Brane Calculi

Presented by Jesús F. Almansa

jfa@brics.dk

Mobile Calculi Course

BRICS, University of Aarhus

December 2004

Motivation

Biological Systems

- Imprecise descriptions
- Complex...

Need to be formalized.

Motivation

Biological Systems

- Imprecise descriptions
- Complex...

Need to be formalized.
In particular, membranes have their own dynamics.

Motivation

Biological Systems

- Imprecise descriptions
- Complex...

Need to be formalized.
In particular, membranes have their own dynamics.

- Motivation
- Design

Motivation

Biological Systems

- Imprecise descriptions
- Complex...

Need to be formalized.
In particular, membranes have their own dynamics.

- Motivation
- Design

Previous work:
P-System: dismatch with reality
BioSpy: calculate with molecules
BioAmbients: calculate with molecules, add membranes
Brane Calculi: calculate *on* membranes

The Papers

Bitonal Membrane Systems,
Interactions of Biological Membranes
Luca Cardelli

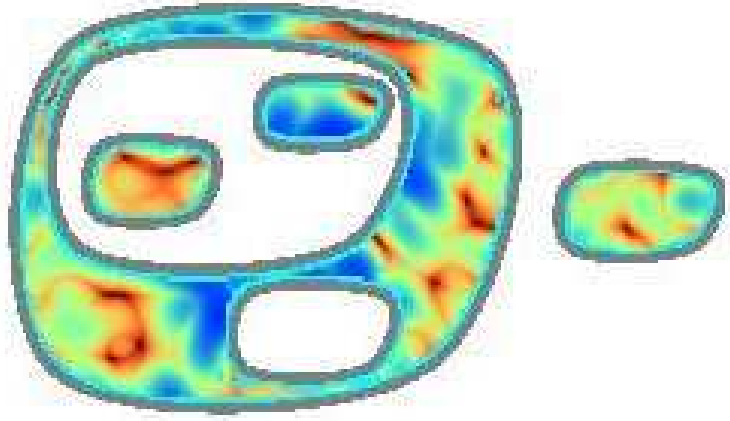
Brane Calculi,
Interactions of Biological Membranes
Luca Cardelli

Membrane Systems

Finite set of simple, closed and smooth curves

Membrane Systems

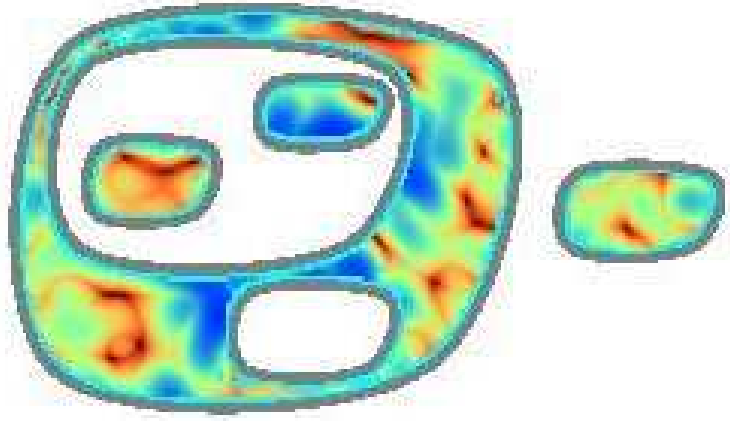
Finite set of simple, closed and smooth curves



Membrane Systems

Finite set of simple, closed and smooth curves

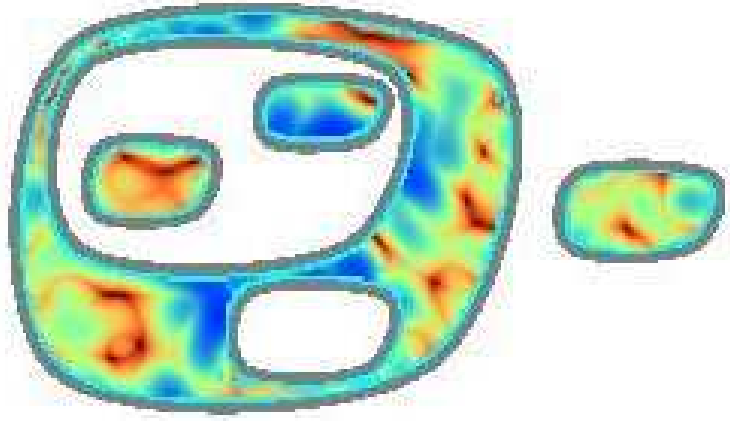
Alternated Orientation: Bitonality



Membrane Systems

Finite set of simple, closed and smooth curves

Alternated Orientation: Bitonality

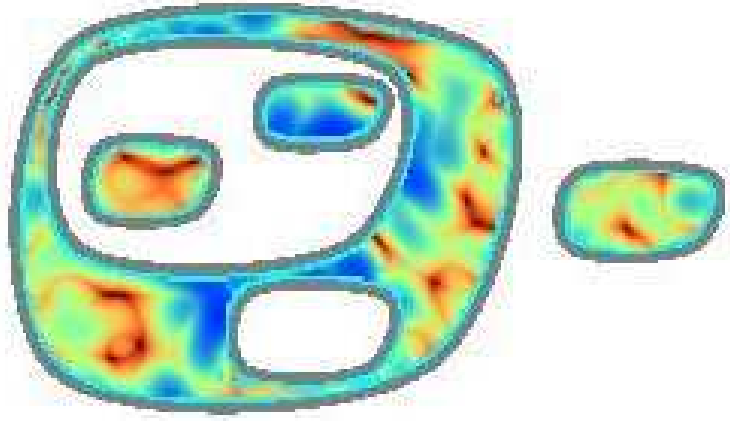


Reactions: (Instantaneous) transformations bitonality-preserving "locally"

Membrane Systems

Finite set of simple, closed and smooth curves

Alternated Orientation: Bitonality

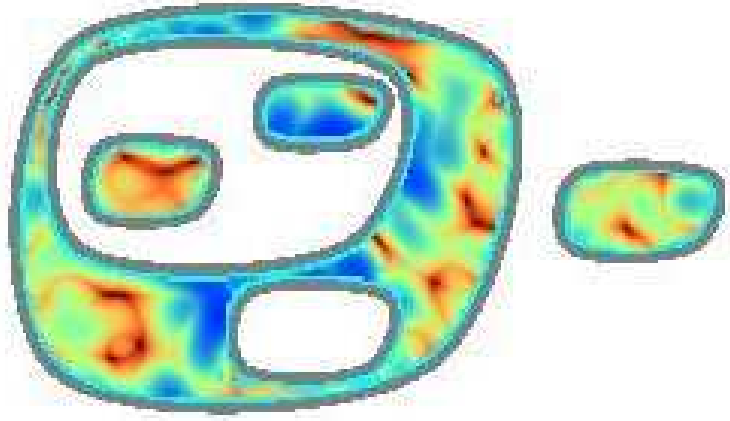


Reactions: (Instantaneous) transformations bitonality-preserving "locally"

Membrane Systems

Finite set of simple, closed and smooth curves

Alternated Orientation: Bitonality



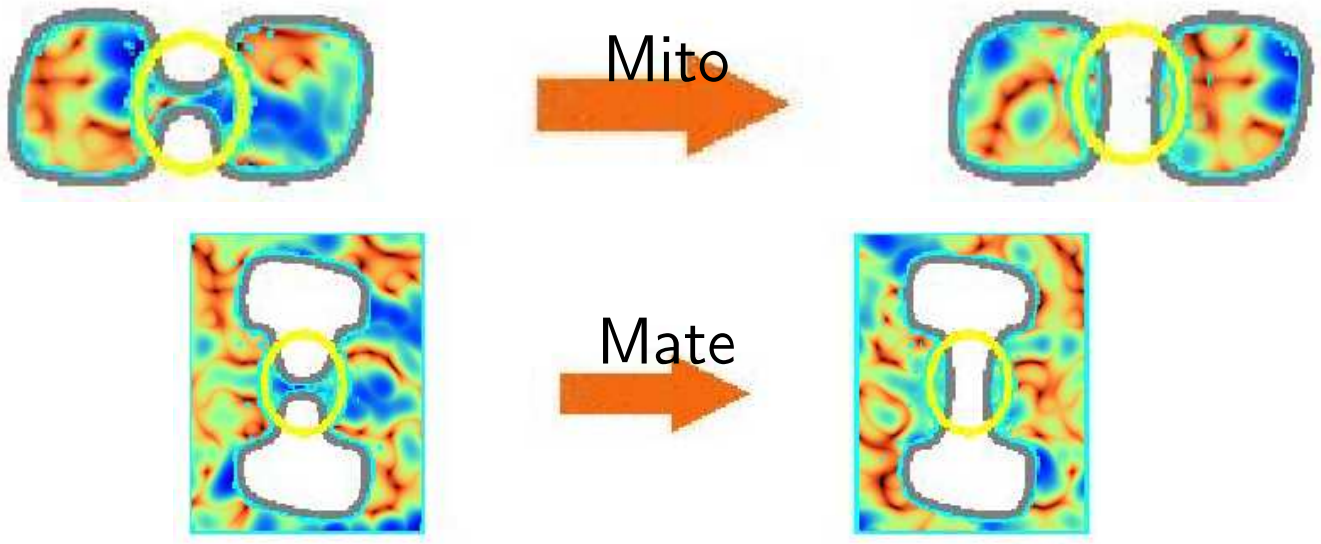
Reactions: (Instantaneous) transformations bitonality-preserving "locally"

Some bio-reactions are atonal, but abs-atonality is mostly unrealistic. Hence, ruled-out.

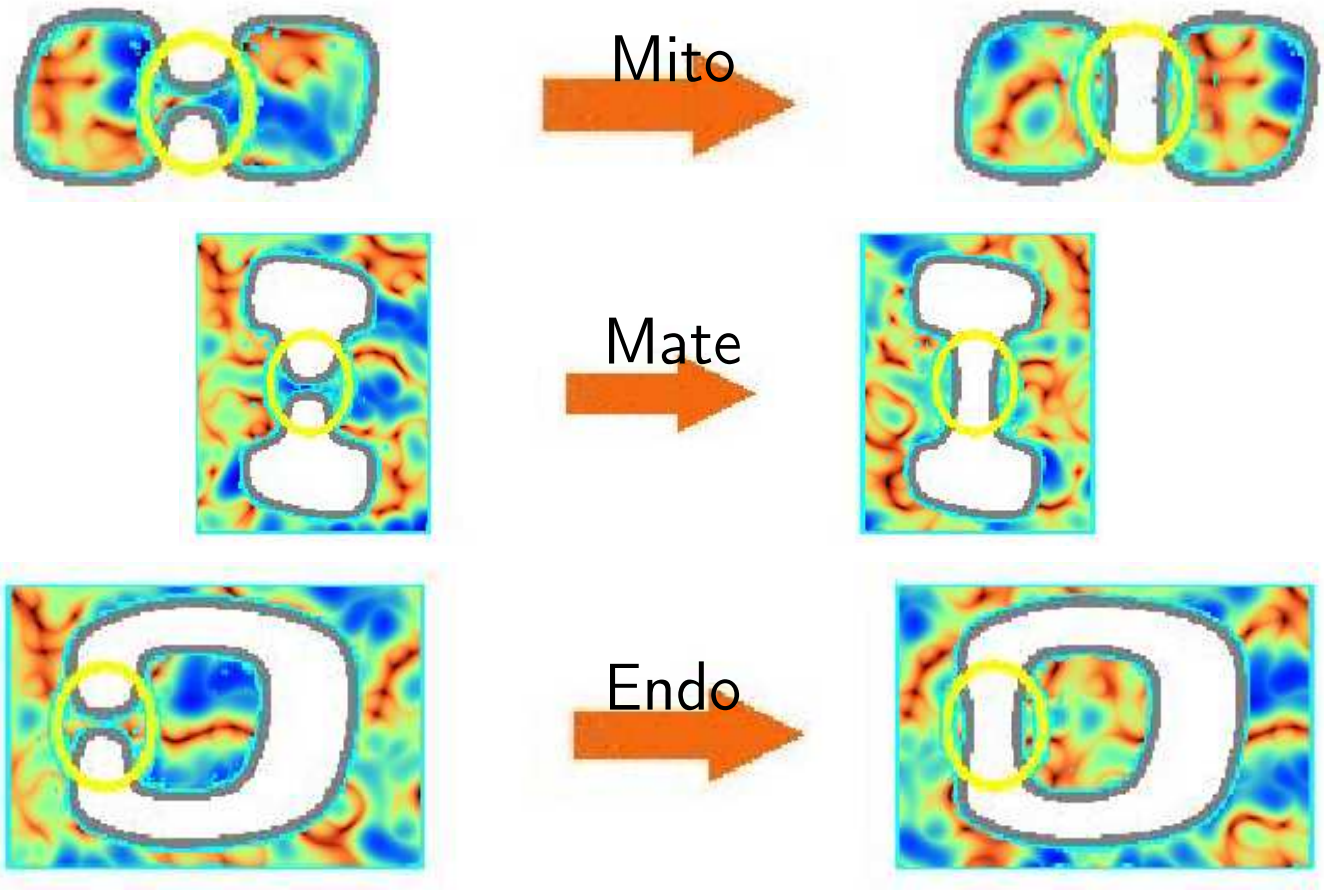
Membrane Reactions

Membrane Reactions

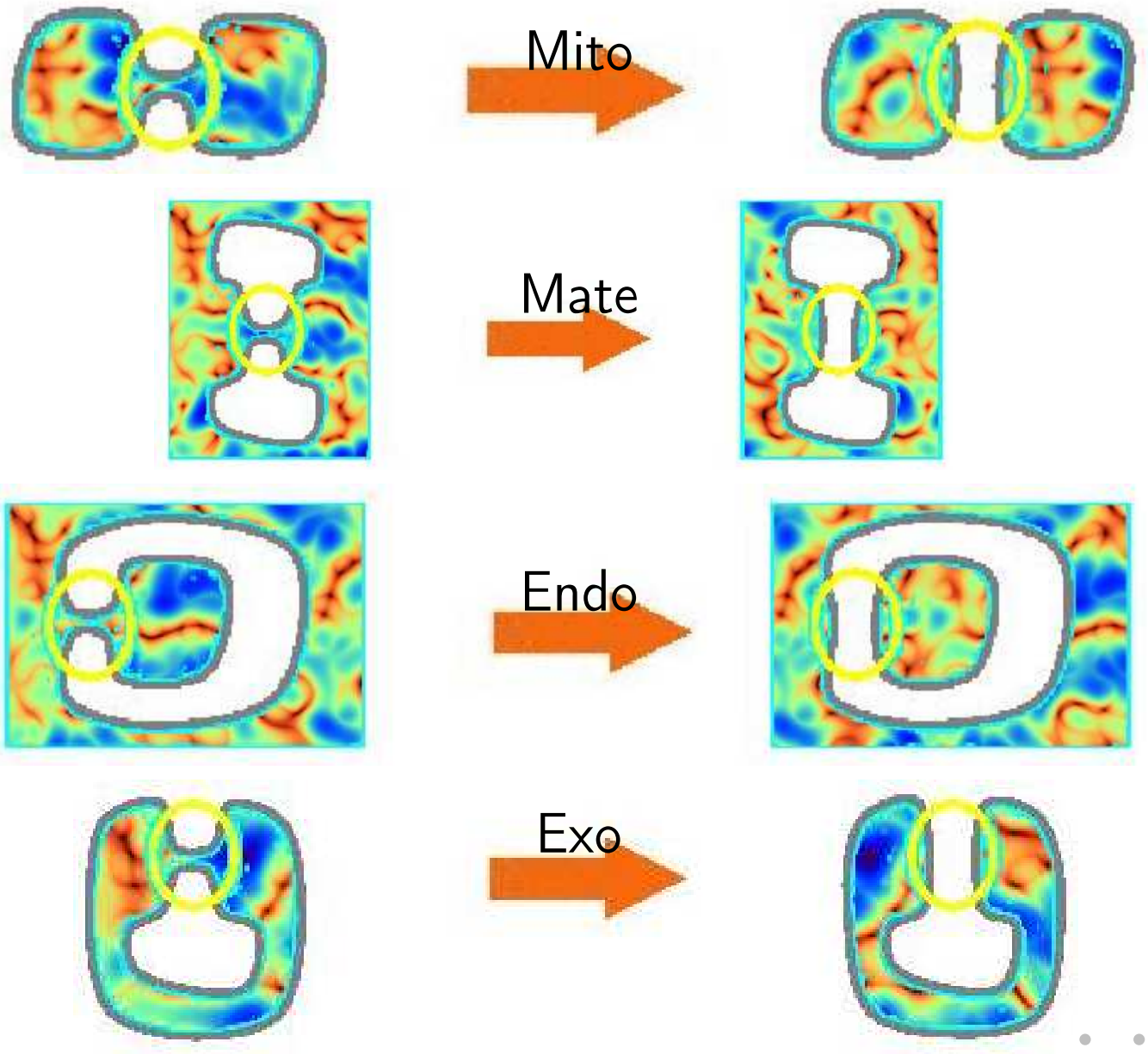
Membrane Reactions



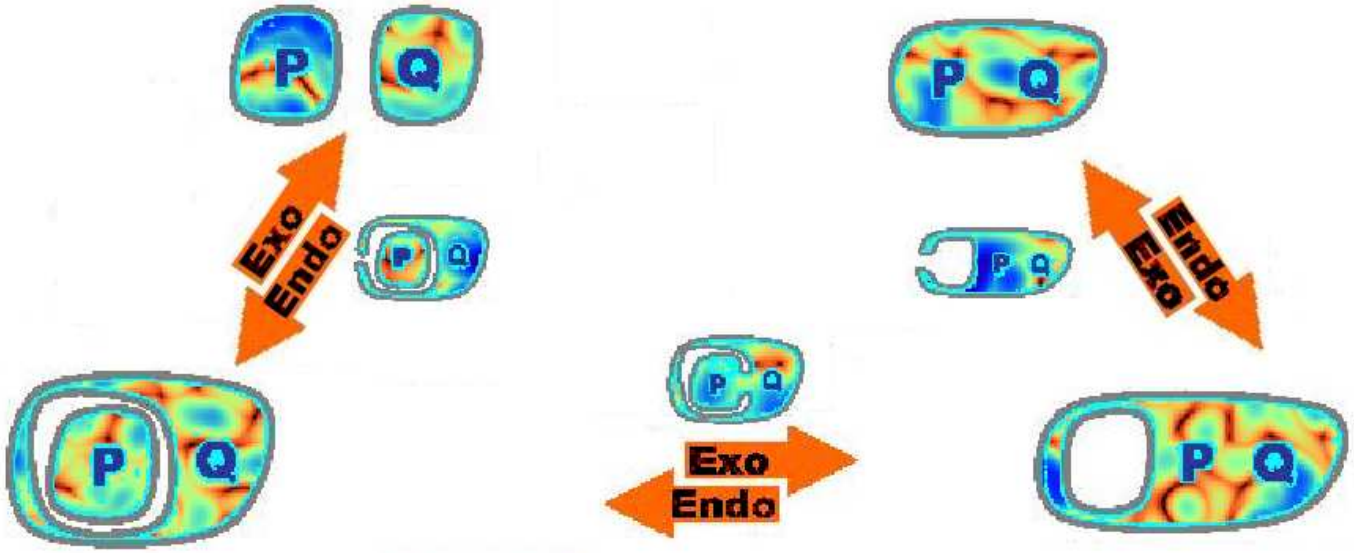
Membrane Reactions



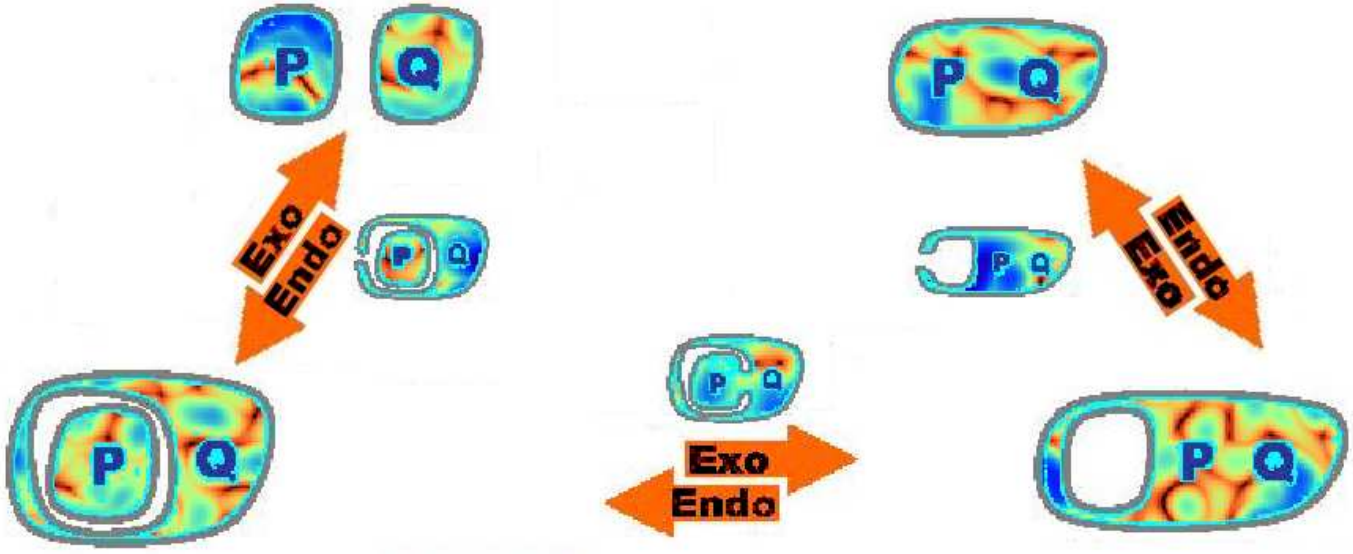
Membrane Reactions



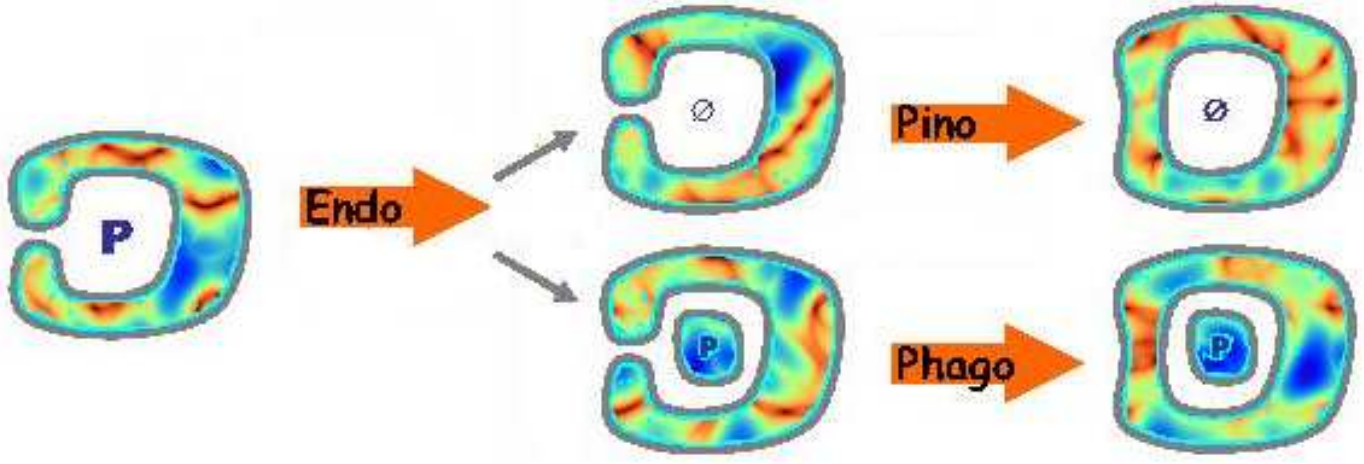
{Endo,Exo} is complete



{Endo,Exo} is complete



Moreover, Endo is splitted:



The Leap to Abstraction

Endo is not spontaneous, but regulated by membranes (i.e. its embedded proteins)

The Leap to Abstraction

Endo is not spontaneous, but regulated by membranes (i.e. its embedded proteins)

A Formalization:

- Actions "on" membranes, not "inside".

The Leap to Abstraction

Endo is not spontaneous, but regulated by membranes (i.e. its embedded proteins)

A Formalization:

- Actions "on" membranes, not "inside".
- Action/co-action interaction style.

The Leap to Abstraction

Endo is not spontaneous, but regulated by membranes (i.e. its embedded proteins)

A Formalization:

- Actions "on" membranes, not "inside".
- Action/co-action interaction style.
- A calculus of membrane reactions.

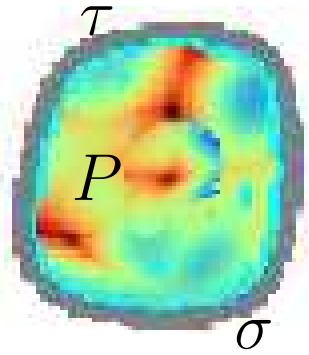
Syntax

Systems	P, Q	$::=$	$\diamond \mid P \circ Q \mid !P \mid \sigma(\!\mid P \mid\!)$
Branes	σ, τ	$::=$	$0 \mid \sigma \mid \tau \mid !\sigma \mid a.\sigma$
Actions	a, b	$::=$	

Syntax

Systems	P, Q	$::=$	$\diamond \mid P \circ Q \mid !P \mid \sigma(\!\mid P \mid\!)$
Branes	σ, τ	$::=$	$0 \mid \sigma \mid \tau \mid !\sigma \mid a.\sigma$
Actions	a, b	$::=$	

$\tau|\sigma(\!\mid P \mid\!)$



Brane with σ, τ and contents P

Congruence $\equiv$, Reactions $\rightarrow$

$(P, \circ, \diamond)$ comutative monoid
$(\sigma, |, 0)$ comutative monoid
the usual...

Congruence $\equiv$, Reactions $\rightarrow$

$(P, \circ, \diamond)$ comutative monoid
$(\sigma, |, 0)$ comutative monoid
the usual...

$$\frac{P \rightarrow Q}{P \circ R \rightarrow Q \circ R}$$

$$\frac{P \rightarrow Q}{\sigma(\!|P|\!) \rightarrow \sigma(\!|Q|\!)}$$

$$\frac{P \equiv P' \quad P' \rightarrow Q' \quad Q' \equiv Q}{P \rightarrow Q}$$

Congruence $\equiv$, Reactions $\rightarrow$

$(P, \circ, \diamond)$ comutative monoid
$(\sigma, |, 0)$ comutative monoid
the usual...

$$\frac{P \rightarrow Q}{P \circ R \rightarrow Q \circ R}$$

$$\frac{P \rightarrow Q}{\sigma(\!|P|\!) \rightarrow \sigma(\!|Q|\!)}$$

$$\frac{P \equiv P' \quad P' \rightarrow Q' \quad Q' \equiv Q}{P \rightarrow Q}$$

plus the effect of actions

Actions

Actions $\quad a, b \quad ::= \quad \rhd_n \mid {}^{\perp}_{n}\!\rhd(\sigma) \mid \lhd_n \mid \lhd^{\perp}_{n} \mid \circledcirc(\sigma)$

Actions

Actions $a, b \quad ::= \quad \rhd_n \mid {}^{\perp}_{n}\!\rhd(\sigma) \mid \lhd_n \mid \lhd^{\perp}_{n} \mid \circledcirc(\sigma)$

Phago:

$$\rhd_n .\sigma | \sigma_0(\!|P|\!) \circ {}^{\perp}_{n}\!\rhd(\rho).\tau | \tau_0(\!|Q|\!) \longrightarrow \tau | \tau_0(\!|\rho(\!|\sigma | \sigma_0(\!|P|\!)|\!) \circ Q|\!)$$

Actions

Actions $\quad a, b \quad ::= \quad \rhd_n \mid {}^{\perp}_{n}\!\rhd(\sigma) \mid \lhd_n \mid \lhd^{\perp}_{n} \mid \circledcirc(\sigma)$

Phago:

$$\rhd_n .\sigma|\sigma_0(\!|P|\!) \circ {}^{\perp}_{n}\!\rhd(\rho).\tau|\tau_0(\!|Q|\!) \longrightarrow \tau|\tau_0(\!|\rho(\!|\sigma|\sigma_0(\!|P|\!)|\!) \circ Q|\!)$$

Exo:

$$\lhd^{\perp}_{n} .\tau|\tau_0(\!|\lhd_n .\sigma|\sigma_0(\!|P|\!) \circ Q|\!) \longrightarrow P \circ \sigma|\sigma_0|\tau|\tau_0(\!|Q|\!)$$

Actions

Actions $a, b \quad ::= \quad \rhd_n \mid {}^{\perp}_{n}\!\rhd(\sigma) \mid \lhd_n \mid \lhd^{\perp}_{n} \mid \circledcirc(\sigma)$

Phago:

$$\rhd_n .\sigma|\sigma_0(\!|P|\!) \circ {}^{\perp}_{n}\!\rhd(\rho).\tau|\tau_0(\!|Q|\!) \longrightarrow \tau|\tau_0(\!|\rho(\!|\sigma|\sigma_0(\!|P|\!)|\!) \circ Q|\!)$$

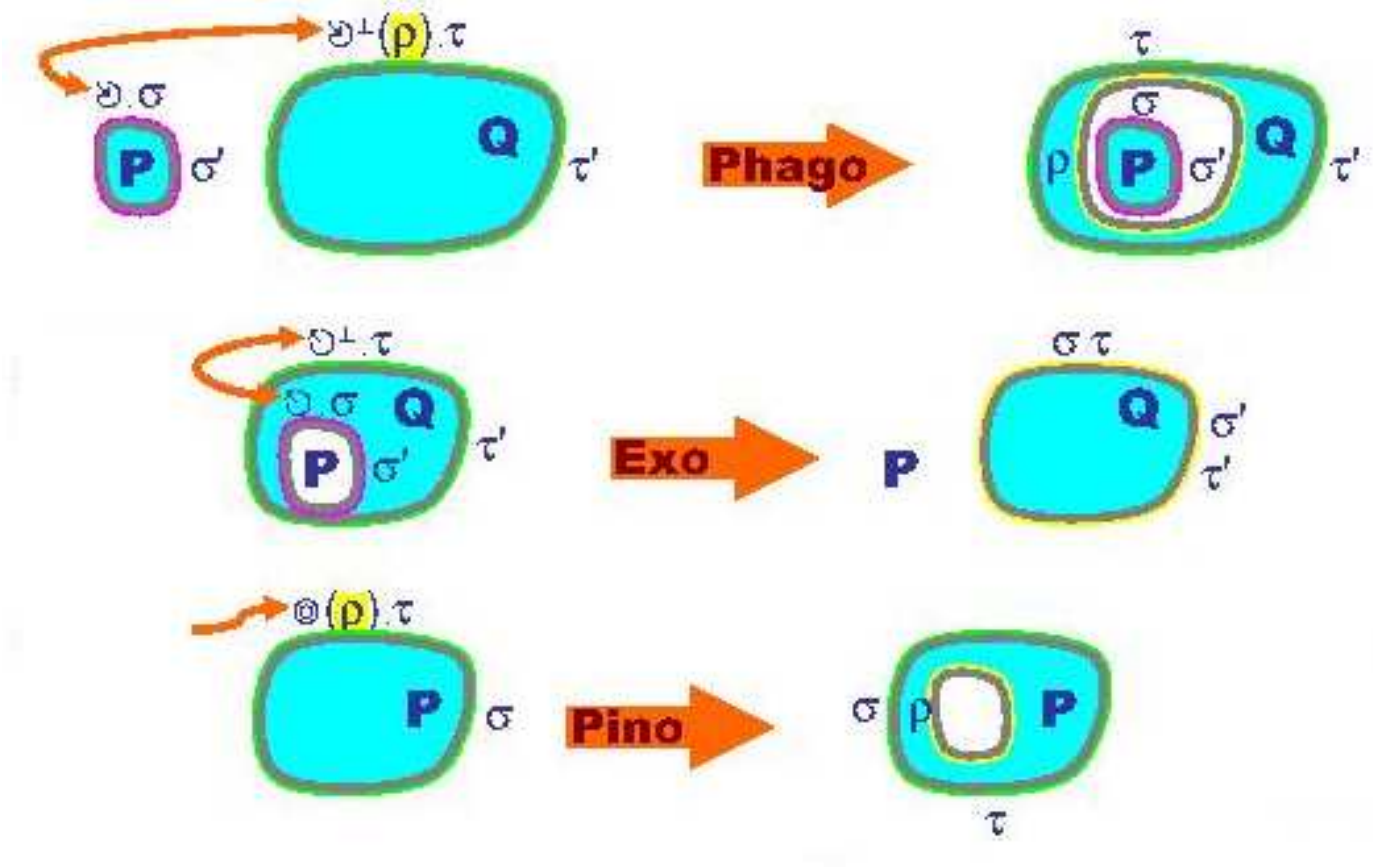
Exo:

$$\lhd^{\perp}_{n} .\tau|\tau_0(\!|\lhd_n .\sigma|\sigma_0(\!|P|\!) \circ Q|\!) \longrightarrow P \circ \sigma|\sigma_0|\tau|\tau_0(\!|Q|\!)$$

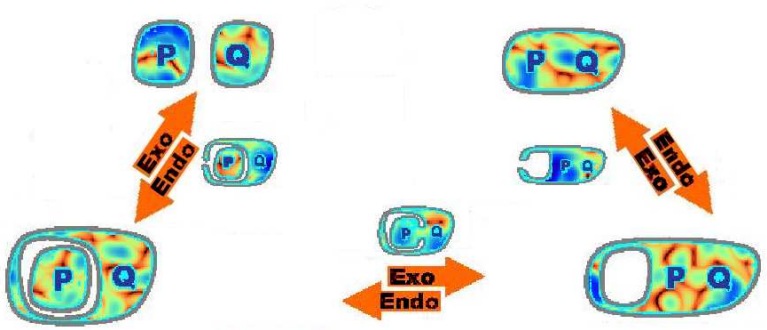
Pino:

$$\circledcirc(\rho).\sigma|\sigma_0(\!|P|\!) \longrightarrow \sigma|\sigma_0(\!|\rho(\!|\diamond|\!) \circ P|\!)$$

Actions Depicted



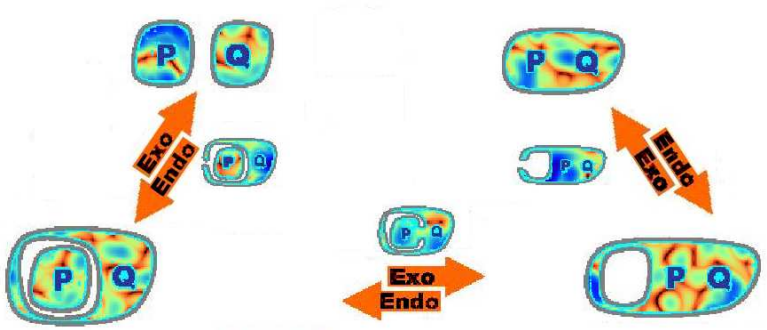
Example: Mate



Proposition:

$$\sigma_0|mate_n.\sigma(\!|P|\!) \circ \tau_0|mate_n^{\perp}.\tau(\!|Q|\!) \rightarrow^* \sigma_0|\sigma|\tau_0|\tau(\!|P \circ Q|\!)$$

Example: Mate



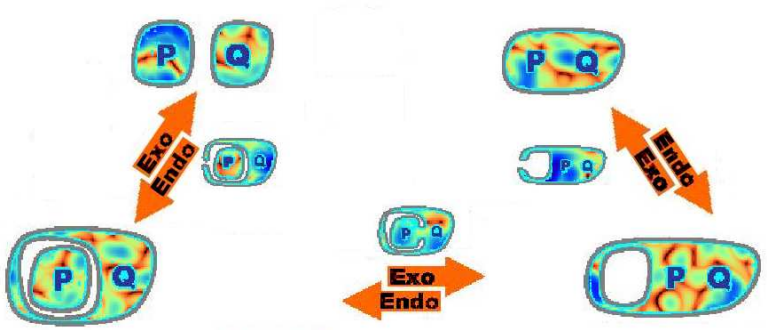
$$mate_n \stackrel{\text{def}}{=} \rhd n \,.\, \lhd_{n'} .\sigma$$

$$mate_n^{\perp} \stackrel{\text{def}}{=} {}^{\perp}_{n}\rhd (\lhd^{\perp}_{n'} \,.\, \lhd_{n''}).\, \lhd^{\perp}_{n''} .\tau$$

Proposition:

$$\sigma_0 | mate_n.\sigma (\!| P |\!) \circ \tau_0 | mate_n^{\perp}.\tau (\!| Q |\!) \rightarrow^* \sigma_0 | \sigma | \tau_0 | \tau (\!| P \circ Q |\!)$$

Example: Mate



$$mate_n \stackrel{\text{def}}{=} \triangleright n \,.\, \triangleleft_{n'} . \sigma$$

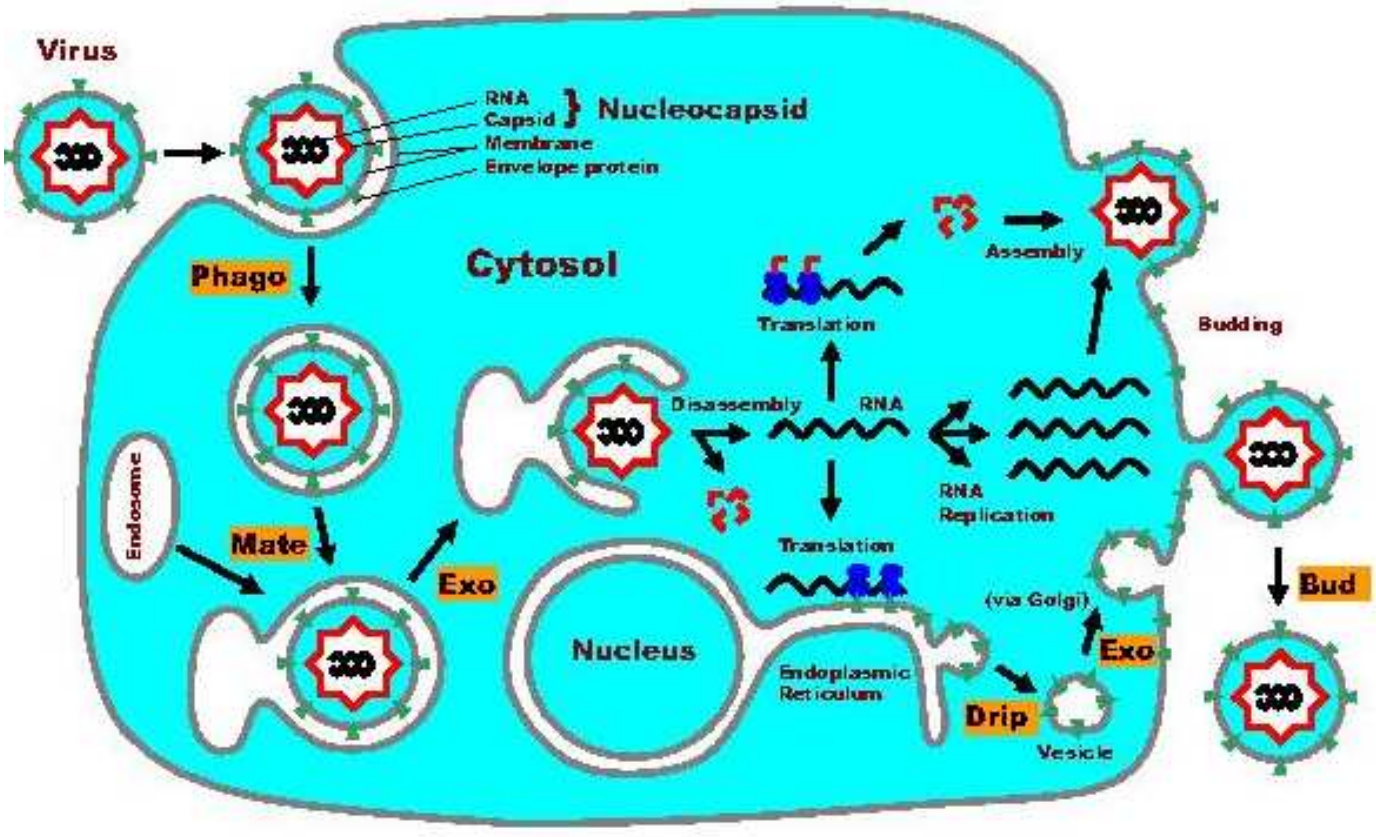
$$mate_n^{\perp} \stackrel{\text{def}}{=} {}^{\perp}_{n}\triangleright (\triangleleft^{\perp}_{n'} \,.\, \triangleleft_{n''}) .\, \triangleleft^{\perp}_{n''} \,.\tau$$

Proposition:

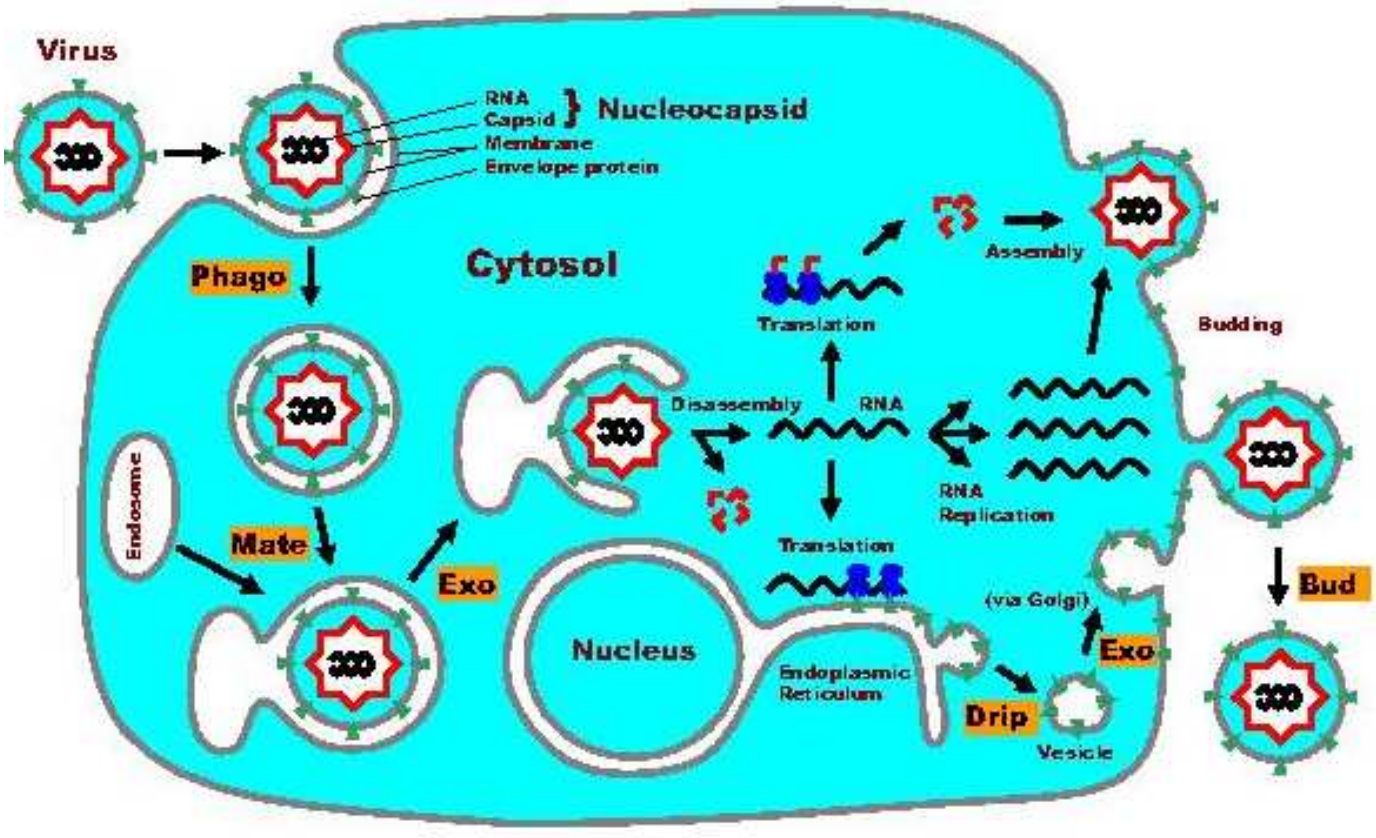
$$\sigma_0 | mate_n . \sigma (\!| P |\!) \circ \tau_0 | mate_n^{\perp} . \tau (\!| Q |\!) \rightarrow^* \sigma_0 | \sigma | \tau_0 | \tau (\!| P \circ Q |\!)$$

Homework: Drip (Mito with 0), Bud (Mito with 1)

Example: Viral Reproduction



Example: Viral Reproduction



Almost... molecules are needed

Nice, but...

Nice, but... what kind of calculus is this?

Nice, but... what kind of calculus is this?

- Purely combinatorial

Nice, but... what kind of calculus is this?

- Purely combinatorial
  communication could be added...
  $a, b ::=$
  $...o2o_n \mid o2o_n^{\perp}(m) \mid s2s_n \mid s2s_n^{\perp}(m) \mid p2c_n \mid p2c_n^{\perp}(m)$
  assuming $\tau\{l \leftarrow m\}$

Nice, but... what kind of calculus is this?

- Purely combinatorial
  communication could be added...
  $a, b ::=$
  $...o2o_n \mid o2o_n^{\perp}(m) \mid s2s_n \mid s2s_n^{\perp}(m) \mid p2c_n \mid p2c_n^{\perp}(m)$
  assuming $\tau\{l \leftarrow m\}$

  ...and name restriction...

Nice, but... what kind of calculus is this?

- Purely combinatorial
  communication could be added...
  $a, b ::=$
  $...o2o_n \mid o2o_n^{\perp}(m) \mid s2s_n \mid s2s_n^{\perp}(m) \mid p2c_n \mid p2c_n^{\perp}(m)$
  assuming $\tau\{l \leftarrow m\}$

  ...and name restriction...
  ...and choice...

Nice, but... what kind of calculus is this?

- Purely combinatorial
  communication could be added...
  $a, b ::=$
  $...o2o_n \mid o2o_n^{\perp}(m) \mid s2s_n \mid s2s_n^{\perp}(m) \mid p2c_n \mid p2c_n^{\perp}(m)$
  assuming $\tau\{l \leftarrow m\}$

  ...and name restriction...
  ...and choice...
  ...and all π?

Nice, but... what kind of calculus is this?

- Purely combinatorial
  communication could be added...
  $a, b ::=$
  $...o2o_n \mid o2o_n^{\perp}(m) \mid s2s_n \mid s2s_n^{\perp}(m) \mid p2c_n \mid p2c_n^{\perp}(m)$
  assuming $\tau\{l \leftarrow m\}$

  ...and name restriction...
  ...and choice...
  ...and all π?
- No equivalence

Nice, but... what kind of calculus is this?

- Purely combinatorial
  communication could be added...
  $a, b ::=$
  $...o2o_n \mid o2o_n^{\perp}(m) \mid s2s_n \mid s2s_n^{\perp}(m) \mid p2c_n \mid p2c_n^{\perp}(m)$
  assuming $\tau\{l \leftarrow m\}$

  ...and name restriction...
  ...and choice...
  ...and all π?
- No equivalence
- Biologically meaningful?

Comparative Exercise: Security Applications

Ambients in the air...

Comparative Exercise: Security Applications

Ambients in the air...Pure and Safe

Comparative Exercise: Security Applications

Ambients in the air...Pure and Safe

$$
\begin{array}{lll}
P & ::= & (\nu\ n)P \mid 0 \mid P \circ Q \mid !P \mid n[P] \mid Cap.P \\
Cap & ::= & in\ n \mid \vec{in}\ n \mid out\ n \mid \vec{out}\ n \mid open\ n \mid \vec{open}\ n \mid
\end{array}
$$

Comparative Exercise: Security Applications

Ambients in the air...Pure and Safe

$$
\begin{array}{lll}
P & ::= & (\nu\ n)P \mid 0 \mid P \circ Q \mid !P \mid n[P] \mid Cap.P \\
Cap & ::= & in\ n \mid \vec{in}\ n \mid out\ n \mid \vec{out}\ n \mid open\ n \mid \vec{open}\ n \mid
\end{array}
$$

to be explored...