Utility Analysis of Simple FEC Schemes for VoIP

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Abstract. Forward Error Correction (FEC) is an attractive solution for recovering from packet losses in real-time applications. However, in many such applications, due to delay constraints, complex FEC schemes that may enable to approach the Shannon channel capacity cannot be used. In those cases, much simpler FEC schemes are implemented, in which a compressed copy of a packet is concatenated to some subsequent packet. In this paper we analyze the utility gained by such schemes in the case where losses are due to queueing congestion in the network. Loss probabilities in presence of FEC are obtained using sophisticated tools based on Ballot theorems. Our explicit expressions allows us to study numerically several approaches for adding FEC.

1 Introduction

Forward Error Correction (FEC) is considered to be an attractive solution for recovering from packet losses in real-time applications (over high speed networks [8, 12]) without increasing latency. Though there are different proposals for FEC schemes, in this paper we shall focus on a simple FEC scheme that has been proposed [4] in the IETF (Internet Engineering Task Force) and implemented in audio tools like Freephone [1] and Rat [15]. The scheme consists in adding a redundant copy of the original packet to the tail of the subsequent packet. If a packet is lost in the network then it can be reconstructed from its redundant copy (if it is correctly received). It was suggested to use PCM coding for data in the original packet and a low bit rate coding (like LPC, GSM, or 2-bit ADPCM) for the redundant information. Proposals to enhance the performance of this simple FEC scheme include: (i) increasing the offset ϕ_0 between the original packet and its redundant copy [10, 9] at the codec, (ii) adding multiple redundant copies of a packet in multiple subsequent ones [10], (iii) adding to a packet a redundant copy computed from a block of some preceding packets [5] etc.

Let the offset between the original packet and its redundant copy at the bottleneck be ϕ. In a recent work [6], treating ϕ as a parameter it has been shown via a queueing analysis that the simple FEC scheme *may* not lead to an improvement in audio quality. The assumptions made in [6] include: (i) a single bottleneck link with dedicated buffer for audio flows implementing FEC and (ii) a linear utility function (A *utility function* is an indication of the variation of the audio quality at the receiver as a function of the transmission rate). The authors showed that for a buffer size K, the ratio of the volume of the redundant information to the volume of the original information α, a Poisson arrival of packets, exponential distribution of service times and for ϕ less than the *scaled* buffer size (in terms of number of packets) $K_\alpha = \frac{K}{(1+\alpha)}$, adding FEC according to the simple FEC scheme always leads to a deterioration in quality caused by increased load in the network due to redundant data. They further showed that the same is also true for the limiting case $\phi \to \infty$, which forms an upper bound on the audio quality.

In their subsequent work [7], the authors showed that improvement in quality is possible with the simple FEC scheme when: (i) the (total) rate of flow(s) adding FEC is small compared to the total rate of the other flow(s), called *exogenous flows* not adding FEC and sharing the same bottleneck and (ii) with particular non-linear utility functions. It was also concluded that the audio quality is always an increasing function of the offset ϕ between the original packet and its copy. They also provided lower and upper bounds on the audio quality for case (i) with Poisson arrivals of packets but with exponential distribution of packet sizes for exogenous flows and deterministic packet sizes for audio flows implementing FEC.

It was argued in [6] that $\phi > K_\alpha$ may not be interesting as it could lead to unacceptable delays at the receiver. But it should be noticed that the actual offset between the original packet and its redundant copy at the bottleneck could become larger than the offset generated at the transmission codec due to multiplexing of different flows (either implementing FEC or not) at the bottle-neck[3]. Moreover, in the case of large bandwidth, when many connections can be supported, a large offset does not generate necessarily a large delay, as the transmission time of a packet at the bottleneck is still very short.

In this paper we first complement the work in [6] and compute the performance for a fixed $\phi > K_\alpha$. It turns out that this case requires much more sophisticated computations than those in [6]. Using these results, we then compute the performances for the case of random offset obtained in situations where several flows are multiplexed. In particular, we obtain expressions for the expected audio quality for general utility functions. We use these results for an extensive numerical investigation using six different utility functions.

In this paper we deal with a Markovian framework: Poisson arrivals of packets and exponential distribution of all packet lengths and with the simple FEC scheme with $\phi_0 = 1$. We derive expressions for the quality function of an audio flow implementing FEC, multiplexed with: (i) other audio flows implementing FEC and/or (ii) exogenous flows not implementing FEC and different utility functions and Φ varying from 1 to ∞. These calculations will finally help us in the evaluation of the expected value of the quality function and the (possible) gain achievable with FEC in a more realistic framework of multiplexing of different flows.

2 Model

In this section we shall use a simple (same as in [6]) $M/M/1/K$ queue to model the network and thus the loss process of audio packets. In other words, we assume that packets arrive at the bottleneck [4] according to a Poisson process with intensity λ and the lengths of the packets are exponentially distributed with parameter μ. Let $\rho = \lambda/\mu$, be the intensity of the audio traffic and with $\rho < 1$, the loss probability of a packet in steady state is given by (see [11]), $\pi_{\rho,K} = \frac{1-\rho}{1-\rho^{K+1}}\rho^K$, and for $\rho = 1$ it is equal to, $\pi_{\rho,K} = \frac{1}{K+1}$.

Let $U(\alpha)$ be the utility function associated with receiving only a compressed version of a packet with a redundancy factor α, $0 \le \alpha \le 1$. We assume that $U(\alpha)$ is non-decreasing in α and $lim_{\alpha \to 0}U(\alpha) = 0$. Let Y_n [5] be a random variable

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[3] In presence of other multiplexed traffic, the offset ϕ is not deterministic anymore: it becomes a random variable Φ, taking values from ϕ_0 (the offset generated at the codec) to ∞.

[4] Most of the losses from a flow occur in the router having the smallest available bandwidth in the chain of routers, so that one may model the whole chain of routers by one single router called the *bottleneck*. This assumption has both theoretical and experimental [14, 2] justification.

[5] subscript n is for the nth packet of the total flow at the queue

defined as,

$$Y_n = \begin{cases} 0 & \text{if packet } n \text{ is lost} \\ 1 & \text{if packet } n \text{ is correctly received} \end{cases}$$

2.1 Case I: All Flows implementing FEC

We consider the case when an audio flow implementing FEC (we shall refer to this flow as the *tagged* flow) shares the bottleneck with other audio flows also implementing same type of FEC and arriving as independent Poisson processes. Let λ_a be the total arrival rate of all other audio flows at the bottleneck. For each flow, the redundant information of a packet is located in the next. Thus $j + 1$ packet of an audio flow will contain in addition to its own useful information some redundant information of the jth packet of the same flow. Let $\bar{\rho} = (\lambda + \lambda_a)/\mu$. We shall consider two scenarios concerning the influence of redundancy on the packet size.

- *Constant packet size model*: We assume that the size of a packet is not affected by adding redundancy and thus redundancy is a overhead. The more the redundancy the less is the *useful* information carried by the packet.
- *Constant amount of useful information*: We assume that the packet size increases when adding redundancy (and thus increases with α) so that the amount of useful information contained in a packet is unchanged. Increasing packet size has an impact both on the service times, and on the number of packets that can be stored at buffers. To account for this we assume [6]:
  - the service time is exponentially distributed with parameter $\mu_\alpha = \frac{\mu}{1+\alpha}$. Thus $\bar{\rho}_\alpha = \bar{\rho}(1 + \alpha)$.
  - The amount of buffering is diminished by $1 + \alpha$. Thus the scaled buffer size is $K_\alpha = \lfloor \frac{K}{1+\alpha} \rfloor$

  The loss probability in the presence of redundancy for this case is $\pi_{\bar{\rho}_\alpha, K_\alpha} = \frac{1-\bar{\rho}_\alpha}{1-(\bar{\rho}_\alpha)^{K_\alpha+1}}(\bar{\rho}_\alpha)^{K_\alpha}$.

In the following analysis we shall denote by $P_\alpha(.)$, the probabilities evaluated at μ_α and K_α. Observe that by taking $\alpha = 0$ in $P_\alpha(.)$ and $\pi_{\bar{\rho}_\alpha, K_\alpha}$ we get the corresponding expressions for the case of constant packet size as $K_0 = K$ and $\bar{\rho}_0 = \bar{\rho}$. Let $P_\alpha(Y_{n+\Phi} = 1, \Phi = \phi | Y_n = 0)$ be the probability that due to multiplexing there are $\phi - 1$ packets between two consecutive packets of the tagged audio flow at the bottleneck and that the $n + \phi$th packet is received correctly given that the nth packet is lost. Without loss of generality we assume that packet n belongs to the tagged audio flow. We define below the expected quality function $Q(\alpha)$ for the two cases.

- Constant packet size

$$Q(\alpha) = \tag{1}$$

$$P_0(Y_n = 1)U(1 - \alpha) + U(\alpha)P_0(Y_n = 0)\sum_{\phi=1}^{\infty} P_0(Y_{n+\Phi} = 1, \Phi = \phi | Y_n = 0)$$

$$= U(1 - \alpha) - \pi_{\bar{\rho}, K}(U(1 - \alpha) - U(\alpha)\sum_{\phi=1}^{\infty} P_0(Y_{n+\Phi} = 1, \Phi = \phi | Y_n = 0)).$$

- Constant amount of useful information: as the amount of useful information carried by a packet remains unchanged we define

$$Q(\alpha) = \tag{2}$$

$$P_\alpha(Y_n = 1)U(1) + U(\alpha)P_\alpha(Y_n = 0)\sum_{\phi=1}^{\infty} P_\alpha(Y_{n+\Phi} = 1, \Phi = \phi|Y_n = 0)$$

$$= U(1) - \pi_{\bar{\rho}_\alpha, K_\alpha}(U(1) - U(\alpha)\sum_{\phi=1}^{\infty} P_\alpha(Y_{n+\Phi} = 1, \Phi = \phi|Y_n = 0)).$$

We will next find the complementary probability $P_\alpha(Y_{n+\Phi} = 0, \Phi = \phi|Y_n = 0)$. In steady state we need to evaluate the probability $P_\alpha(Y_\Phi = 0, \Phi = \phi|Y_0 = 0)$ which in terms of the buffer level is $P_\alpha(X_\Phi = K_\alpha, \Phi = \phi|X_0 = K_\alpha)$, where X_k is the queue size as seen by an arriving kth packet. We will now evaluate $P_\alpha(X_\Phi = K_\alpha, \Phi = \phi|X_0 = K_\alpha)$ for $\phi = \{1, 2, \ldots\}$ using results from ballot theorem.

Let A_0 be the event of the loss of the 0th packet and A_ϕ the event of the loss of the ϕth packet. We consider the following cases:

- Case 1: no packet is lost in the interval between the happening of A_0 and $A_\phi \Rightarrow$ the first lost packet after the 0th packet is the ϕth packet.
- Case 2: $i(1 \leq i \leq \phi - 1)$ packets are lost between A_0 and A_ϕ.

We consider the two cases separately. The main difficulty in the case when the redundancy-offset is larger than the (scaled) buffer, i.e., $\phi > K_\alpha$ is that paths with this property may cause the buffer to become empty (even several times). This makes the analysis substantially more complex than in [6].

To proceed we first consider a general path that starts when there are K_α packets in the buffer, ends with K_α packets in the buffer, contains η events (arrivals of Poisson process with parameter $\lambda + \lambda_a$ and departures) and no packets are lost. Let ζ_η be the probability of such a path. We will now express $P_\alpha(X_\Phi = K_\alpha, \Phi = \phi|X_0 = K_\alpha)$ for the two cases in terms of this probability.

- Case 1: for the ϕ packet to be the first lost packet there must be an epoch after the arrival of the $\phi - 1$ packet that the buffer is full and the next event is an arrival. Upto that epoch the number of departures must be equal to the number of arrivals. The number of arrivals upto that epoch is $\phi - 1$ and so the number of departures is $\phi - 1$ to yield a total number of $\eta = 2(\phi - 1)$ events. Thus the probability for this case is: $\zeta_{2(\phi-1)}\frac{\lambda}{\lambda+\lambda_a+\mu_\alpha}$. However we also require that all the $\phi - 1$ arrivals should be from other audio flows (coming as a Poisson process with intensity λ_a). Hence the joint probability that there are $\phi - 1$ packets from other flows (due to multiplexing) at the bottleneck and the ϕth packet is lost is $\zeta_{2(\phi-1)}\frac{\lambda}{\lambda+\lambda_a+\mu_\alpha}\left(\frac{\lambda_a}{\lambda_a+\lambda}\right)^{\phi-1}$.
- Case 2: First we look at $i = 1$. Let the lost packet be the jth arrival $(1 \leq j \leq \phi - 1)$. Thus there should be no loss from the instant of the loss of 0th packet till the epoch before the arrival of the jth packet. Since the queue size is K_α at A_0 and since the jth packet is the first to be lost, there must be an epoch after the arrival of the $j - 1$ packet that the buffer is again full and the next event is an arrival. Again, upto this epoch, the number of departures must be equal to the number of departures. The number of arrivals upto that epoch is $j - 1$, so the number of departures is $j - 1$ yielding a total number of $\eta = 2(j - 1)$. Thus the probability of the path is: $\zeta_{2(j-1)}\frac{\lambda_a}{\lambda+\lambda_a+\mu_\alpha}\zeta_{2(\phi-j-1)}\frac{\lambda}{\lambda_a+\lambda+\mu_\alpha}$. Also, since we require that all the $\phi - 1$ arrivals should be from other audio flows we have the probability of the path as $\zeta_{2(j-1)}\frac{\lambda_a}{\lambda+\lambda_a+\mu_\alpha}\zeta_{2(\phi-j-1)}\frac{\lambda}{\lambda_a+\lambda+\mu_\alpha}\left(\frac{\lambda_a}{\lambda+\lambda_a}\right)^{\phi-2}$ Since j can take values from 1 to $\phi - 1$, the total probability for this case is $\left(\frac{\lambda_a}{\lambda+\lambda_a}\right)^{\phi-2}\sum_{j=1}^{\phi-1}\zeta_{2(j-1)}\frac{\lambda_a}{\lambda+\lambda_a+\mu_\alpha}\zeta_{2(\phi-j-1)}\frac{\lambda}{\lambda+\lambda_a+\mu_\alpha}$. Now consider the case when $i = 2$. We will first calculate the probability of a single path with two losses before the loss of the ϕ packet. Let the

first loss be of j_1 and second of j_2 packet ($j_1 < j_2, 1 \le j_1 \le \phi - 2, 2 \le j_2 \le \phi - 1$). By previous arguments the probability of such a path is: $\zeta_{2(j_1-1)}\zeta_{2(j_2-j_1-1)}\left(\frac{\lambda_a}{\lambda+\lambda_a+\mu_\alpha}\right)^2\zeta_{2(\phi-j_2-1)}\frac{\lambda}{\lambda+\lambda_a+\mu_\alpha}$. The total probability for this case is (taking into consideration the requirement that all the $\phi - 1$ packets should be from other audio flows):

$$\left(\frac{\lambda_a}{\lambda+\lambda_a}\right)^{\phi-3}\sum_{j_1=1}^{\phi-2}\sum_{j_2=j_1+1}^{\phi-1}\zeta_{2(j_1-1)}\zeta_{2(j_2-j_1-1)}\zeta_{2(\phi-j_2-1)}\frac{\lambda\lambda_a}{(\lambda+\lambda_a+\mu_\alpha)^3}.$$

In general for $i = k (1 \le k \le \phi - 1)$, we can write the probability as:

$$\left(\frac{\lambda_a}{\lambda+\lambda_a}\right)^{\phi-(k+1)}\sum_{j_1=1}^{\phi-k}\sum_{j_2=j_1+1}^{\phi-k+1}\cdots\sum_{j_k=j_{k-1}+1}^{\phi-1}\zeta_{2(j_1-1)}\zeta_{2(j_2-j_1-1)}\cdots$$

$$\cdots\zeta_{2(j_k-j_{k-1}-1)}\zeta_{2(\phi-j_k-1)}\left(\frac{\lambda_a}{\lambda+\lambda_a+\mu_\alpha}\right)^k\frac{\lambda}{\lambda+\lambda_a+\mu_\alpha}.$$

Thus to complete the analysis we need to evaluate the quantity ζ_η where $\eta = 2, 4, 6, \ldots$. To evaluate this we use the result of [13] which is based on ballot theorems. In this paper the authors have obtained the following general expression for $\zeta_\eta(\iota, v)$, i.e., the probability of a path that starts with $\iota(1 \le \iota \le K_\alpha)$ packets in the buffer ends with $v(1 \le v \le K_\alpha)$ packets in the buffer and contains $\eta(0 \le \eta \le 2\phi + \iota - v)$ events (arrivals and departures) and no packets are lost:

$$\zeta_\eta(\iota, v) = \xi_\eta(\iota, v) + \sum_{r=1}^{\mathcal{R}}W_\iota Y^{r-1}[Z(\iota, v)],$$

where $\xi_\eta(\iota, v)$ is the probability of a path that starts with ι packets in the buffer ends with v packets in the buffer and contains η events ($| \iota - v | \le \eta \le 2n + \iota - v$, $1 \le \iota \le K_\alpha$, $1 \le v \le K_\alpha$) and the buffer never empties along this path, $\mathcal{R} = 1 + \frac{\eta-\iota-v}{2}$, W_ι and Z are $\mathcal{R}$-length row and Y is an $\mathcal{R}X\mathcal{R}$ matrix defined as

$$W_\iota = \left(\xi_\iota(\iota, 0), \xi_{\iota+2}(\iota, 0), \ldots, \xi_{\iota+2(\mathcal{R}-1)}(\iota, 0)\right),$$
$$Z(\iota, v) = \left(\xi_{\eta-\iota}(0, v), \xi_{\eta-\iota-2}(0, v), \ldots, \xi_{\eta-\iota-2(\mathcal{R}-1)}(0, v)\right),$$

$$Y = \begin{pmatrix} 0 & \xi_2(0,0) & \xi_4(0,0) & \ldots & \xi_{2(\mathcal{R}-1)}(0,0) \\ 0 & 0 & \xi_2(0,0) & \ldots & \xi_{2(\mathcal{R}-2)}(0,0) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & \xi_2(0,0) \\ 0 & 0 & 0 & \ldots & 0 \end{pmatrix}.$$

For our analysis we need $\zeta_\eta(= \zeta_\eta(K_\alpha, K_\alpha))$. Thus we get,

$$\zeta_\eta(K_\alpha, K_\alpha) = \zeta_\eta = \xi_\eta(K_\alpha, K_\alpha) + \sum_{r=1}^{\mathcal{R}}W_{K_\alpha}Y^{r-1}Z(K_\alpha, K_\alpha),$$

where $\mathcal{R} = 1 + \frac{\eta - 2K_\alpha}{2} = 1 + \frac{\eta}{2} - K_\alpha$. The steps for evaluation of $\xi_\eta(\iota, v)$ are available in [13]. Thus we can write $P_\alpha(X_\Phi = K_\alpha, \Phi = \phi | X_0 = K_\alpha)$ as

$$P_\alpha(X_\Phi = K_\alpha, \Phi = \phi | X_0 = K_\alpha) = \tag{3}$$

$$\zeta_{2(\phi-1)} \frac{\lambda}{\lambda_a + \lambda + \mu_\alpha} \left(\frac{\lambda_a}{\lambda + \lambda_a} \right)^{\phi-1} + \frac{\lambda}{\lambda_a + \lambda + \mu_\alpha}$$

$$\sum_{k=1}^{k=\phi-1} \left[\sum_{j_1=1}^{\phi-k} \sum_{j_2=j_1+1}^{\phi-k+1} \cdots\cdots \sum_{j_k=j_{k-1}+1}^{\phi-1} \zeta_{2(j_1-1)} \zeta_{2(j_2-j_1-1)} \cdots \right.$$

$$\left. \cdots \zeta_{2(j_k-j_{k-1}-1)} \zeta_{2(\phi-j_k-1)} \left(\frac{\lambda_a}{\lambda_a + \lambda + \mu_\alpha} \right)^k \left(\frac{\lambda_a}{\lambda + \lambda_a} \right)^{\phi-(k+1)} \right].$$

Thus knowing $P_\alpha(X_\Phi = K_\alpha, \Phi = \phi | X_0 = K_\alpha)$ we have $P_\alpha(Y_\Phi = 0, \Phi = \phi | Y_0 = 0)$ and thus we have $P_\alpha(Y_\Phi = 1, \Phi = \phi | Y_0 = 0) = P_\alpha(\Phi = \phi | Y_n = 0) - P_\alpha(Y_\Phi = 0, \Phi = \phi | Y_0 = 0) = P_\alpha(\Phi = \phi) - P_\alpha(Y_\Phi = 0, \Phi = \phi | Y_0 = 0, 1 \leq \phi < \infty$. We next find the distribution of Φ. Observe that $\Phi - 1$ is the number of packets of other flows (other audio flows arriving as an independent Poisson process with parameter λ_a). Thus Φ is geometrically distributed: $P_\alpha(\Phi = n) = \gamma^{n-1}(1 - \gamma)$ for $n \geq 1$, with $\gamma = \gamma_1 = \frac{\lambda_a}{\lambda + \lambda_a}$. Knowing this we can evaluate the expected quality function $Q(\alpha)$ using (1),(2).

2.2 Case II: Multiplexing between audio flows implementing FEC and exogenous flows for the constant packet size case

The case when an audio flow implementing FEC shares the bottleneck with other audio flows also implementing the same FEC scheme and with some other exogenous flows not implementing FEC can be analysed similarly for the case of constant packet size. We model the packets of exogenous arriving as an independent (from audio flows) Poisson process with intensity λ_e and hence the total arrival process is again a Poisson process with parameter $\lambda + \lambda_a + \lambda_e$. Further we assume that the distribution of the size of all packets of all flows is exponential with parameter μ. With $\bar{\rho} = \frac{\lambda + \lambda_a + \lambda_e}{\mu}$, the loss probability of a packet is steady state (with $\bar{\rho} < 1$) is $\pi_{\bar{\rho}} = \frac{1-\bar{\rho}}{1-\bar{\rho}^{K+1}} \bar{\rho}^K$, and for $\bar{\rho} = 1$ it is equal to $\pi_{\bar{\rho}} = \frac{1}{(1+K)}$. The expressions for expected quality can be obtained from the expressions for the case without exogenous but with λ_a replaced by $\lambda_a + \lambda_e$ everywhere. For the case of constant amount of useful information it is not clear how we should scale the service rate and the buffer size for this scenario.

3 Numerical Examples with Different Utility Functions

We will next evaluate the expected quality achieved using different utility functions. The utility functions that we consider are used in [6, 7] and are of the form proposed in [10, 16]. We define: $U_1(\alpha) = \alpha$, $U_2(\alpha) = \sqrt{\alpha}$, $U_3(\alpha) = \alpha^{1/10}$, $U_4(\alpha) = esc(\alpha - \alpha_0) \left(\frac{1-\cos(\pi\alpha)}{2} \right)^{1/10}$, $U_5(\alpha) = \left(\frac{1-\cos(\pi\alpha)}{2} \right)^{1/3}$ and $U_6(\alpha) = 1$ if $\alpha > 0$ otherwise $U_6(\alpha) = 0$. All utilities are concave starting at some minimum and reach $U(\alpha) = 1$ for $\alpha = 1$. The least concave is the linear utility function U_1, which is thus proportional to the amount of information which is well received. The most concave function is U_6. Although it does not represent a utility of any real application, it can be used to obtain an upper bound on the gain achieved with employing FEC. Indeed, we see that U_6 is larger than any other

utility, so it follows that in both models described by (1) and (2), it should give the best quality. The utility U_4 is zero for $\alpha \leq \alpha_0$. This is typical for real time applications with a minimum hard constraint. For example, the constraint may represent the fact that the throughput of existing codecs for voice applications cannot go beneath some bound. We shall plot the quality (expected) function with these six utility functions under different scenarios. For the plots we denote by M the actual buffer size and by ρ that total load. Also note that **in all the plots that follow the utility functions are represented as:** $* - U_1$, $+ - U_2$, $. - U_3$, $o - U_4$, $- U_5$ **and finally** $- - U_6$.

1. *A single audio flow implementing FEC traversing the bottleneck (no multiplexing)*: When a single audio flow traverses the bottleneck (i.e., $\lambda_a = \lambda_e = 0$), then the spacing between a packet and its redundancy at the bottleneck is the same as that generated at the transmission codec. We take $\lambda = 1$ and $\rho = \lambda / \mu$. We next plot the quality function with the six utility functions for this scenario and for the cases when: (i) the packet sizes (and hence the buffer sizes) remains unchanged after adding redundancy and the quality function is given by (1) in Figs. 1, 2 (ii) when the packet sizes changes after adding redundancy and the quality function is given by (2) in Figs. 3, 4.
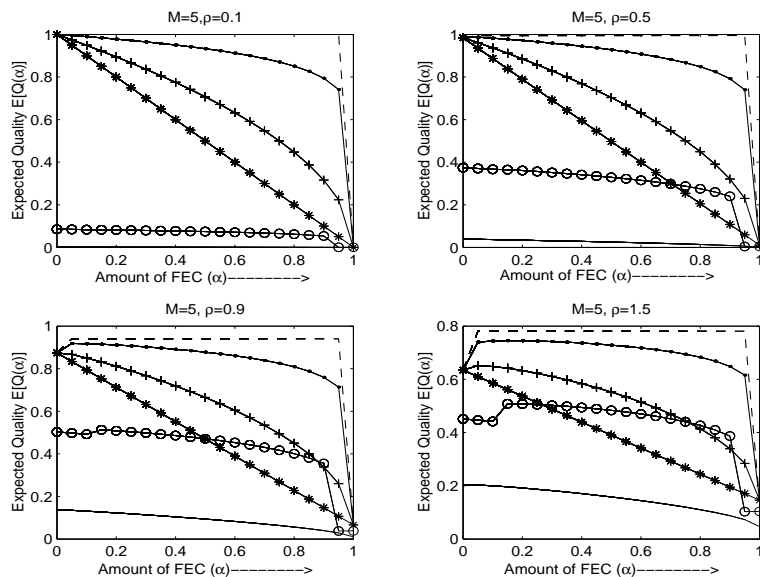

Fig. 1. A single audio flow implementing simple FEC traversing the bottleneck with quality function given by (1).

We observe that
 – as expected, U_6 gives an upper bound for the quality. More generally, for two utility functions, U_j and U_i, if $U_i \geq U_j$ for all α, then the corresponding quality is larger for both cases (1) and (2). This is confirmed in the figures, taking into account that $U_6 \geq U_3 \geq U_2 \geq U_1$.
 – For large buffer size ($M = 500$) the quality is almost the same for all $\rho \leq 0.9$; the reason is that the loss probabilities are very small and almost all contribution for the quality is from the utility of unlost packets.
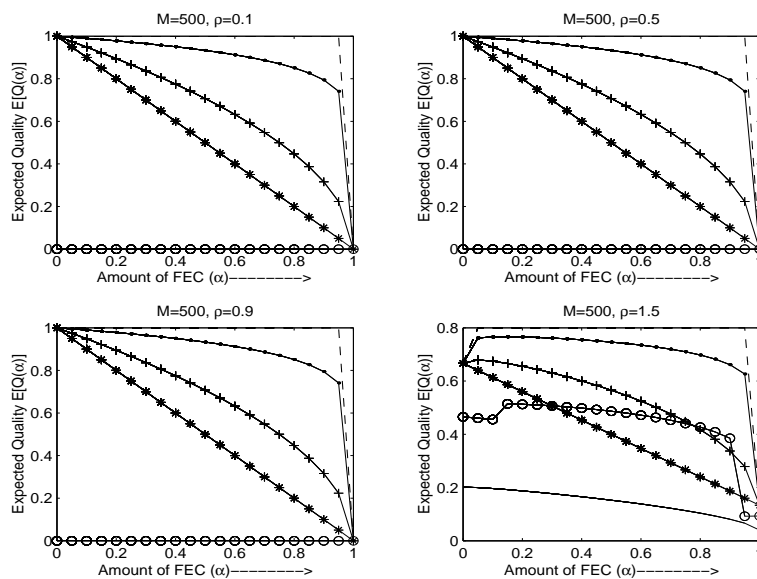
Fig. 2. A single audio flow implementing simple FEC traversing the bottleneck with quality function given by (1).
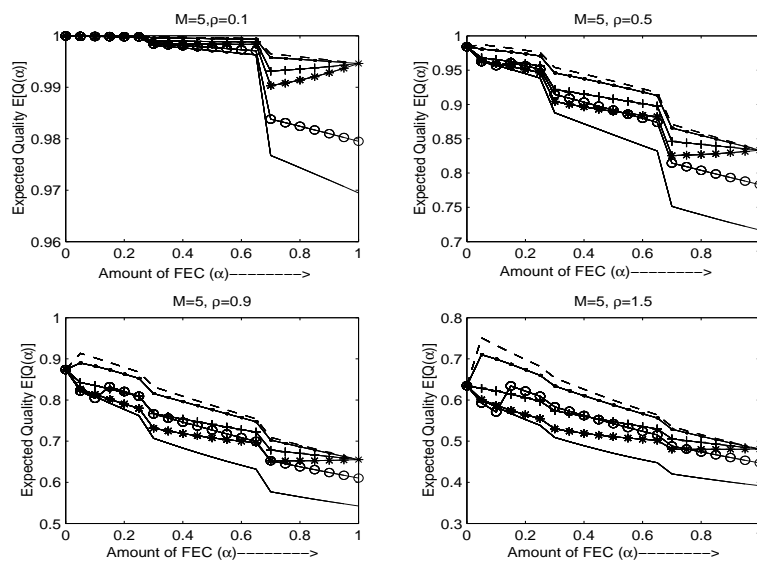

Fig. 3. A single audio flow implementing simple FEC traversing the bottleneck with quality function given by (2).
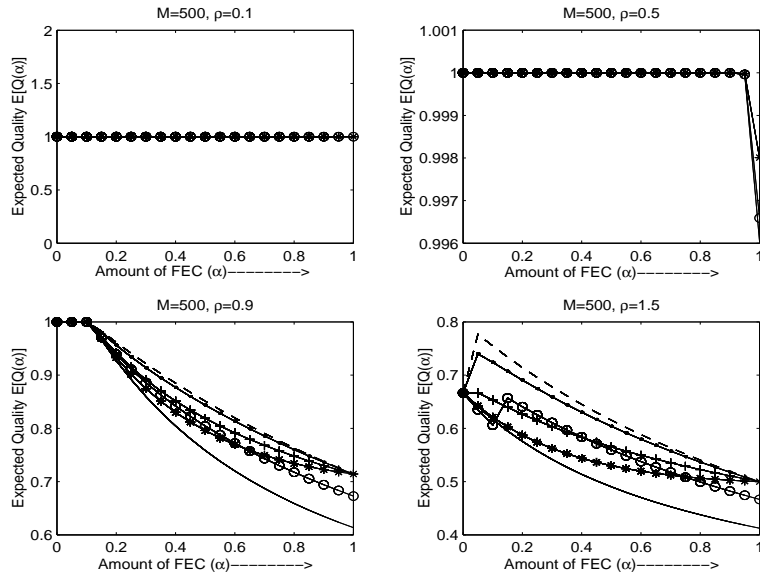
Fig. 4. A single audio flow implementing simple FEC traversing the bottleneck with quality function given by (2).

- For $\rho > 1$ we see that we gain by adding FEC (for any buffer size, and for both cases described by (1) and (2)), for utilities U_3, U_4, U_6. For small buffer size $M = 5$ we gain also for $\rho = 0.9$ when using utility U_3, U_4, U_6 for both the cases but for very low α, 0.1 or less.
- The linear utility function always decreases with FEC for any ρ and any M.
- The quality is higher with constant information model than with constant packet size model.

2. *An audio flow implementing simple FEC sharing the bottleneck with other audio flows implementing the same FEC:* This is the case I (Sec. 2.1) of our analysis. We take $\lambda = 0.1$ and $\lambda_a = 0.9$ and $\rho = (\lambda + \lambda_a)/\mu$. We plot the *expected* quality function with the six utility functions for this scenario and for the cases when: (i) the packet sizes (and hence the buffer sizes) remains unchanged after adding redundancy and the quality function is given by (1) in Figs. 5, 6 (ii) when the packet sizes changes after adding redundancy and the quality function is given by 7, 8. Due to multiplexing Φ takes values in $\{1, 2, \ldots\}$. For our numerical calculations we restricted to $\phi = \{1, 2, \ldots, 8\}$ as for $\phi \geq 9$, the contribution to $Q(\alpha)$ was negligible. Thus for buffer size of 5 we will have the spacing exceeding the buffer size. Observe that the plots (5) and (6) are almost similar to plots (1) and (2) respectively. From (7) and (8) We observe that for large $\rho(> 1)$, the buffer size does not affect the expected quality.
Also from all the plots it is observed that U_5 always gives a lower bound on expected quality for large $\alpha(\geq 0.2)$.

4   Conclusion

In this paper we studied the (possible) gain obtained with a simple FEC scheme when the losses are due to buffer overflow at the bottleneck. We obtained the
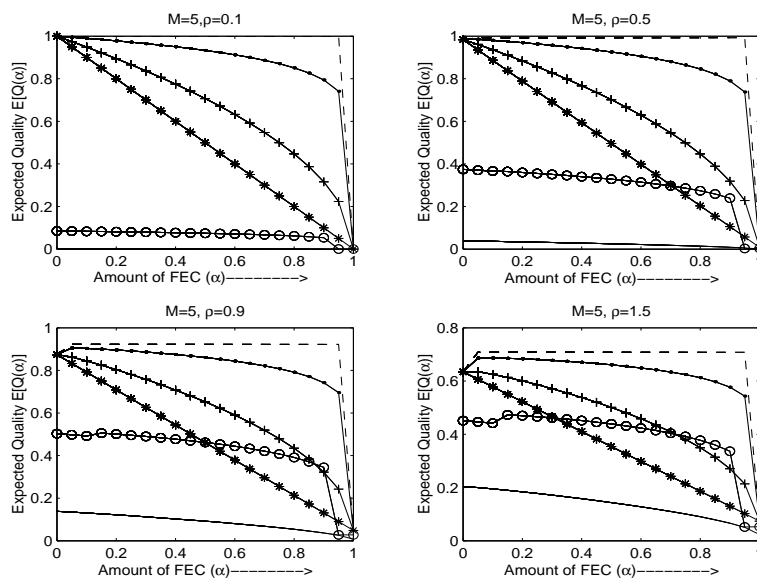
Fig. 5. An audio flow implementing simple FEC sharing the bottleneck with other audio flows implementing the same FEC scheme with quality function given by (1).
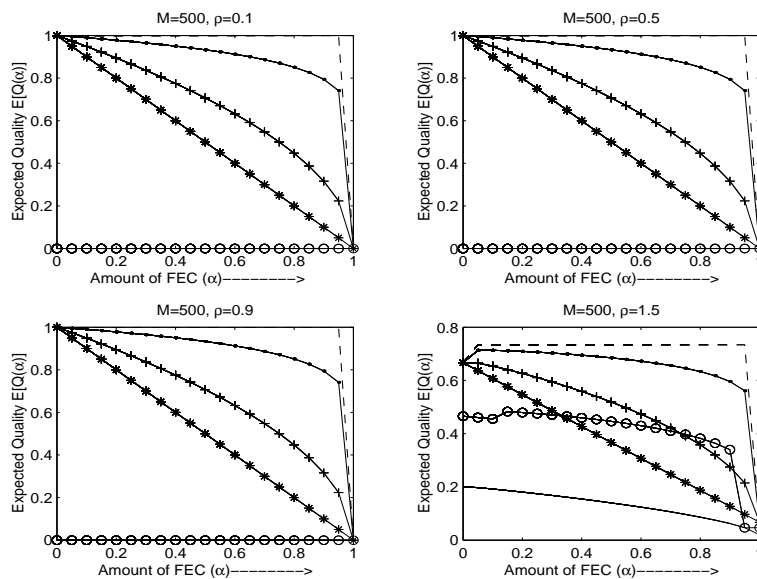

Fig. 6. An audio flow implementing simple FEC sharing the bottleneck with other audio flows implementing the same FEC scheme with quality function given by (1).
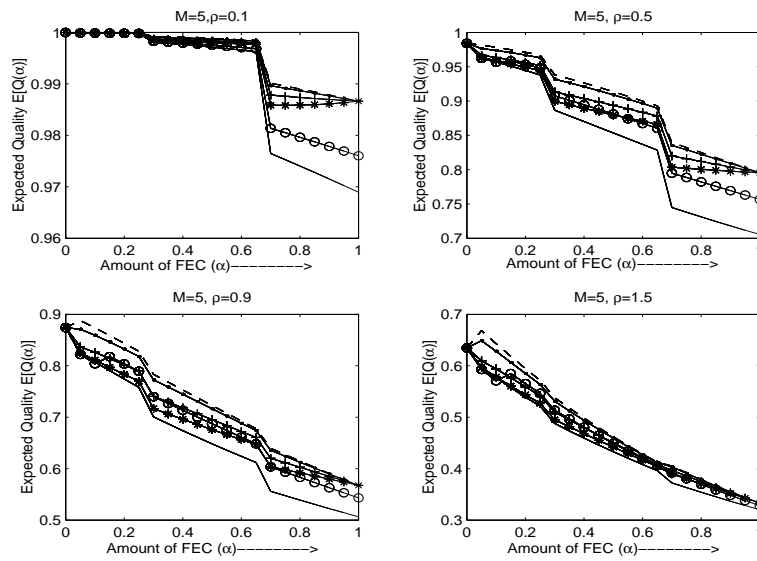
Fig. 7. An audio flow implementing simple FEC sharing the bottleneck with other audio flows implementing the same FEC scheme with quality function given by (2).

Fig. 8. An audio flow implementing simple FEC sharing the bottleneck with other audio flows implementing the same FEC scheme with quality function given by (2).
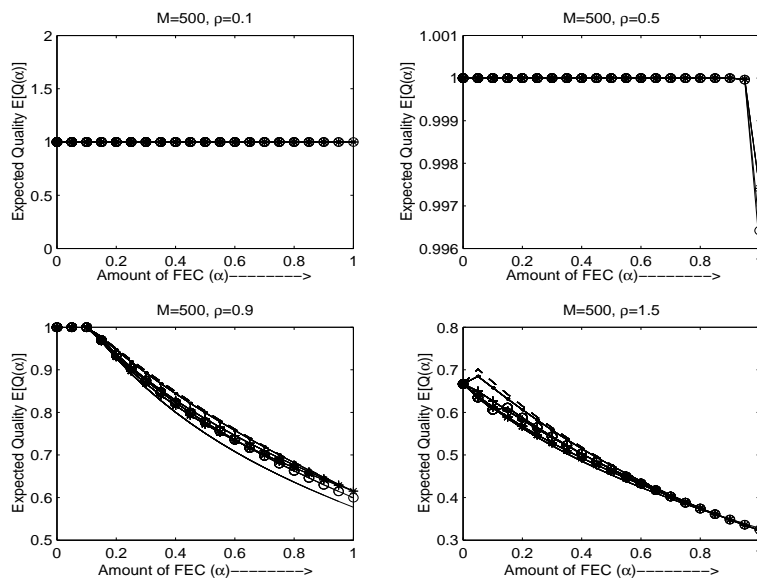
loss probabilities in the presence of FEC using ballot theorem. To this end we generalize the analysis in [6] (which was for $\phi < K_\alpha$) and computed the loss probability for a fixed ϕ taking values from 0 to ∞. Using these results we obtained the expressions for the expected audio quality for general utility functions and then utilised the tools developed for a detailed numerical studies with six utility functions under various scenarios of multiplexing at the bottleneck. Our future work is to analyse delay aware utility functions [3] and to do utility analysis of other more intelligent and efficient FEC schemes to quantize the (possible) gain.

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