Formal proofs on collisions for Bézier curves

Yves Bertot
Yves.Bertot@inria.fr

Bézier curves are defined parametrically by polynomials, with properties that make their usage noticeably intuitive and practical: A Bézier curve of degree n is determined by $n+1$ control points, so that the curve lays inside the convex hull of these control points (when the parameter is taken between 0 and 1). An illustration of this property is shown on the wikipedia pages for this concept (see e.g. the French page).

Moreover, an algorithm designed by Paul de Casteljau makes it possible to construct the control points for sub-intervals, thus reducing progressively the convex hulls and making possible to find the collision points with obstacles by dichotomy and to prove the absence of collision when it is the case.

The objective of this internship is to design such a collision detection algorithm and to prove its correctness. Preliminary studies have already been performed concerning Bernstein polynomials, in particular concerning their use to isolate the roots of a polynomial and the guarantee to find a correct answer in a finite number of steps using the dichotomy process.

This internship requires a part of programming with symbolic representations of mathematical objects (especially polynomials) and recursive techniques of search by dichotomy, together with the corresponding step of formal proof. We expect the formal proof work to be performed using the Coq proof system [1], more precisely with the Mathematical Components library [4]. This work will give the opportunity of further work in the direction of NURBS (Non-Uniform Rational Basis Splines).

We also wish to derive outreach material from this experiment, since Bézier curves are a very "visual" concept of mathematics, with applications in computer aided design and art, amenable to show the beauty of some rather simple mathematical operations (barycenters, convex hulls, etc). The concept of collision avoidance should also be a nice framework to explain the usefulness of formal proofs.

There will be opportunities to continue research work in the framework of a PhD thesis.

This internship will be supervised by Yves Bertot.

References

[1] Bertot, Y., Castran, P.: Interactive Theorem Proving and Program Development, Coq'Art: The Calculus of Inductive Constructions. Springer. 2004.

[2] Latombe, J.-C.: Robot Motion Planning. Kluwer Academic Publishers. 1991.

[3] LaValle, S. M.: Planning Algorithms. Cambridge University Press. 2006. http://planning.cs.uiuc.edu/

[4] Mahboubi A., Tassi E.: Mathematical Components. To appear. https://math-comp.github.io/mcb/