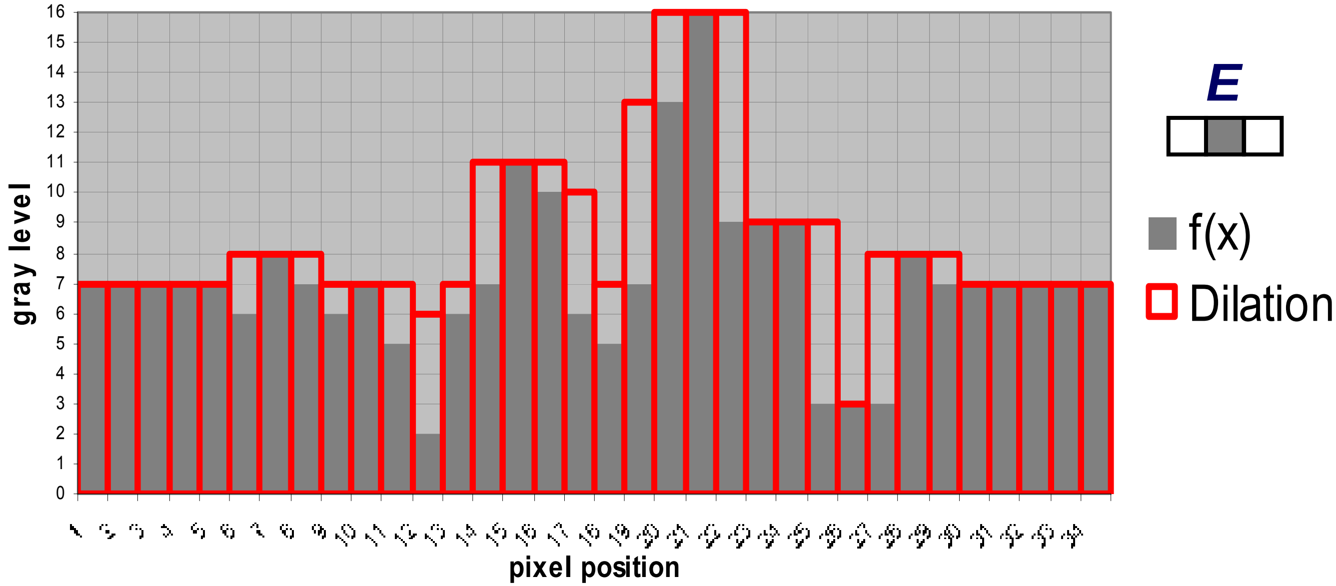
Mathematical morphology for grey-scale and hyperspectral images

Dilation for grey-scale images

- ***Dilation:*** replace every pixel by the *maximum* value computed over the neighborhood defined by the structuring element



Dilation for grey-scale images

- ***Dilation:*** replace every pixel by the *maximum* value computed over the neighborhood defined by the structuring element
- Consequence:
  - Features that are *brighter* than their immediate surroundings are *enlarged*
  - Features that are *darker* than their immediate surroundings are *shrinked*
  - Effect driven by the size and shape of the SE

$\delta_5(X)$ →

SE: disk of radius = 5

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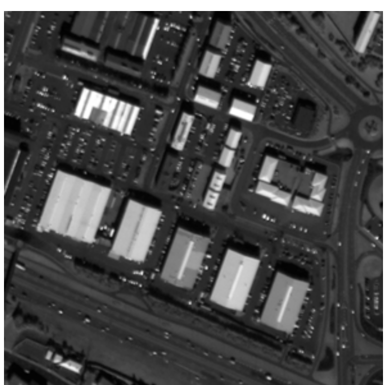

$\delta_{15}(X)$

SE: disk of radius = 15

Erosion for grey-scale images

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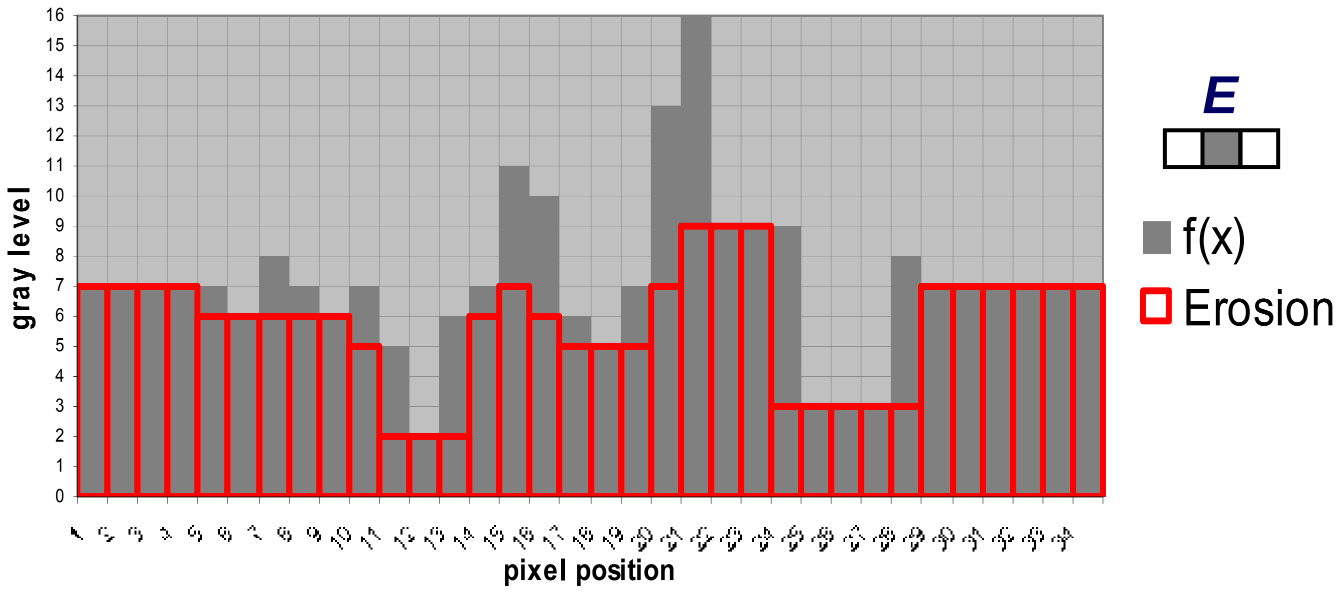

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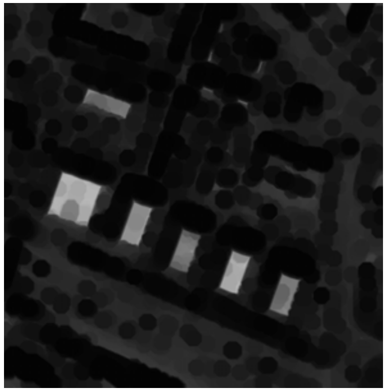
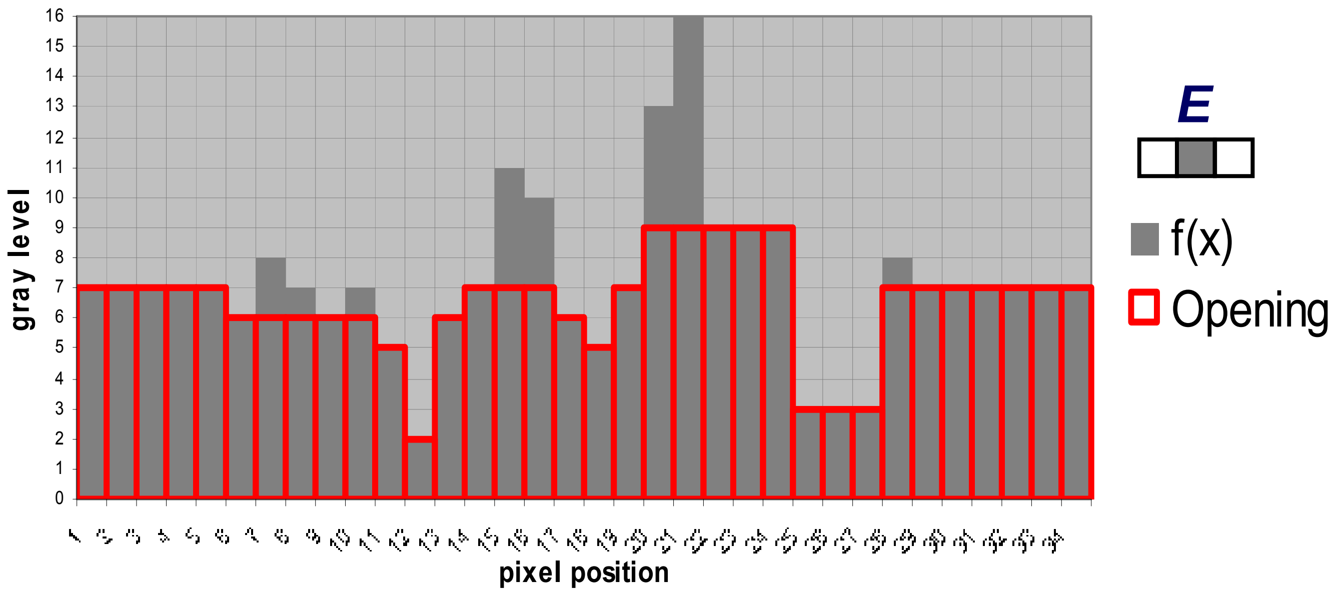


Opening for grey-scale images

- ***Opening:*** erosion followed by a dilation with the symmetrical structuring element



Opening for grey-scale images

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- Consequence:
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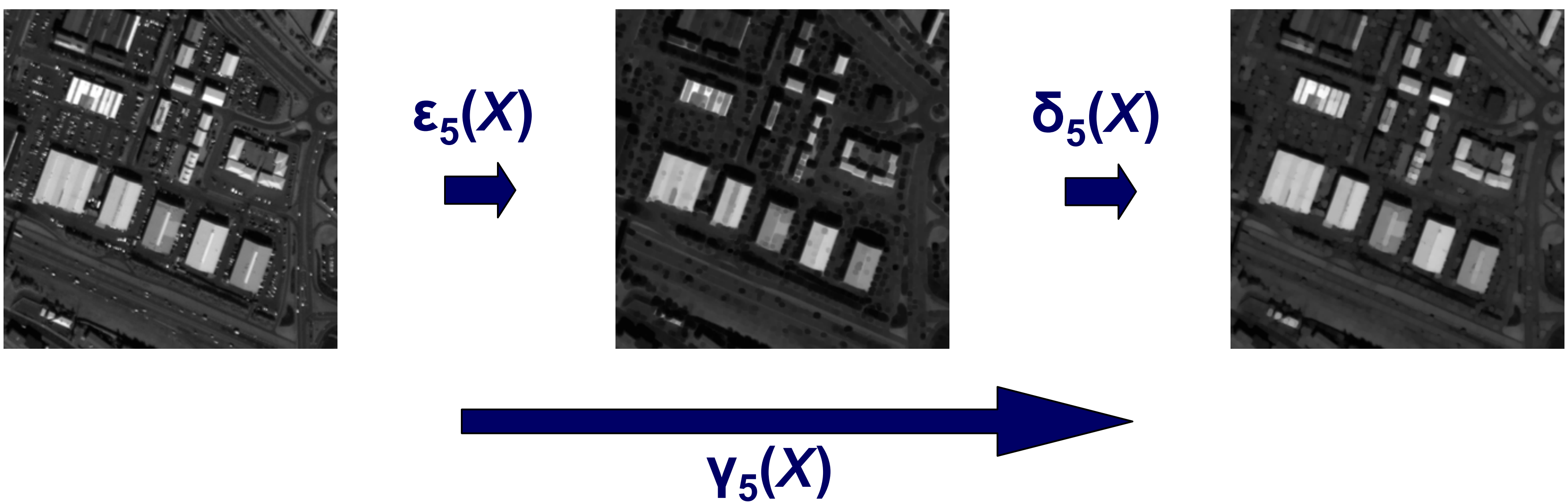

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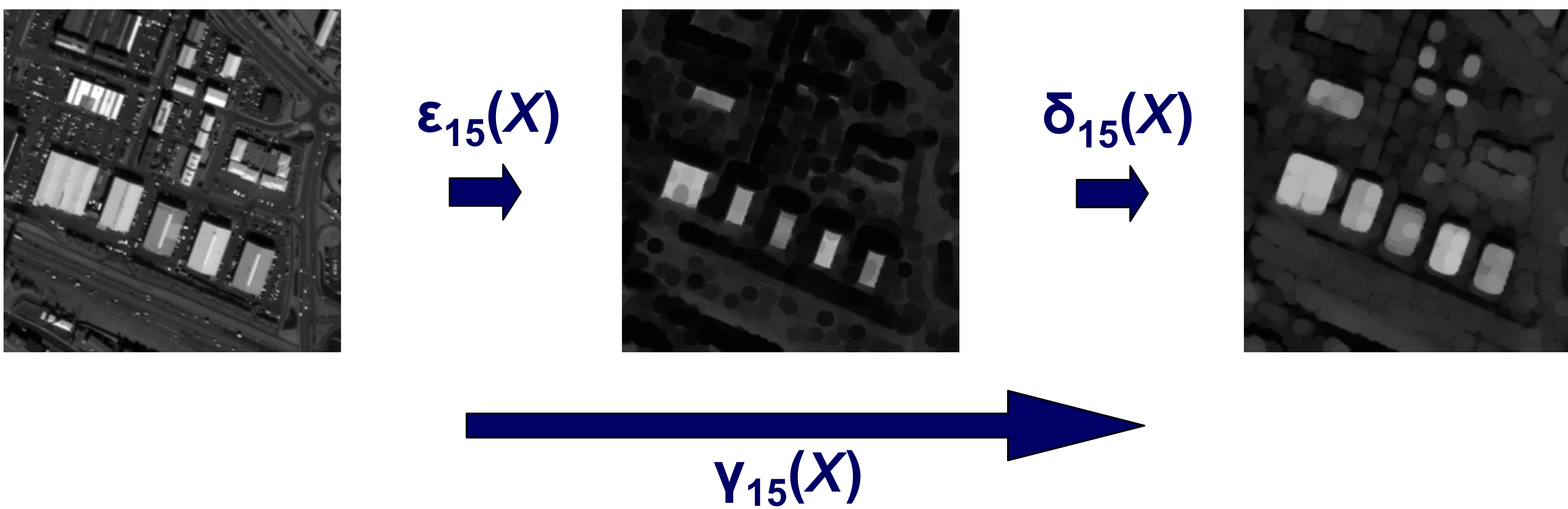
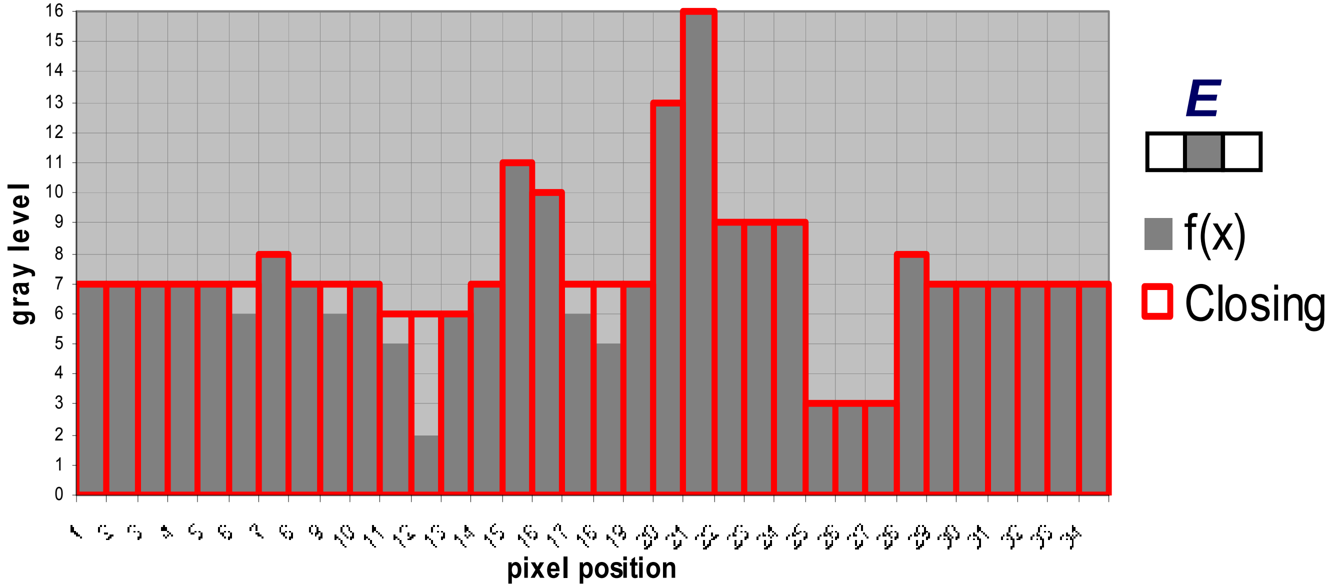


Closing for grey-scale images

- ***Closing:*** dilation followed by an erosion with the symmetrical structuring element



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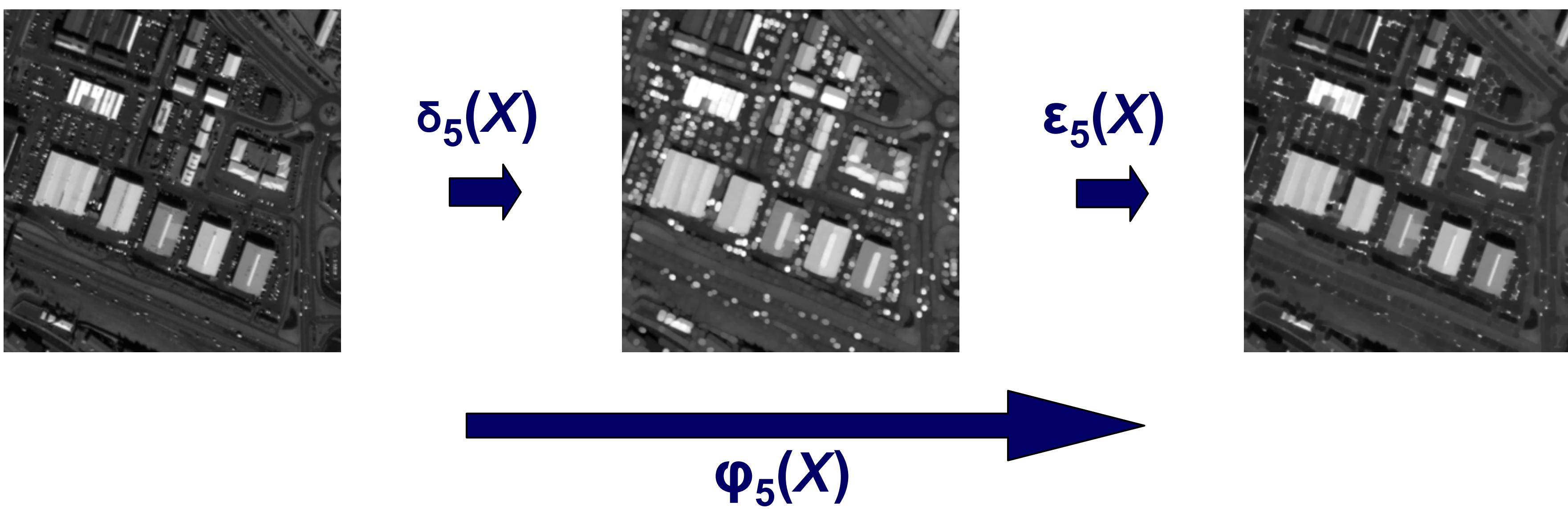

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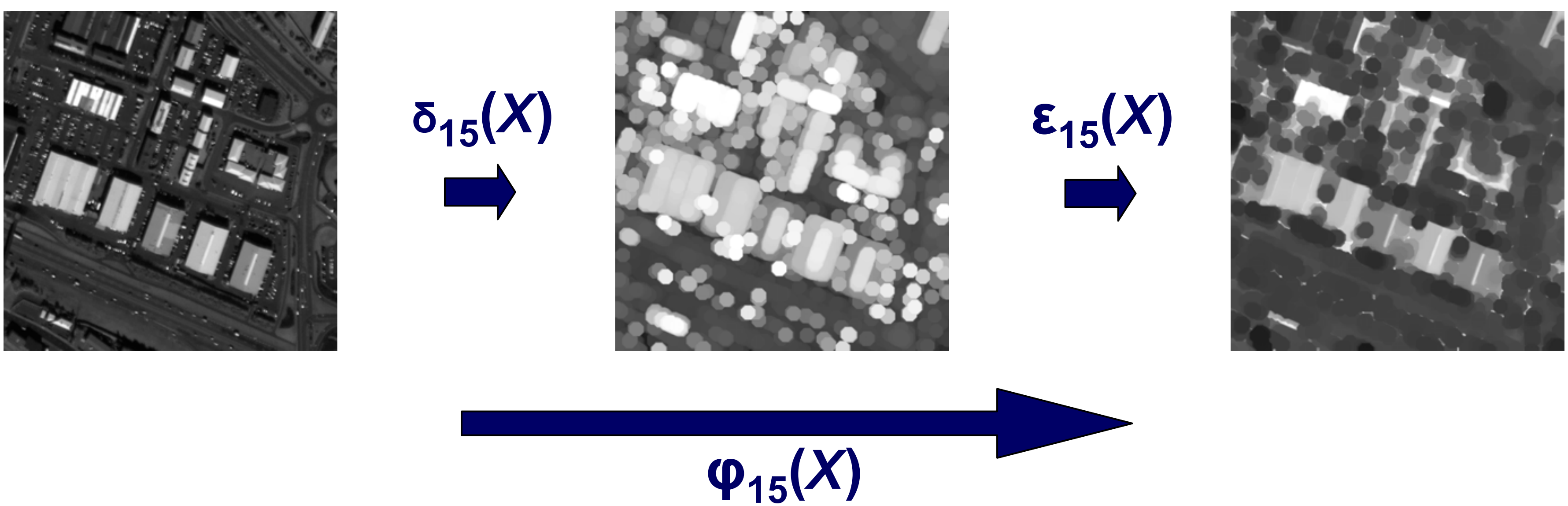
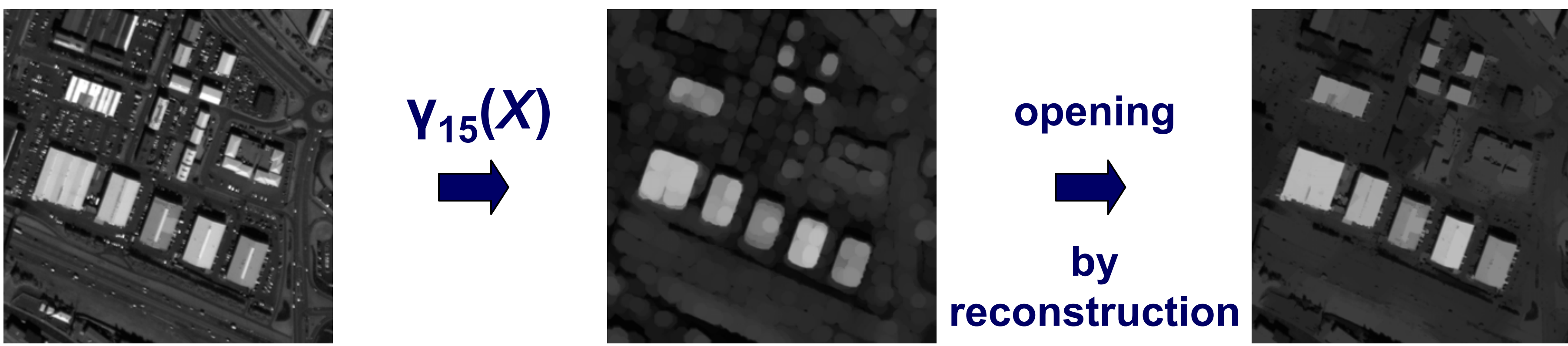


Geodesic reconstruction

- *Connected* operators
- Same properties, with no shape noise

- Opening by reconstruction:
  - Preserves the shape of the objects that are not removed by erosion

Original image: X
Im1 := $\varepsilon_{15}(X)$
Im2 := $\delta_{15}(\text{Im1})$
Im3 := min(Im2, X)
Im1 := Im3
CV



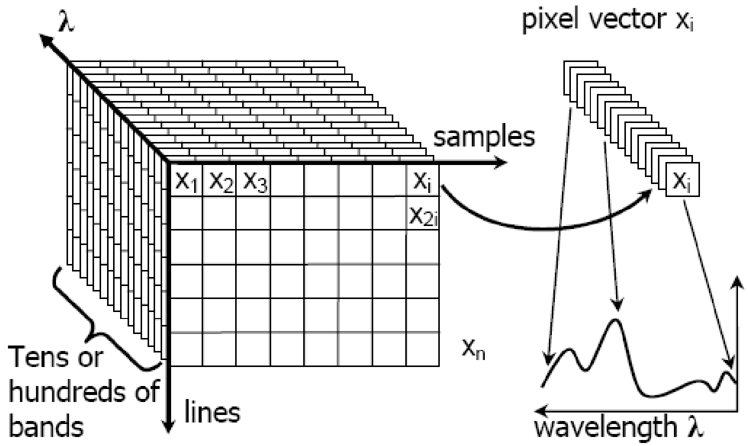
To read: P. Soille, Morphological Image Analysis, 2nd ed. Springer-Verlag, 2003.

Mathematical morphology for hyperspectral images

Multivariate mathematical morphology

- ***Hyperspectral image:***

  every pixel = spectrum
  = vector of ***very high dimension***



- ***Problem:***
  Mathematical morphology requires a ***complete lattice*** structure.
  Every set of pixels has one ***infimum*** and one ***supremum***.

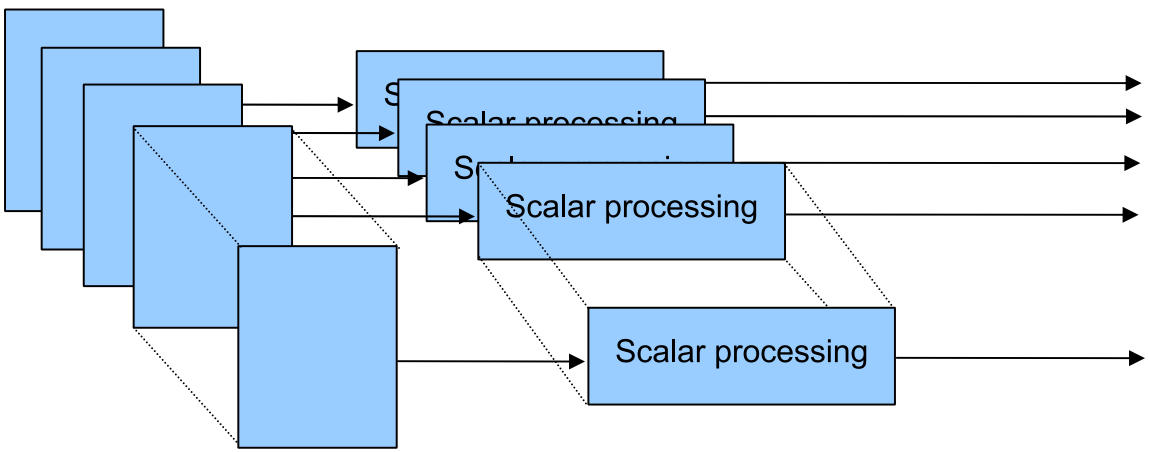
Shall one process hyperspectral data in a vector way ?

If yes : ***How can one order vectors ?***

If no : ***How shall one proceed ?***

Multivariate mathematical morphology

- ***Marginal*** approach:
  Hyperspectral image = ***set of grey level images*** that are processed ***separately***



Multivariate mathematical morphology

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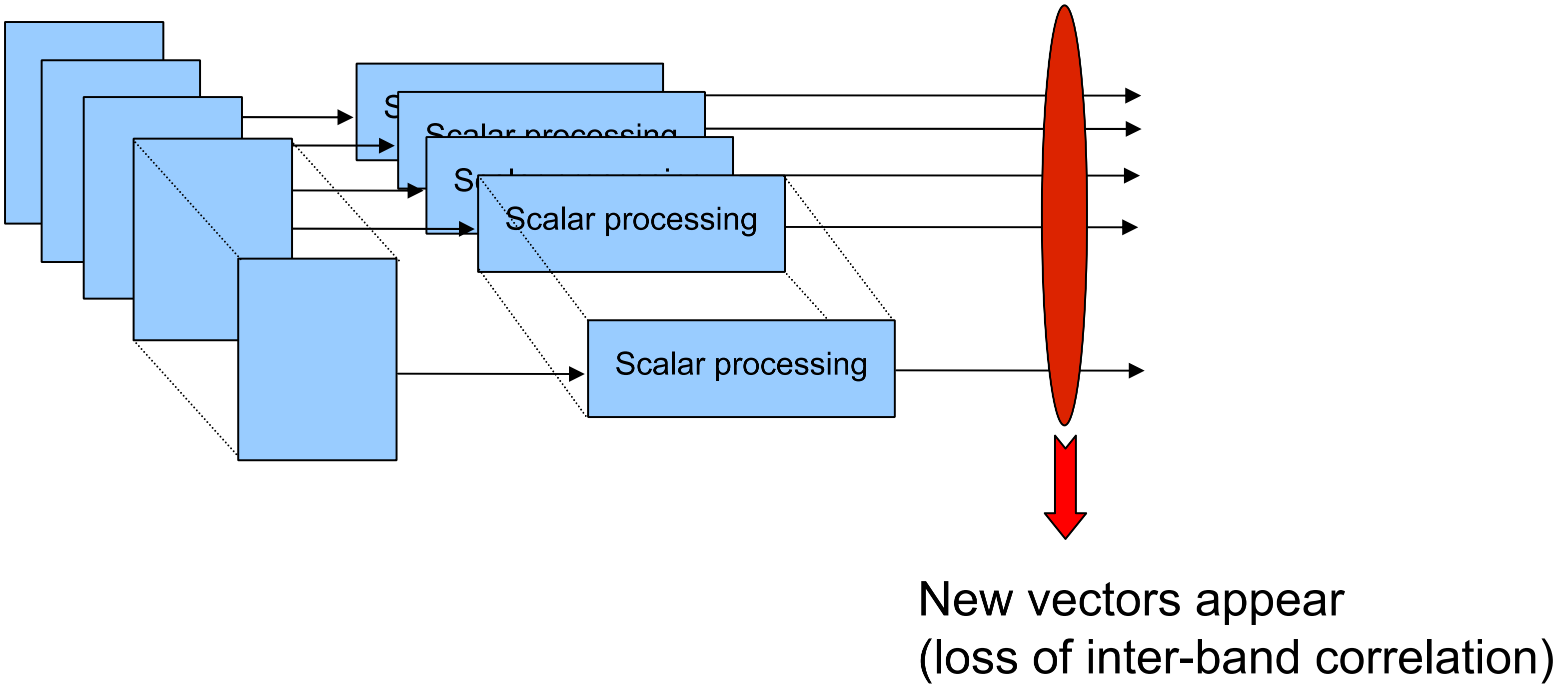

Multivariate mathematical morphology

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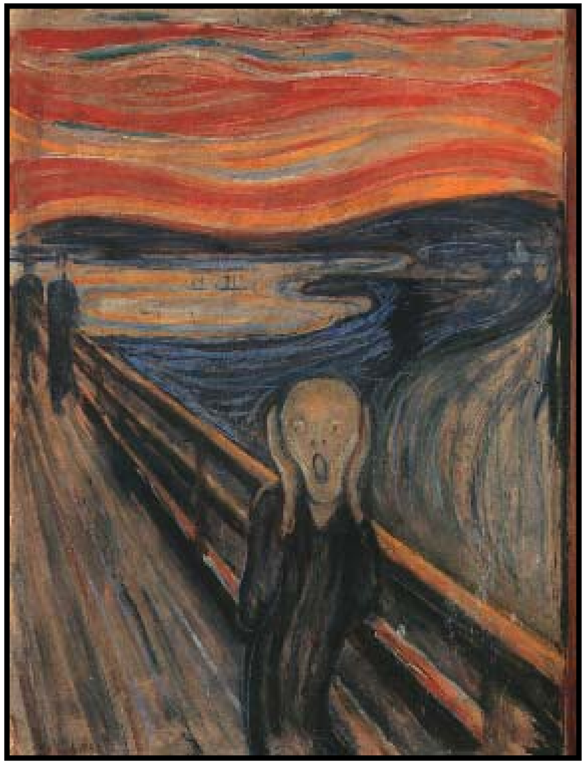

Original image

Marginal opening

Multivariate mathematical morphology

- ***Marginal*** approach:
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Original image

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Multivariate mathematical morphology

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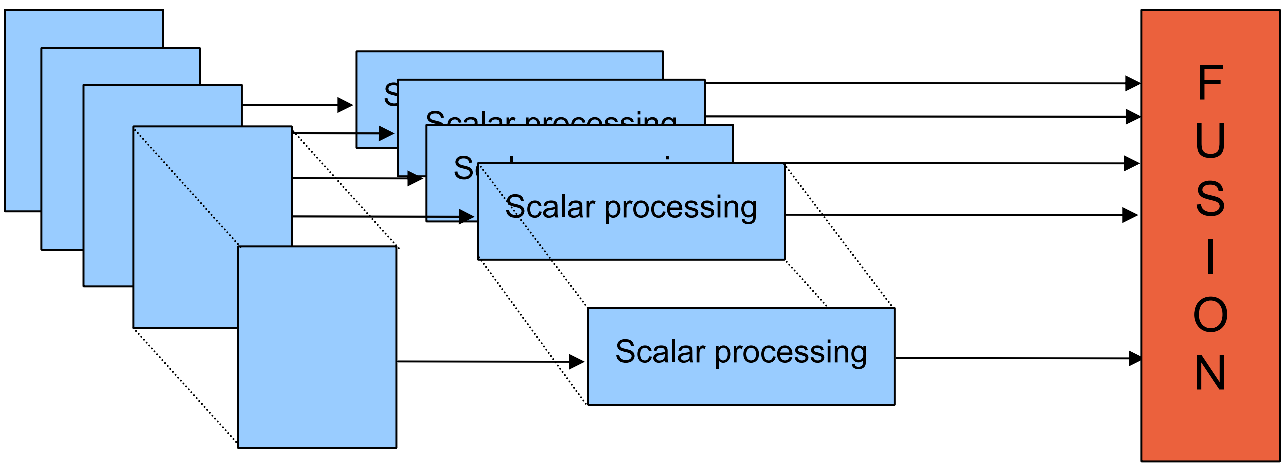
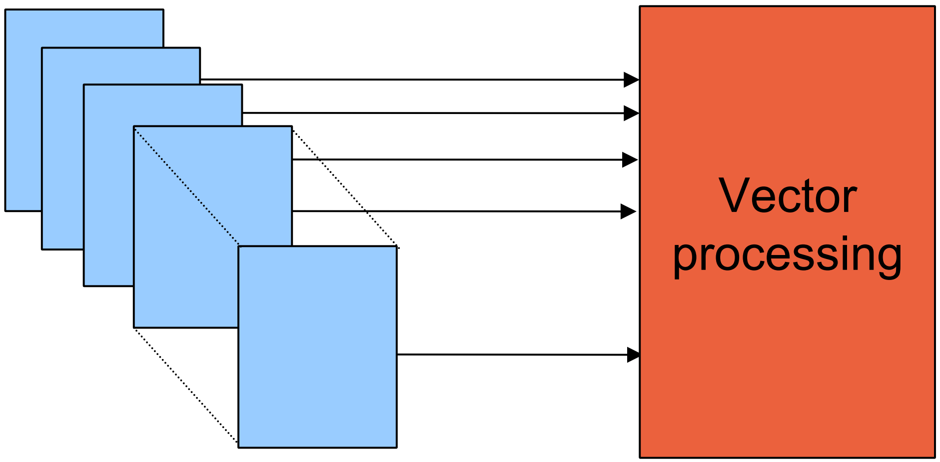


Multivariate mathematical morphology

- ***Vector*** approach:
  hyperspectral image = one ***single image***, every pixel is one (very long) vector

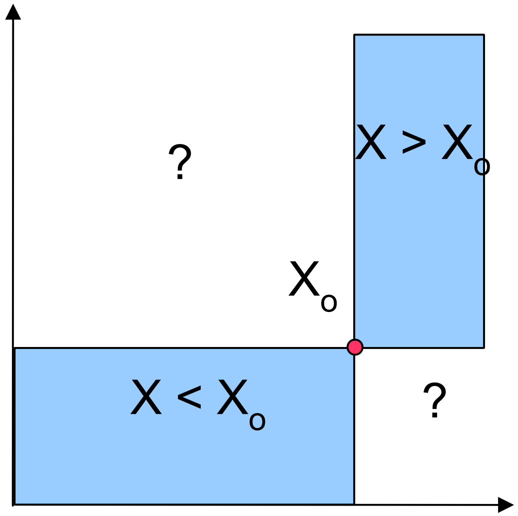


Multivariate mathematical morphology

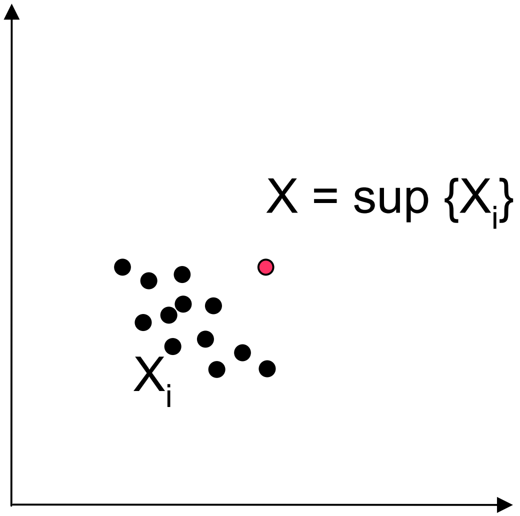
- ***Vector*** approach

  define an order on vectors?

  ➢ Canonical order $(X \leq Y) \Leftrightarrow (X(i) \leq Y(i) , \forall i)$



Partial order



False “colors”

Multivariate mathematical morphology

- ***Vector*** approach

define an order on vectors?

- Canonical order $(X \leq Y) \Leftrightarrow (X(i) \leq Y(i), \forall i)$
- General formalism :

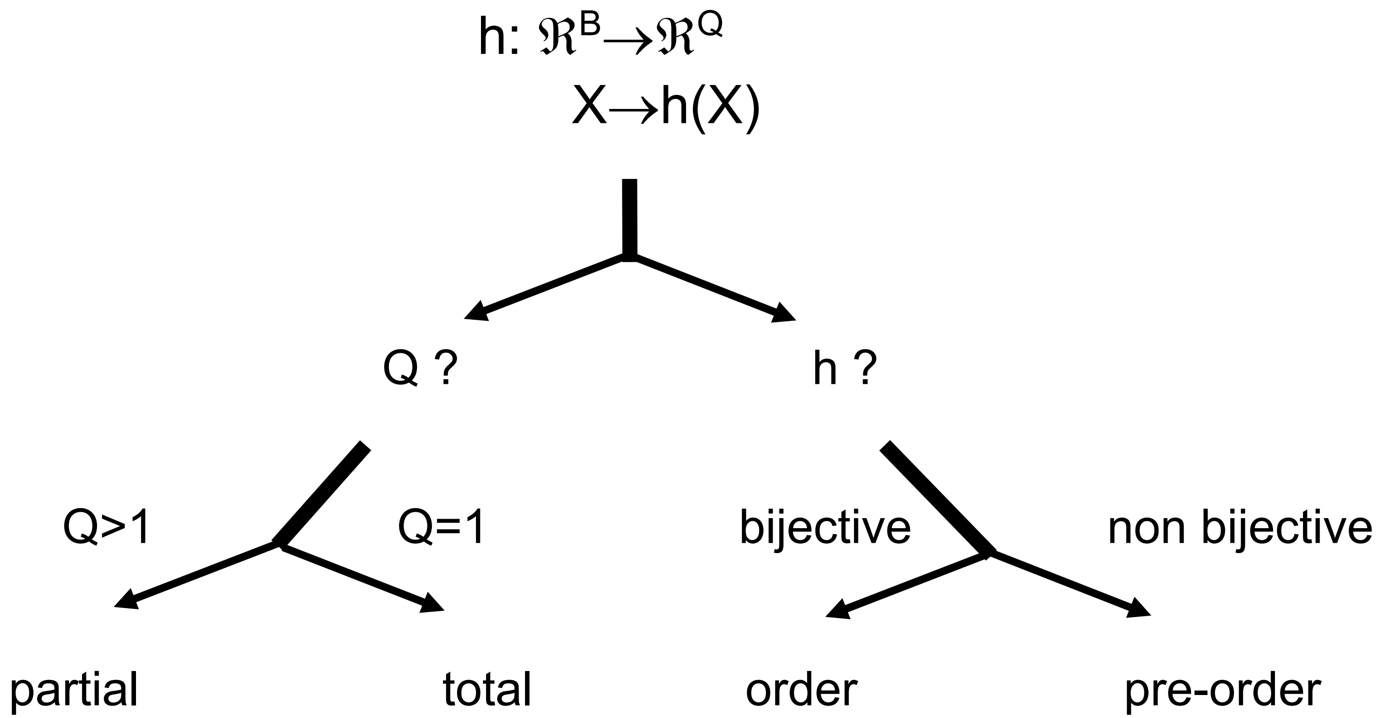
Change space for ordering

transform *h* such that:

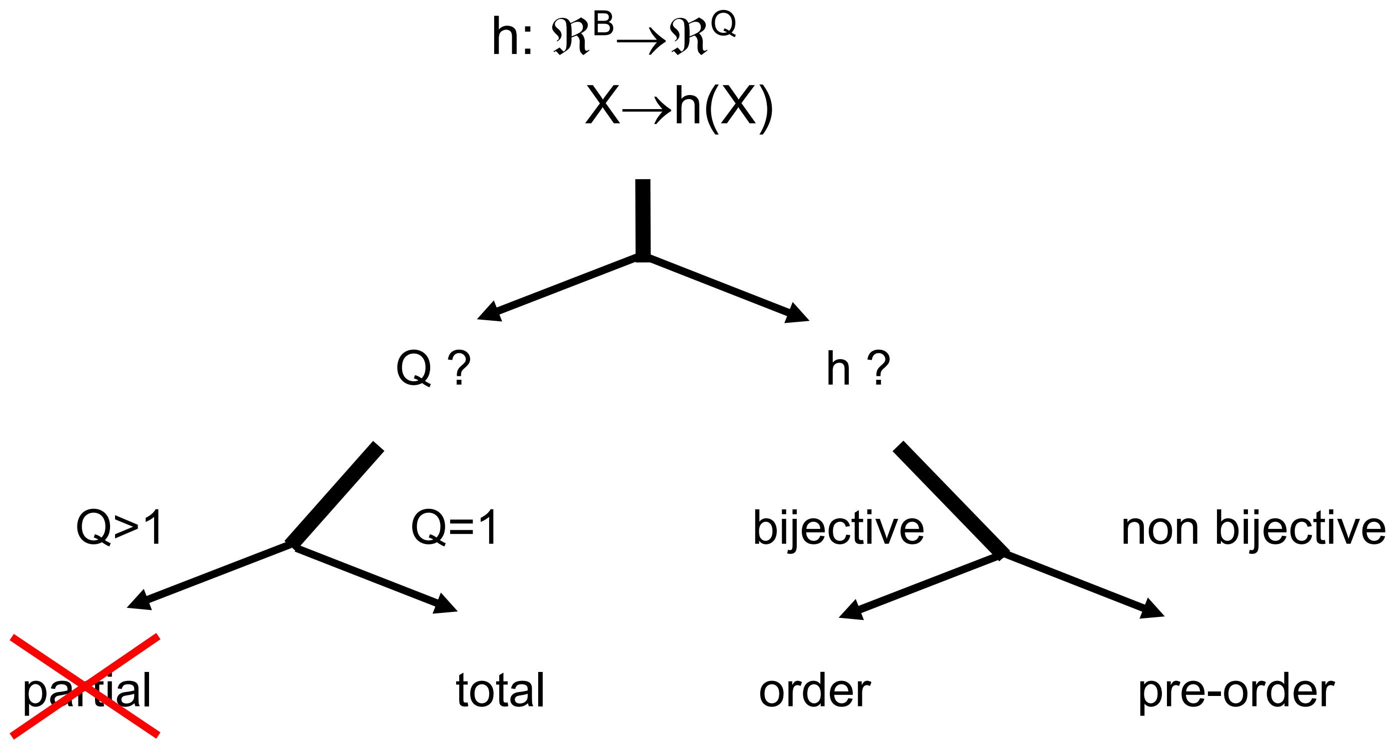
$$h: \Re^B \rightarrow \Re^Q$$
$$X \rightarrow h(X)$$

$$(X \leq Y) \Leftrightarrow (h(X) \leq h(Y))$$

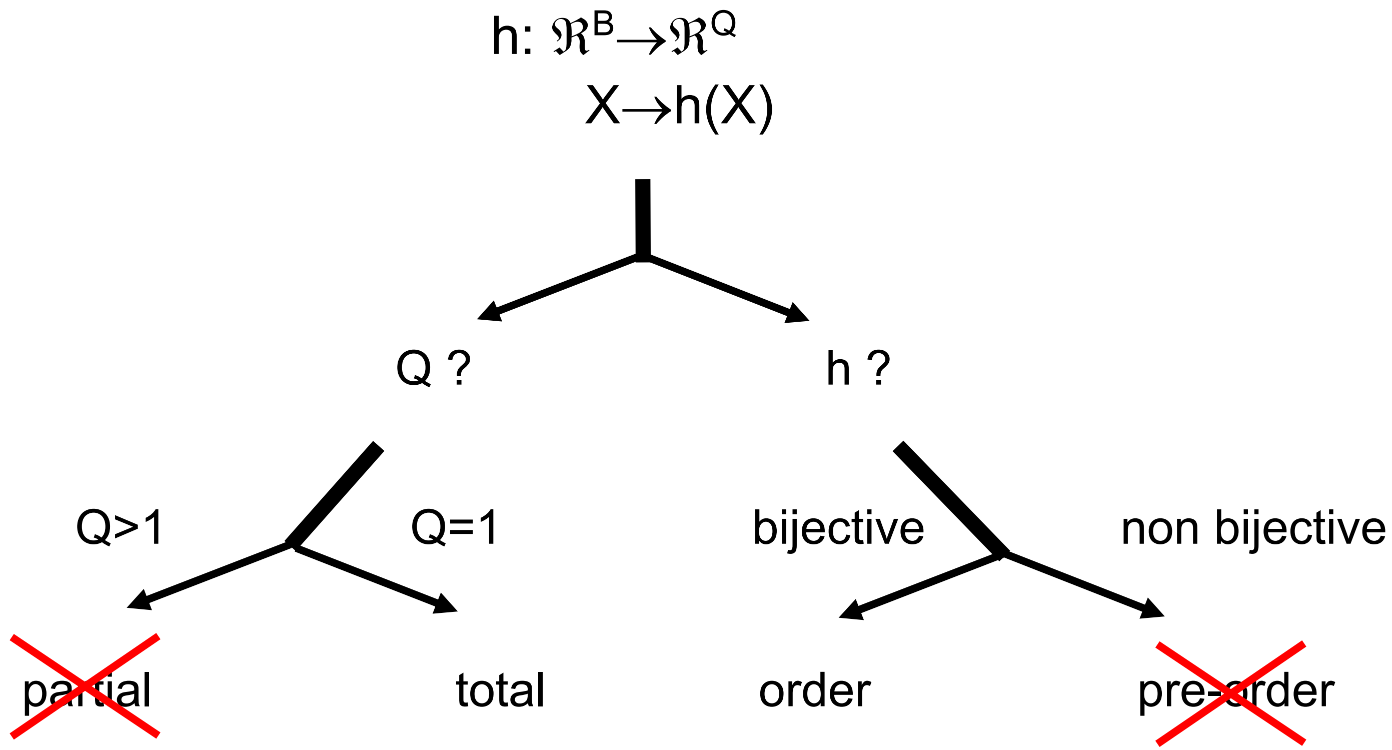
Multivariate mathematical morphology



Multivariate mathematical morphology



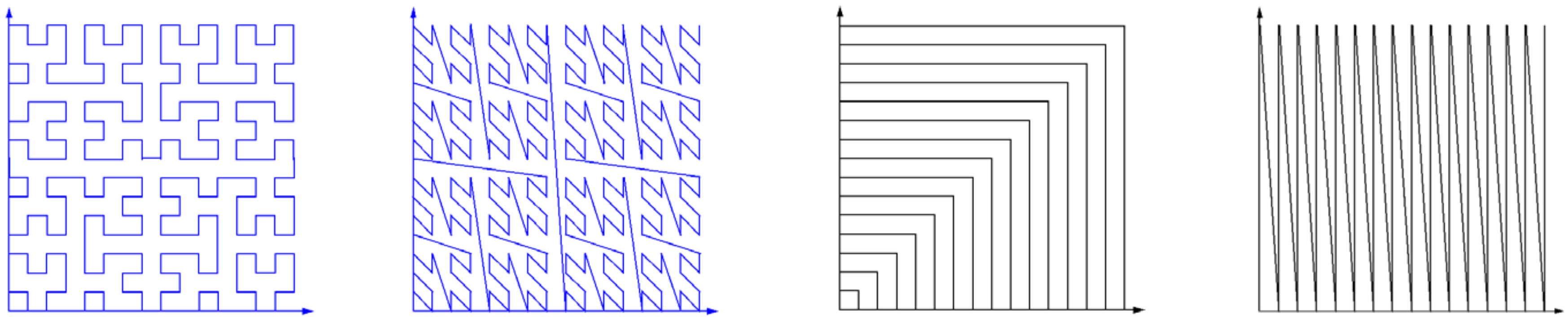
Multivariate mathematical morphology



Multivariate mathematical morphology

- Total ordering relation ⇔ bijective function h, $Q=1$

  ⇔ space filling curve

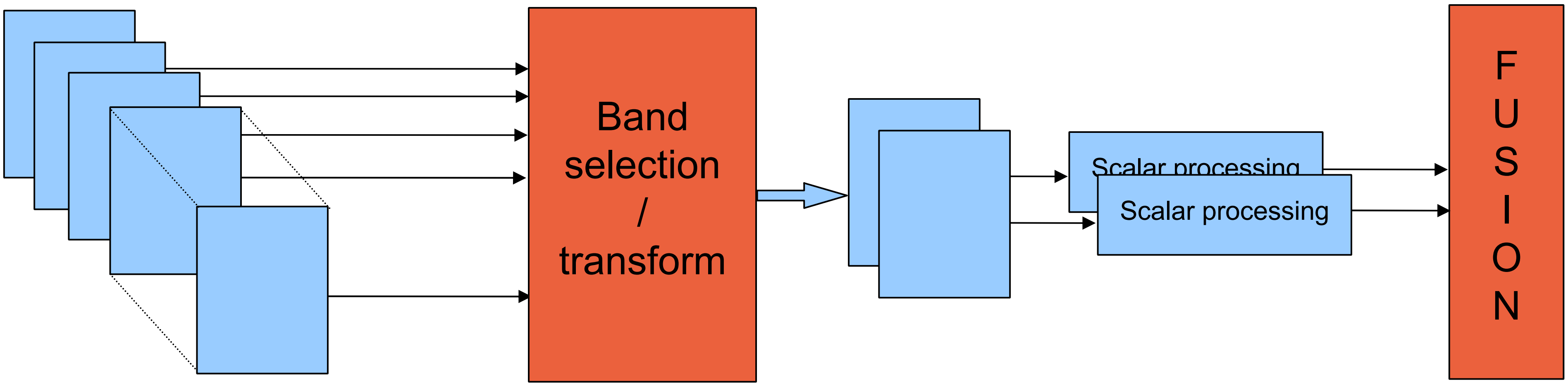


- Problem: such a mapping *h* cannot be linear

  distortion of the space topology

Multivariate mathematical morphology

- ***Mixed*** approach



- Band selection/transform: Principal Component Analysis (PCA)
  Independent Component Analysis (ICA)
  Decision Boundary Feature Extraction (DBFE)

- Fusion: Concatenate marginal results
  Decision fusion

Summary: MM for grey-scale and hyperspectral images

- MM operators can be directly extended to grey-scale images using max (sup) and min (inf) operators

- MM requires a complete lattice structure: total ordering is required
  - How shall one extend MM to hyperspectral images?
    - Marginal approach: each band is processed separately
    - Vector approach: each pixel is one vector
    - Mixed approach
      - Band selection/transform
      - Decision fusion