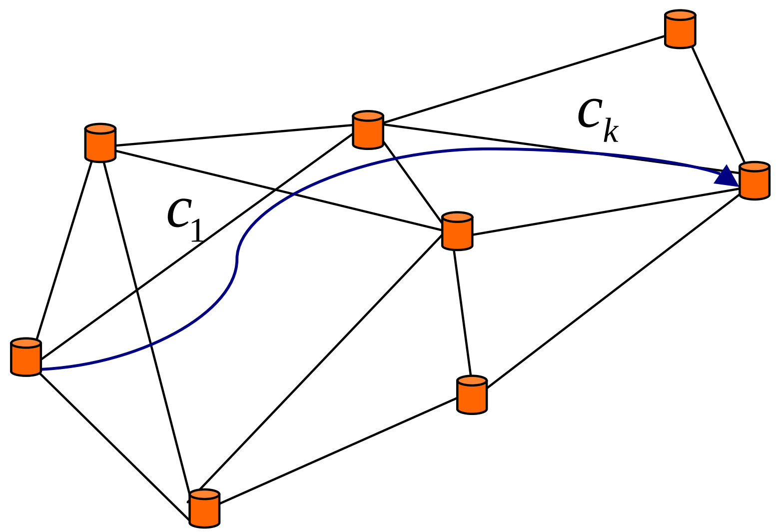
$$B_j = B(\rho_j, C_j) \quad \text{for some twice differentiable } B$$

$$\rho_j = (1 - B_j)^{-1} \sum_r A_{jr} \nu_r \prod_j (1 - B_j)^{A_{jr}}$$

Theorem (Implied Costs)

$$\frac{dW}{d\nu_r} = (1 - L_r)\left\{U_r'(x_r) - \sum_j A_{jr} c_j\right\}$$

$$c_k = \frac{1}{(1 - B_k)^2} \frac{\partial B_k}{\partial \nu_{[k]}} \sum_r A_{kr} x_r \left\{U_r'(x_r) - \sum_j A_{jr} c_j\right\}$$



Corollary (Shadow prices)

- If *B* is Erlang's formula, the *c's* are also the shadow prices

$$c_k = \frac{dW}{dC_k}$$

$$c_k = \frac{\eta_k}{(1-B_k)} \sum_r A_{kr} x_r \left\{ U_r'(x_r) - \sum_j A_{jr} c_j \right\}$$

$$\eta_k \overset{def}{=} Erl(\rho_k, C_k) - Erl(\rho_k, C_k - 1)$$

System with Cost function (Kelly/Gibbens)

- Suppose there is a cost function, $C_j(y)$ giving rate at which cost incurred at resource *j* with load *y*, objective:

$$\text{Maximise} \quad \sum_r U(x_r) - \sum_j C_j\left(\sum_r A_{jr} x_r\right) \qquad \text{over} \quad \mathrm{x} \geq 0$$

Solution the same (for convex *C*) with

$$\mu_j = p_j\left(\sum_r A_{jr} x_r\right) \qquad \text{where} \quad p_j(y) \overset{def}{=} \frac{dC_j(y)}{dy}$$

Example

$$C_j(y) = E(Y - c_j)^+ \quad \Rightarrow \mu_j = \frac{dC_j}{dy} = \Pr\{Y \geq c_j\}$$

if Y Poisson and $E(Y) = y$.

Also,

$$E(x_r I\{Y \geq c\}) = x_r \Pr\{Y \geq c_j\}$$

Hence mark packet if exceed capacity

If $U(x_r) = w_r \operatorname{Log}(x_r)$, then

$$w_r = x_r \Pr\{Y \geq c_j\}$$

Adaptive (prop fair) scheme

Good reasons (eg Nash Arbitration) for choosing

$$U_r(x_r) = w_r \operatorname{Log}(x_r)$$

which suggests the adaptive scheme

$$\frac{d}{dt} x_r(t) = \kappa \left(w_r(t) - x_r(t) \sum_{j \in r} \mu_j(t) \right)$$

where $\mu_j(t) = p_j\left(\sum_{j \in r} x_r(t)\right)$, $\frac{d}{dy} C_j(y) = p_j(y)$

Example - elastic control

$$x_{t+1} = x_t + \kappa\left(w_t - f\left(t\right)\right)$$

w_t reflects willingness to pay,

$f\left(t\right)$ is feedback received from the network

eg

$f\left(t\right) = x_t$ if (resource/ bottleneck overloaded) else $= 0$

Example Strategies

- Willingness to pay

$$x_{t+1} = x_t + \kappa\left(w_t - f(t)\right)$$

w_t willingness to pay,

$$f(t) = x_t \, \mathrm{P_{sat}} = x_t \, \Pr\left\{\sum x_r \geq C\right\}$$

$$= x_t \, \Pr\{Y \geq C\}$$

- TCP-like

$$x_{t+1} = x_t + \kappa\left(w_t - f(t)\frac{x_t}{2}\right)$$

$$f(t) = x_t \, \mathrm{P_{loss}} = x_t \frac{E[Y - C]^+}{E[Y]}$$

- TCP-like with shadow price

$$x_{t+1} = x_t + \kappa\left(w_t - f(t)\frac{x_t}{2}\right)$$

$$f(t) = x_t \, \mathrm{P_{sat}} = x_t \, \Pr\{Y \geq C\}$$

Large Deviation Analysis

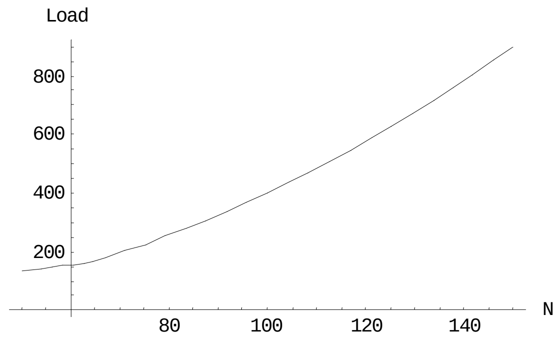
$$\frac{P_{loss}}{P_{sat}} = \frac{1}{E[Y]\left(e^{s^*} - 1\right)}$$

$$s^* = \arg\inf \log\left(\phi(s^*) - s^* C\right)$$

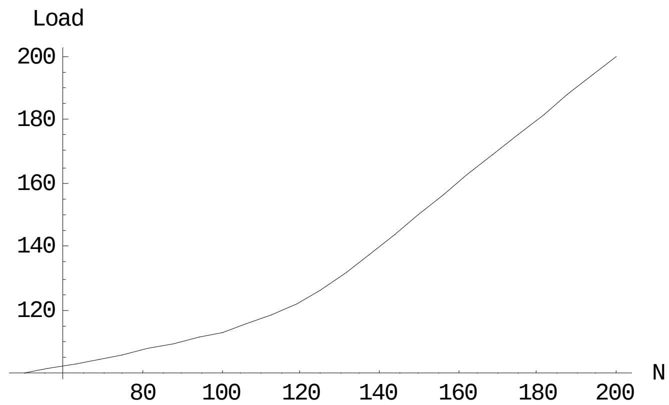
$$\frac{P_{loss}}{P_{sat}} = \frac{1}{C - y}$$

If Y Poisson mean y

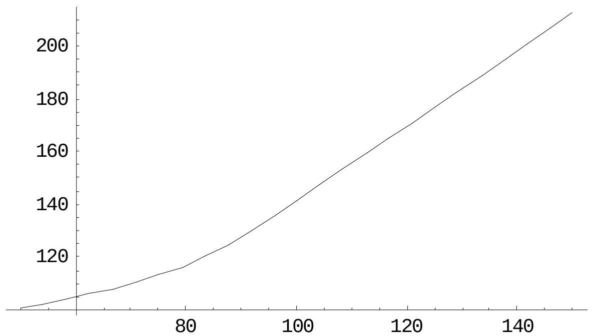
TCP & TCP like schemes



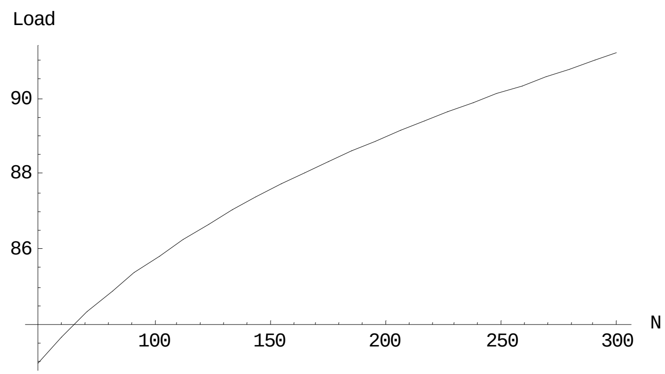
TCP



w=1



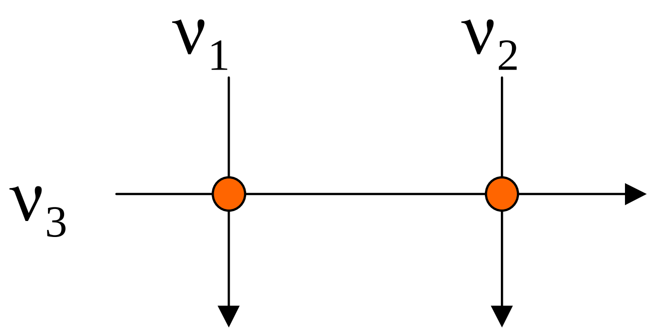
TCP with shadow price



w=0.05

Fairness Example - no control

Offered

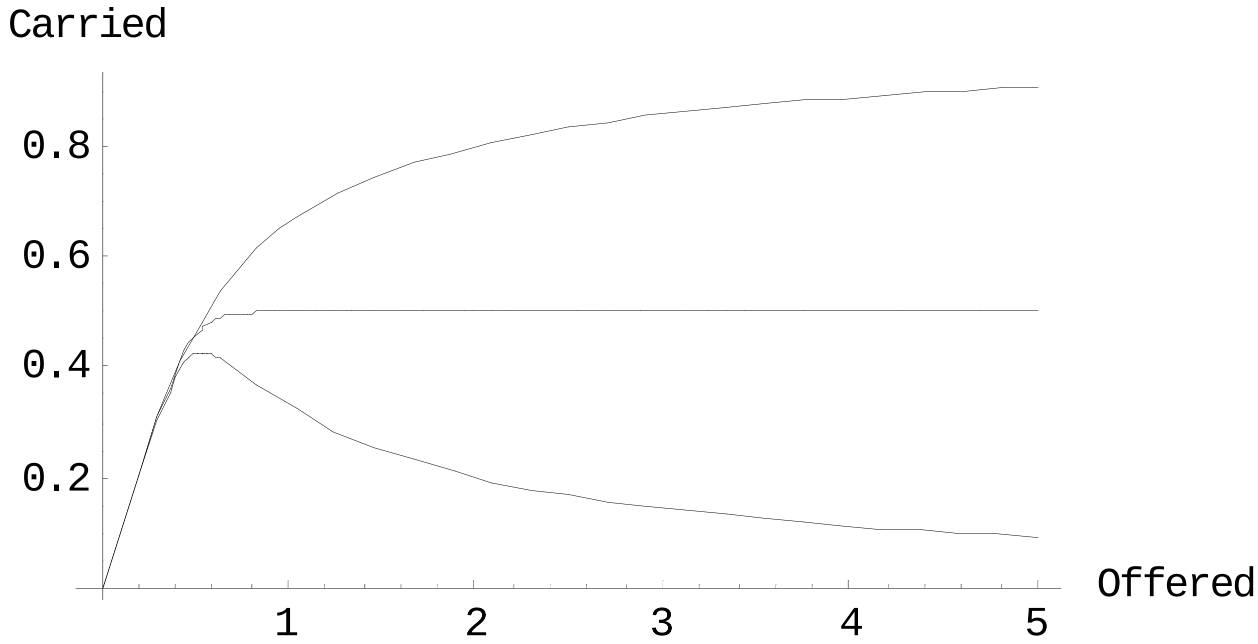


$$v_1 = v_2 = v_3 = \rho C$$

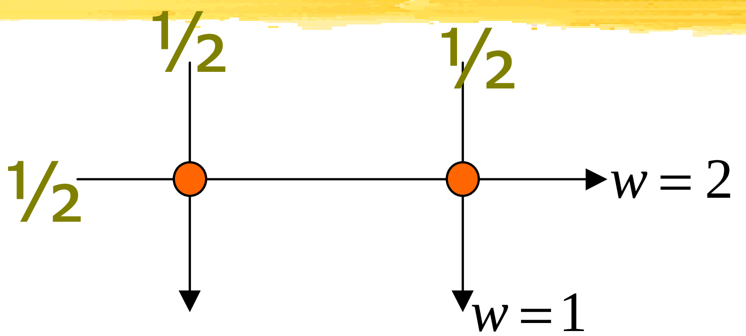
Fixed point equations imply $$\frac{x_1}{v_1} = \frac{1}{2}\left(\sqrt{1+\frac{4}{\rho}} - 1\right) = \frac{x_3}{x_1} \quad \text{for } \rho > \frac{1}{2}$$

and allocation changes from *max-min* fairness to maximum utilisation as ρ increases

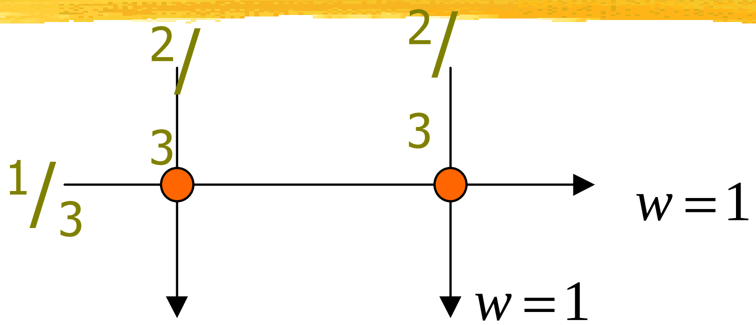
Throughput (no flow control)



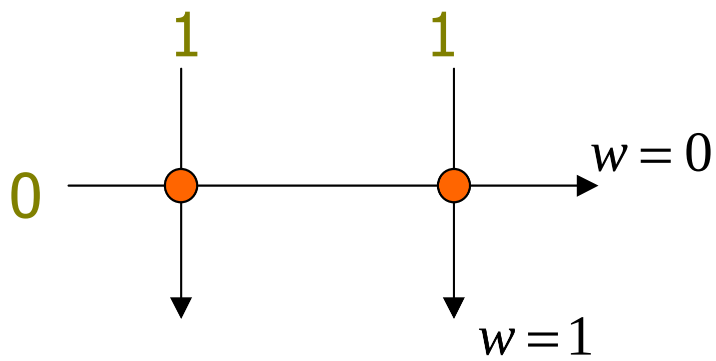
Fairness Examples, prop. fair prices



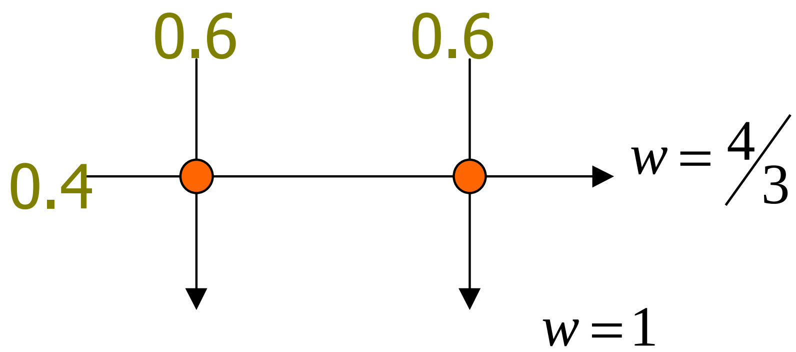
Max-min



Proportional

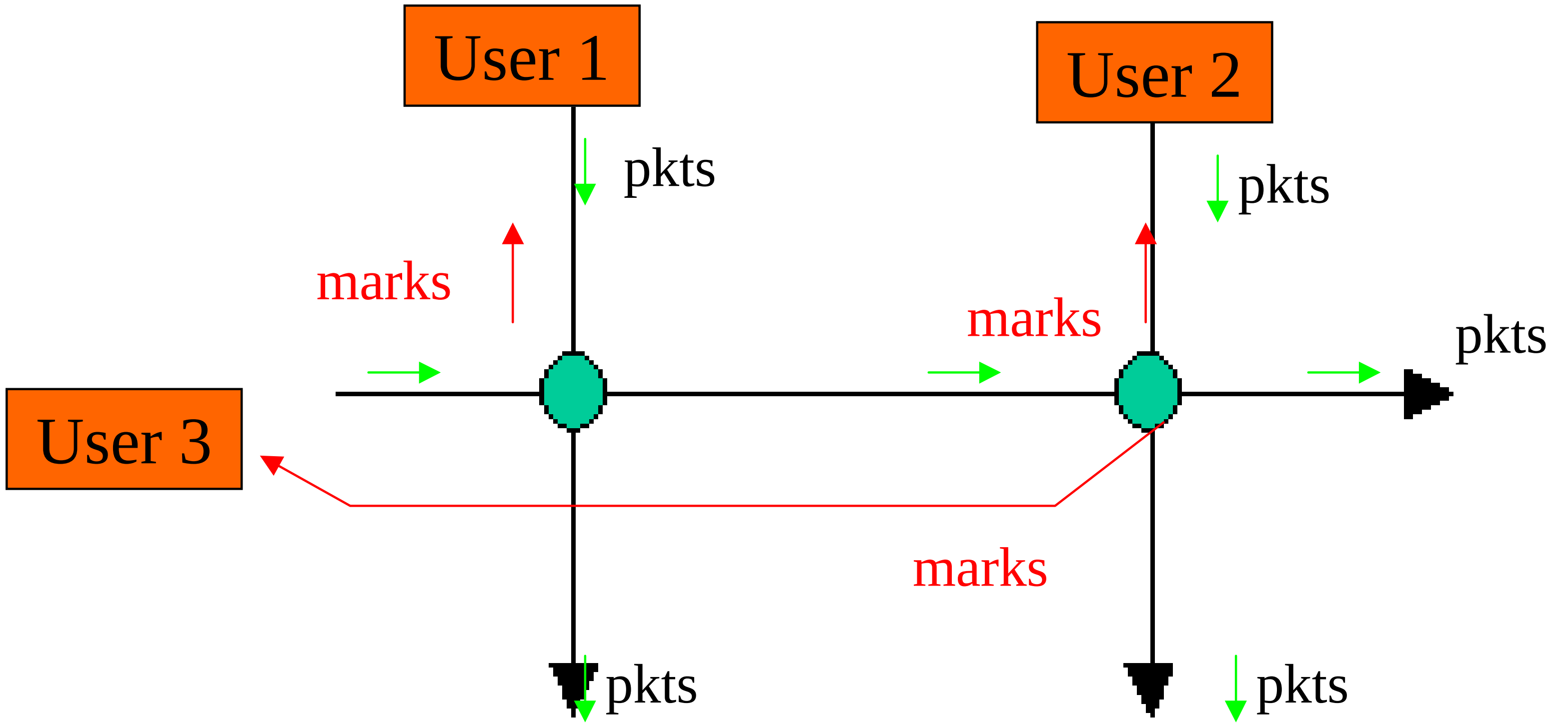


Max load



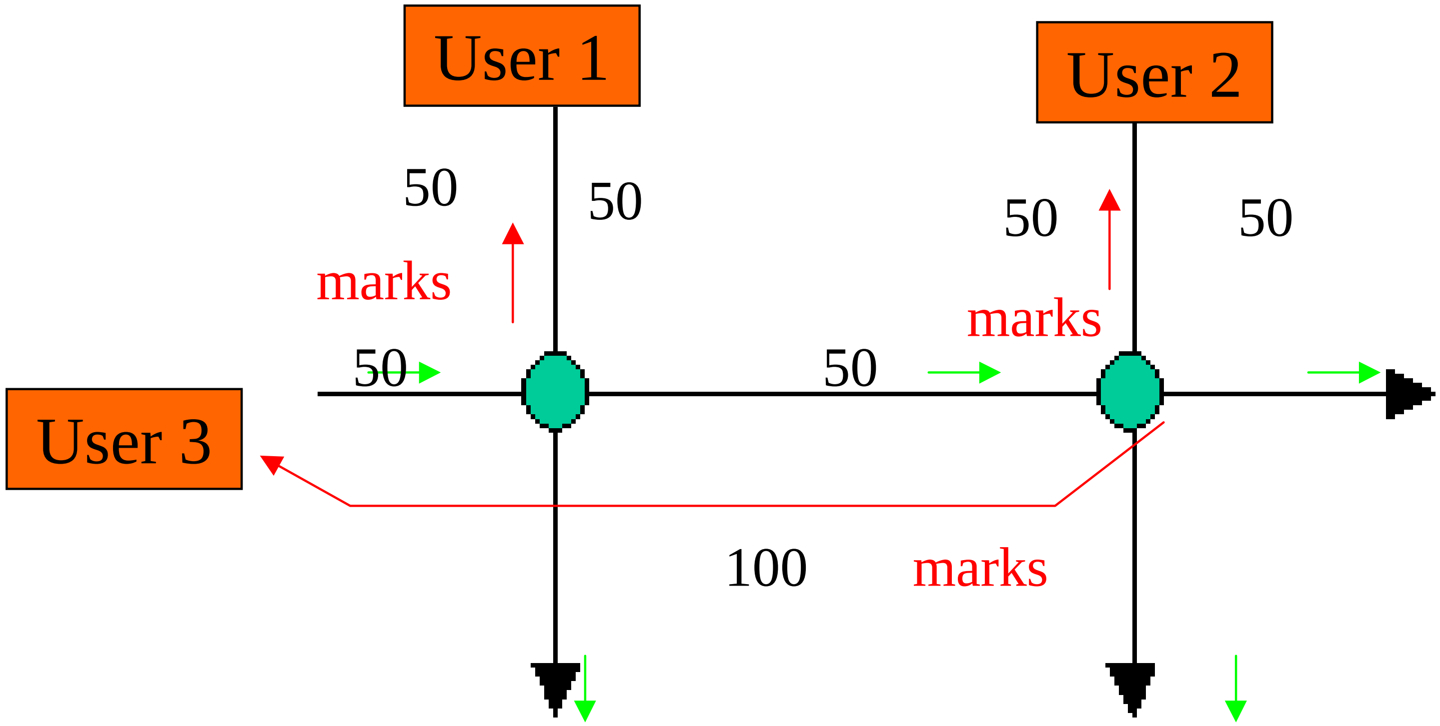
TCP approx

Two-Node Example: Congestion pricing



100 of each user type

Two-Node Example, Delays

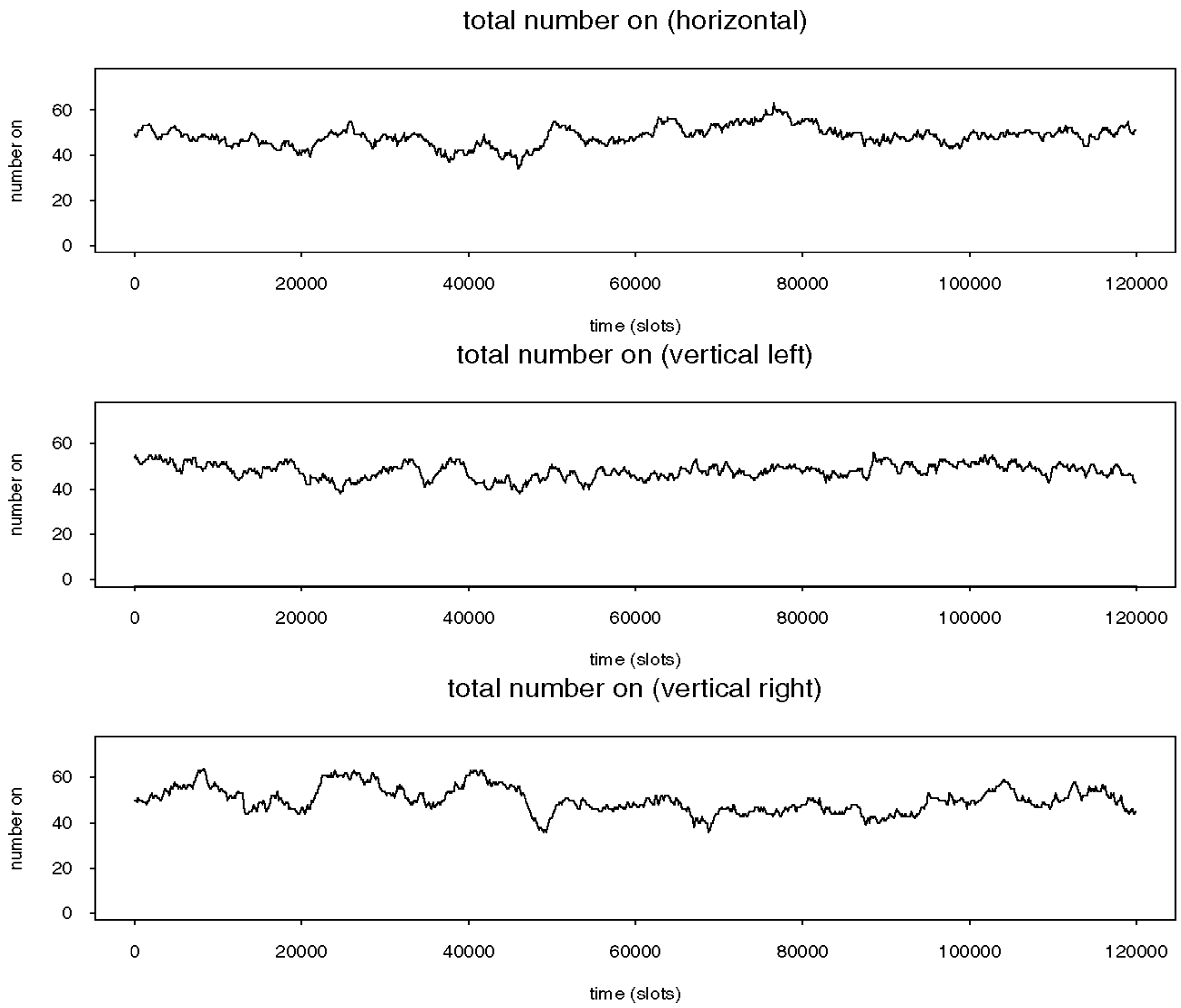


Users types

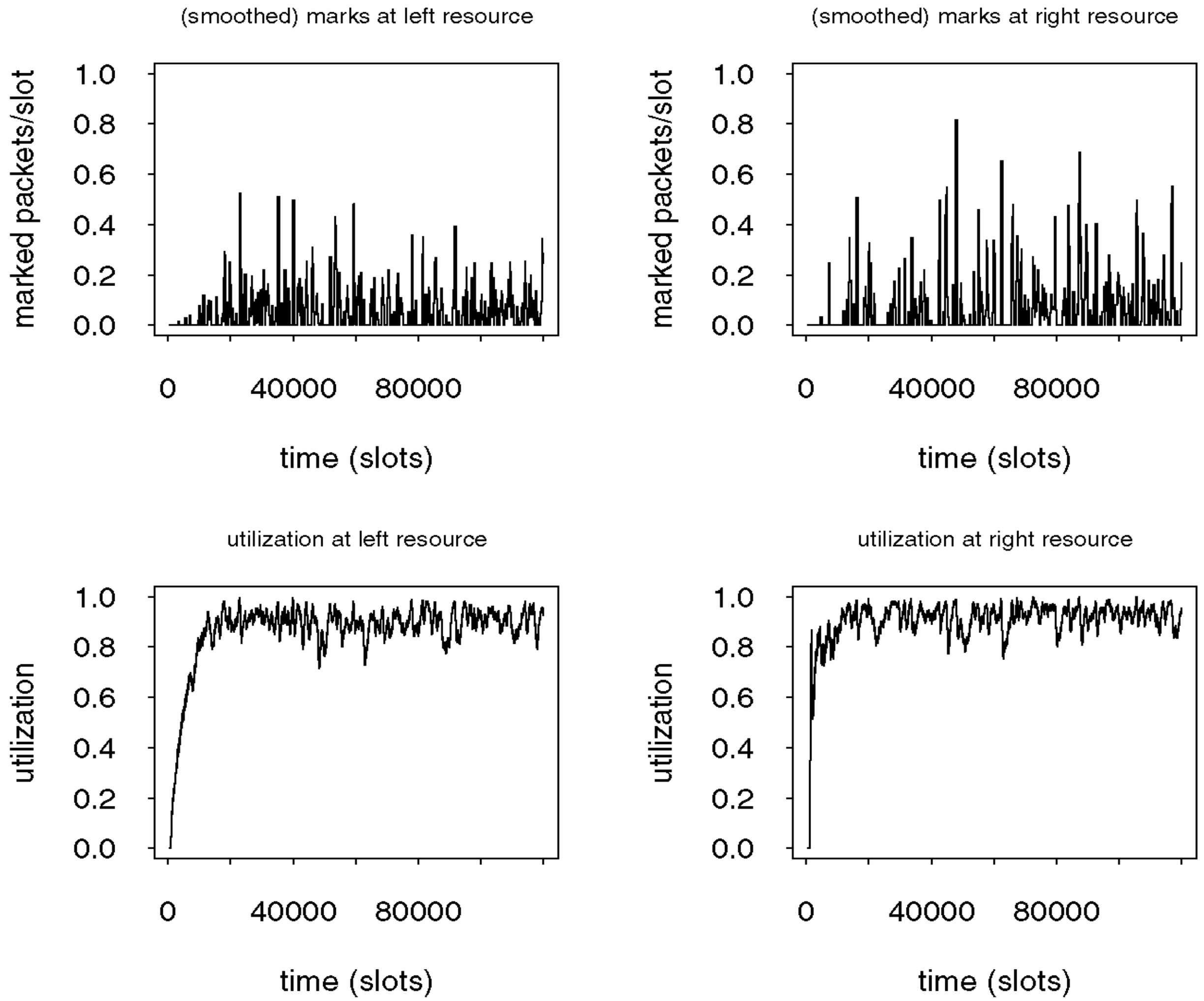
- Users adapt
- Turn on and off (geometric loads)
- Horizontal users have twice worth of vertical users

$$w \in (0.00002, 0.002)$$

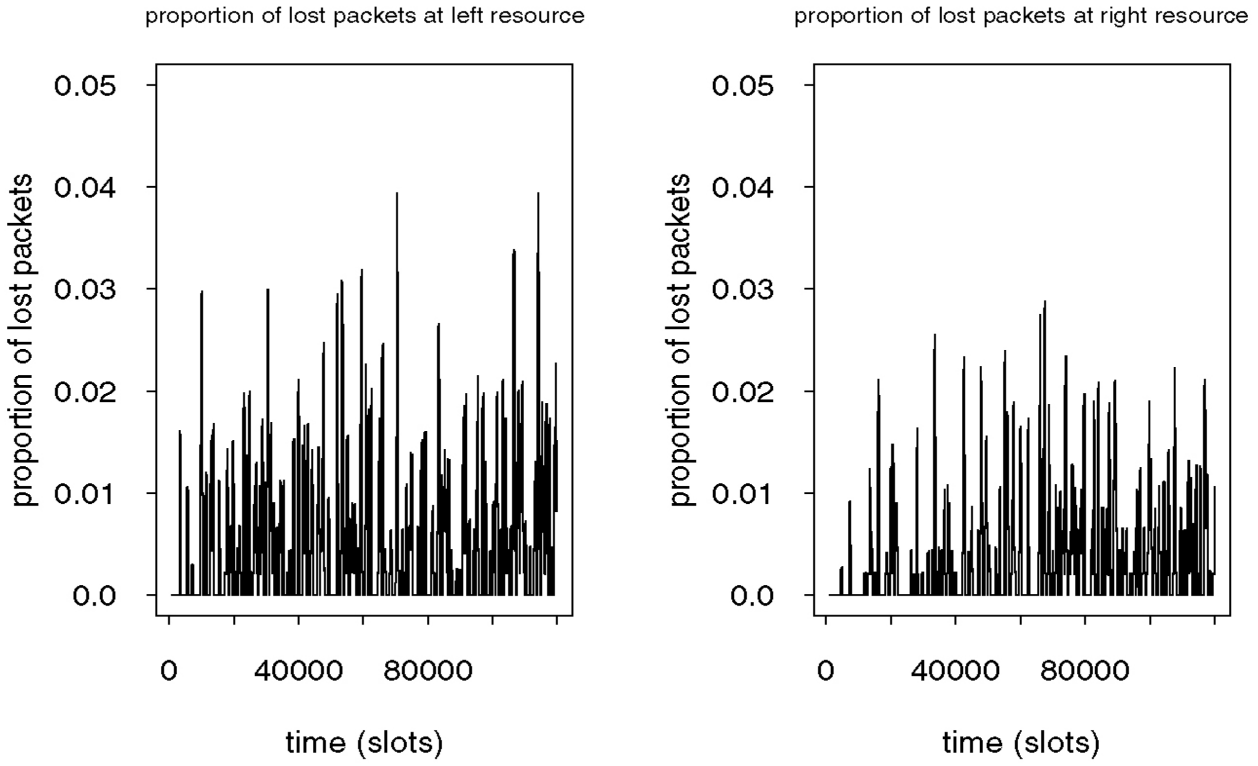
Number active



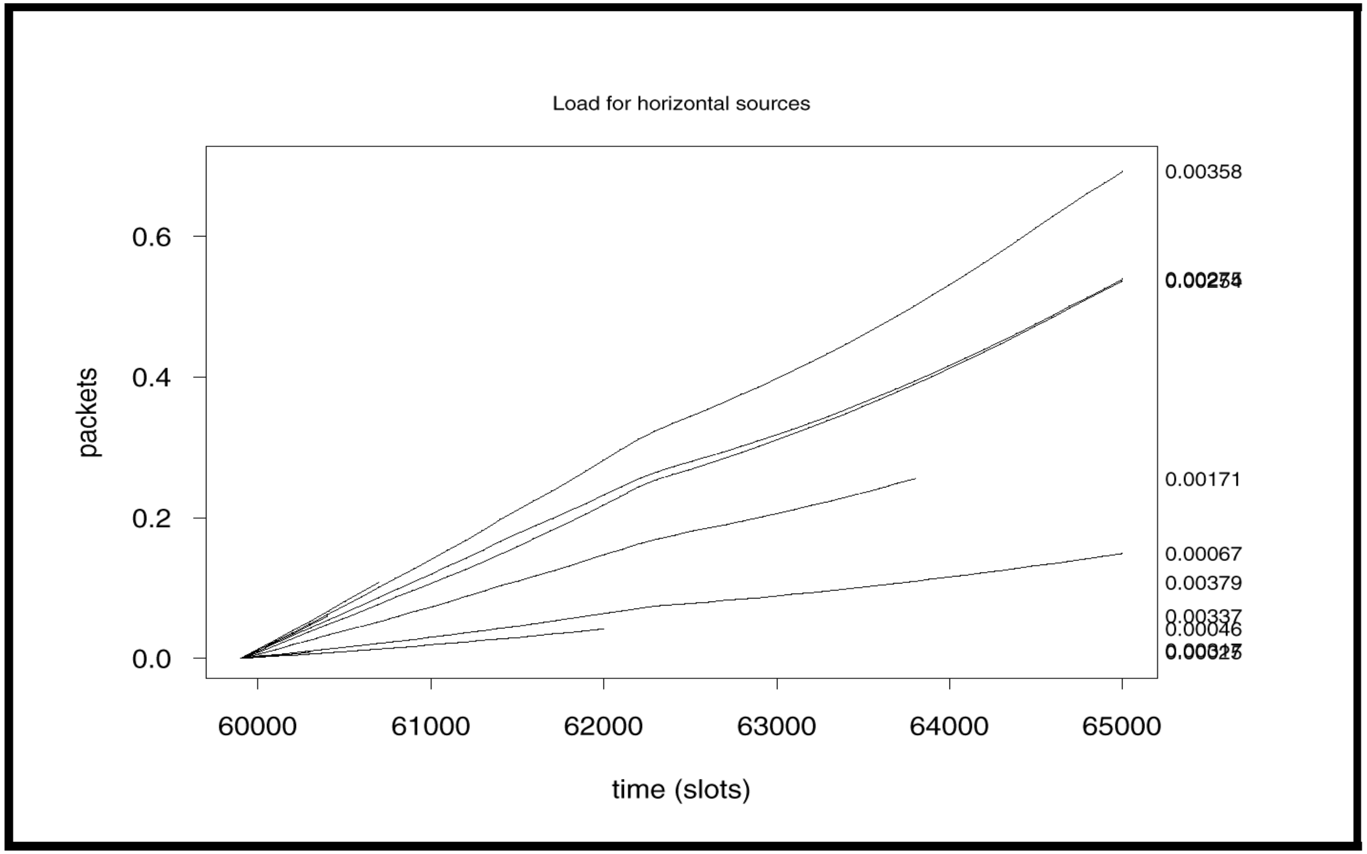
Marking & Utilisation



Lost packets



Througput vs *w*



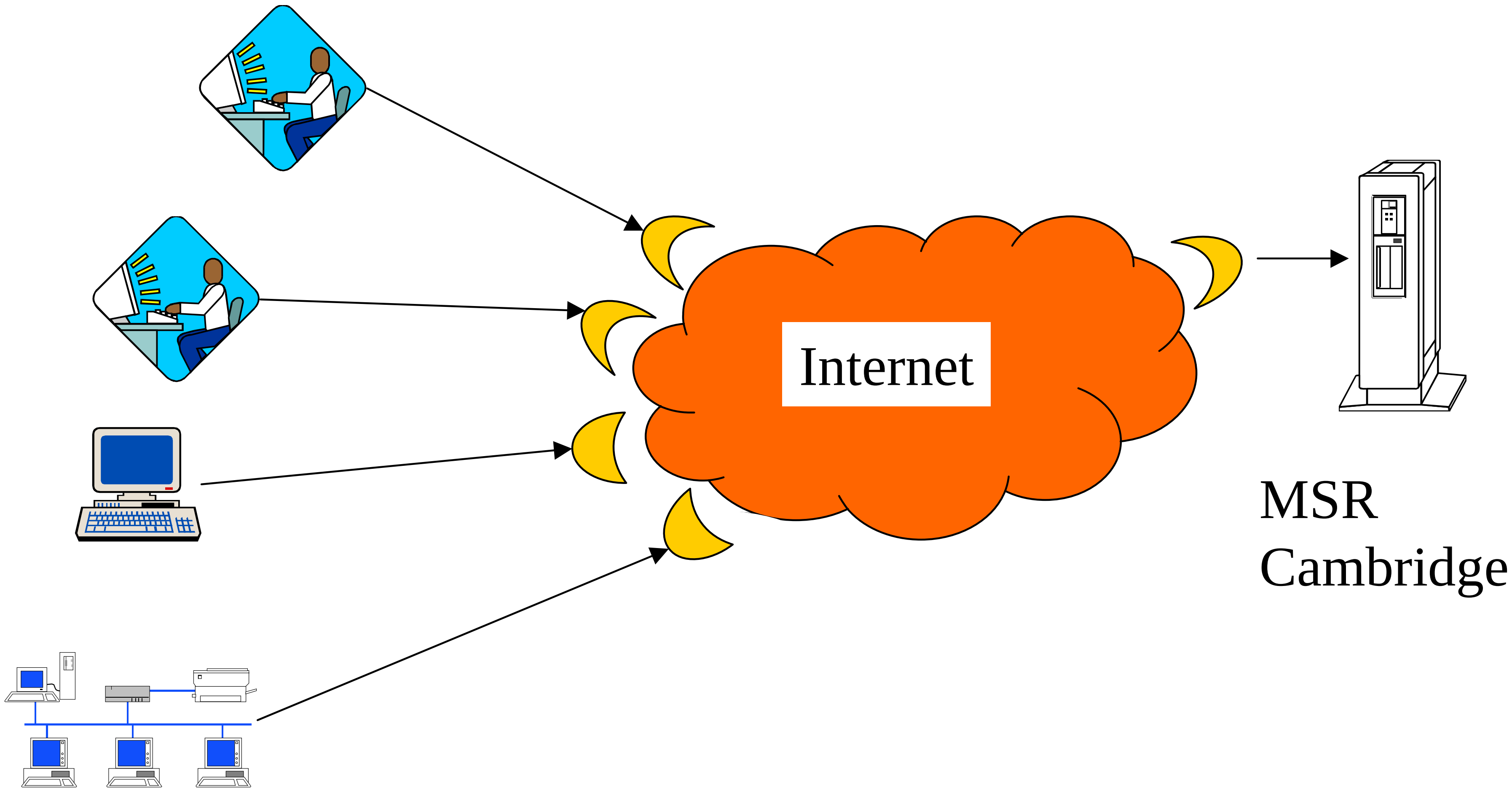
2-node summary

- Very low loss, despite small buffers
- Users who pay more get more!

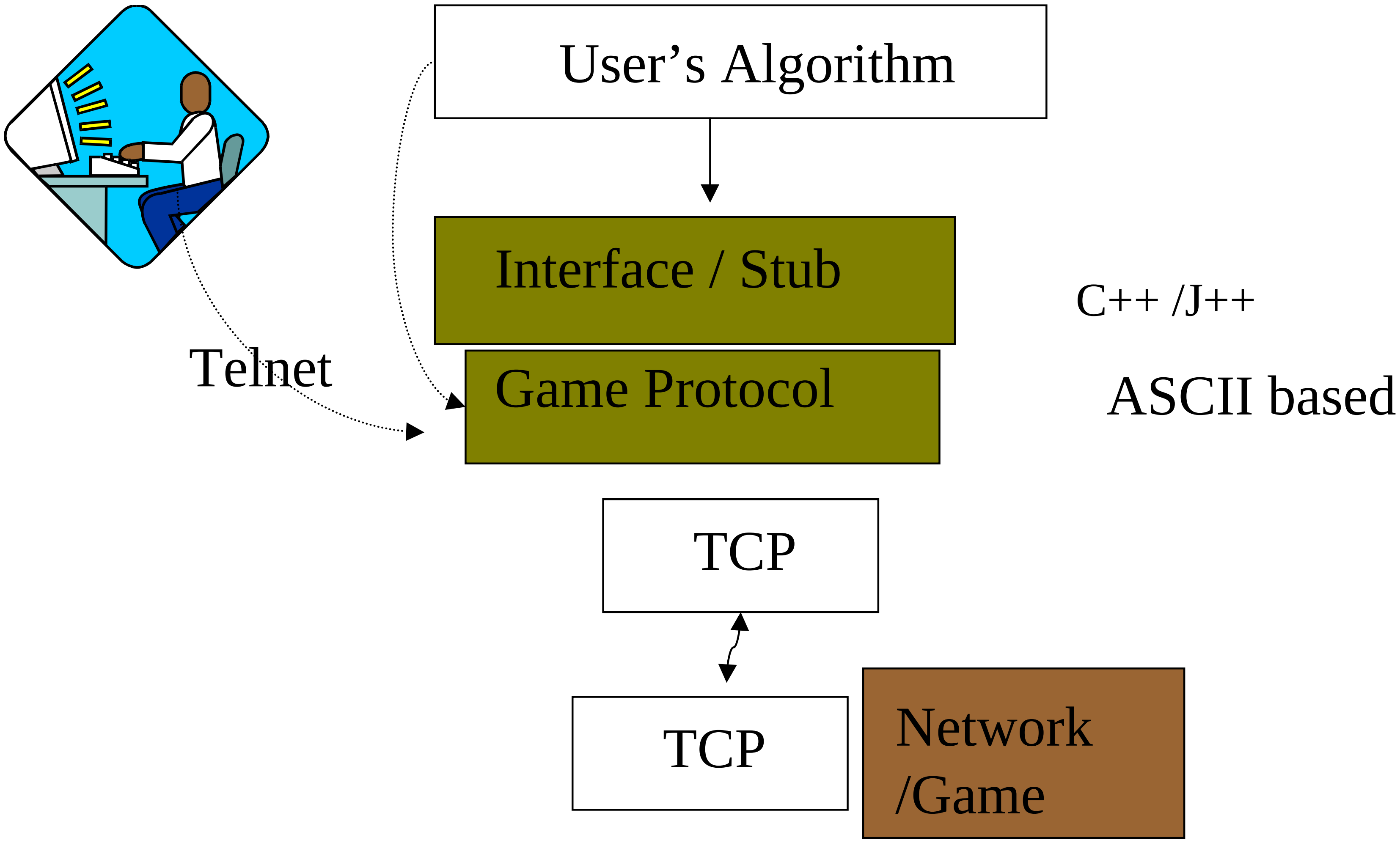
A framework for congestion pricing

- Underlying model very simple
  - Network sends congestion/pricing signals to users
  - Users can react as they wish
- Is this a rich enough framework?
  - Tested with simple models, and via a constrained Java competition
  - But test in a distributed game setting?

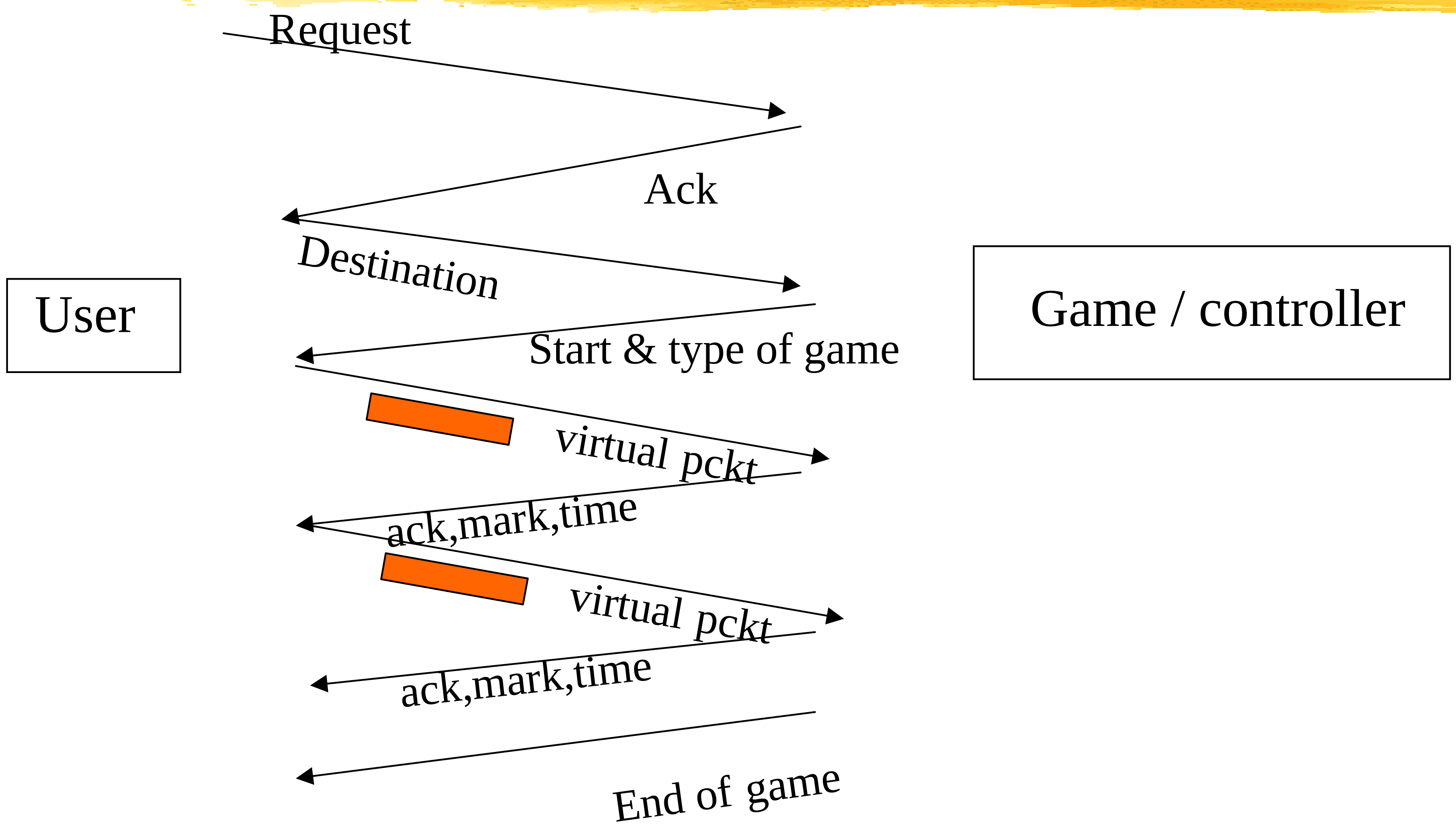
Distributed multi-player game



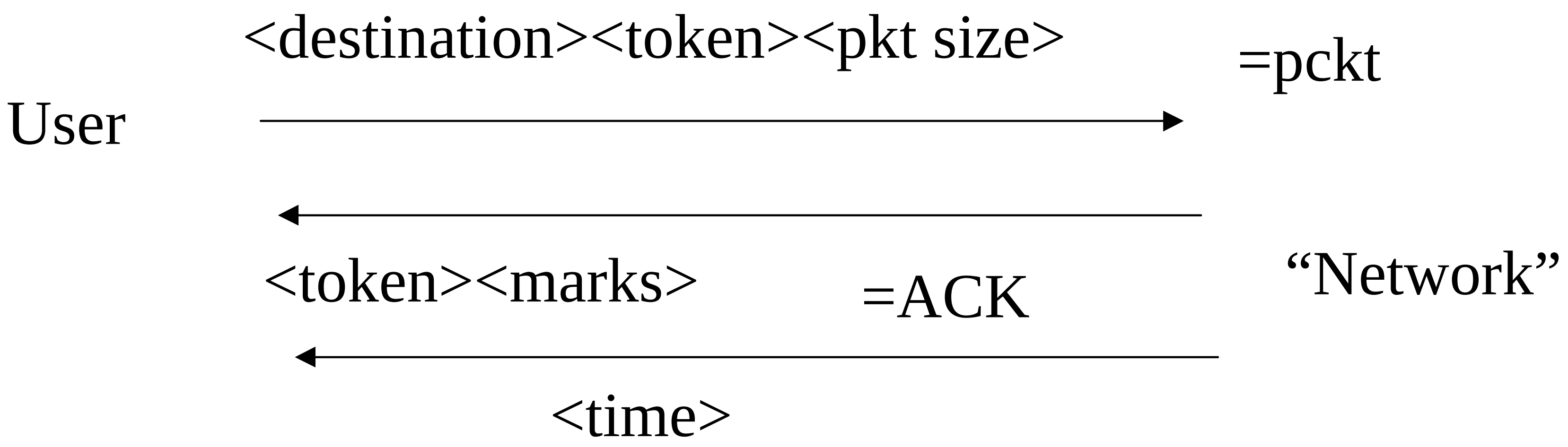
Protocol structures



Information flow



Protocol



Notes:

- *All single word (32bit) unsigned,*
- *pkt size integer*
- *marks integer*
- *Time=sec.μsec*
- *Token generated by user*
- *Corrupted/lost token=packet drop*

Example Objectives

Assumes notified cost per mark

- Maximise (ave. thruput - ave. cost)
- Max Discounted $\sum$(thruput - cost)
- For given utility function, max $\sum$(utility - cost)
- Transfer an amount of data *F*(file) at min cost
- Transfer *F* in set time *T* at min cost
- Transfer *F* as quickly as possible at min cost
- Given fixed budget, maximise transfer

Iterative Approach

- New User plays on test harness
- Plays against controlled load
  - (eg against copies of single game or against sample from random population)
- Plays against other users each with same objective
- Plays against others with multiple objectives

Disciplines

- Computer science
- Control Theory
- Game Theory / econometrics
- Stochastic Decision Theory
- Optimisation / Dynamic Programming

The future ...

- A rich class of differentiated services can be constructed from the simplest of frameworks
- Control shared between the user and the network
- β-version of software exists, will be available
- To see if it works… come and play the game!