# Mathematical morphology

***- with emphasis on analysis of hyperspectral images and remote sensing applications***

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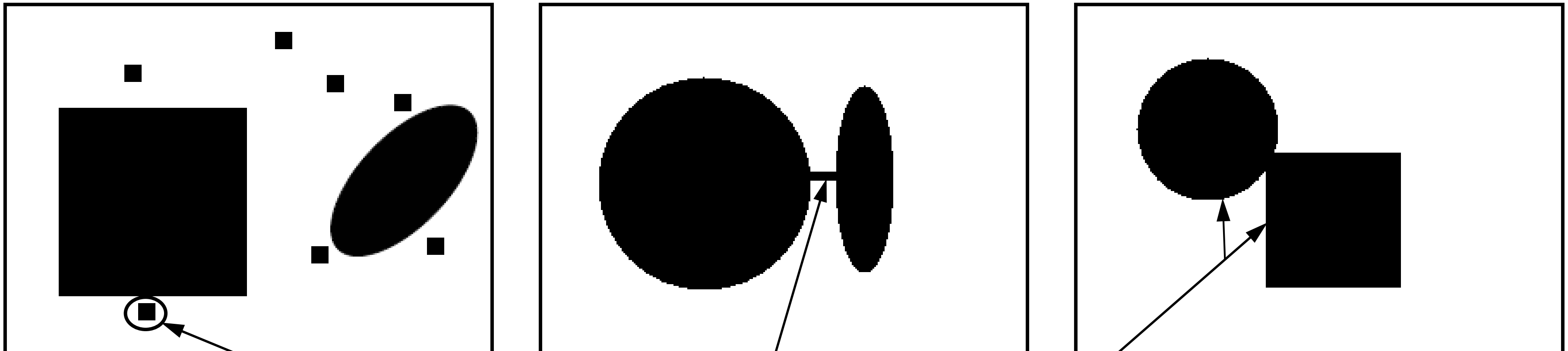
# Outline

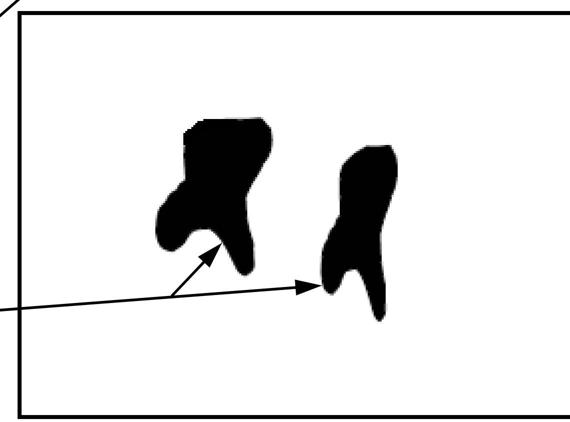
- Introduction
- Basic concepts of mathematical morphology
- Mathematical morphology for grey-scale and hyperspectral images
- Remote sensing application 1: Classification of hyperspectral images of an urban area using morphological profiles
- Remote sensing application 2: Segmentation and classification of hyperspectral images using watershed
- Practical session on mathematical morphology

# Basic concepts of mathematical morphology

# Mathematical morphology: why to use?



- How to remove this noise?
- How to separate these two components?
- How to label differently these two connected shapes?
- How to compare these two shapes?



# Mathematical morphology (MM)

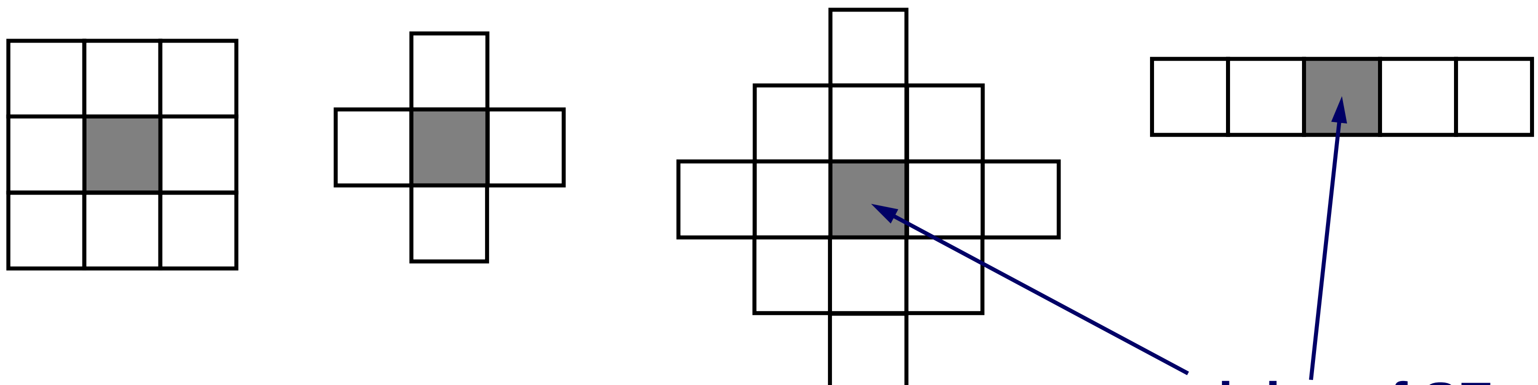
- A theory for the ***analysis of spatial structures***
- ***Morphology:*** aims at analysing the shape and form of objects.
- ***Mathematical:*** analysis is based on:
  - set theory
  - integral geometry
  - lattice algebra

- ***Non-linear*** processing operators (do not blur the edges as convolutions do)
- We'll concentrate on MM for digital images: binary, grey-scale and hyperspectral

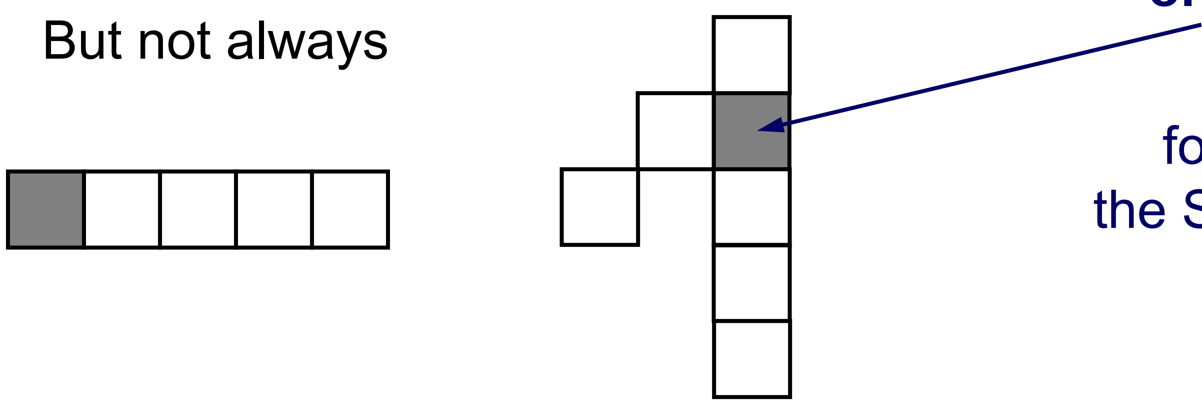
- ***Basic idea:*** locally compare structures within the image with a reference shape called the ***Structuring Element*** (SE)

# Structuring element (SE)

- A small set used to analyse locally the image
- ***Shape*** and ***size*** of SE ← a priori knowledge about the geometry of relevant/irrelevant image structures
- Usually symmetrical, connected, and convex



- But not always



**origins of SEs**

⬇

for positioning of the SE at a given pixel

# Dilation and erosion

***Fundamental morphological operators =***
***2 letters of the morphological alphabet***

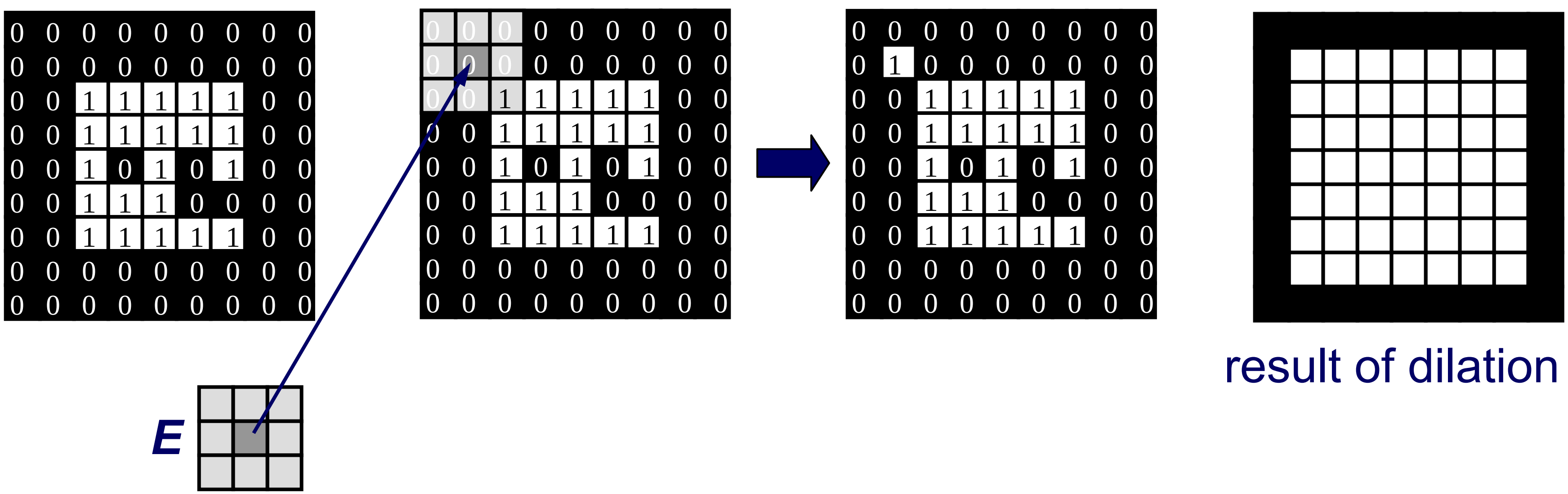
⬇

All other operators are expressed in terms of
***dilations and erosions***

# Dilation for binary images

- “Does the SE ***hit*** the set?”
- ***Dilation*** of a set $X$ by a structuring element $E$ is defined as the locus of points **x** such that $E$ ***hits*** $X$ when its origin is placed at **x**:

$$\delta_E(X) = \{\mathbf{x} \in R^d \mid E_{\mathbf{x}} \cap X \neq \varnothing\}$$



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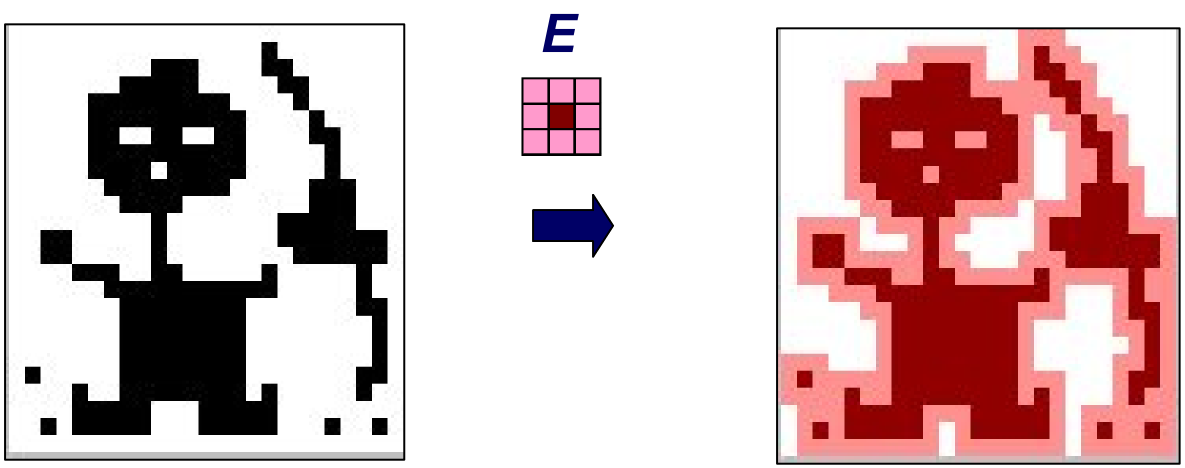
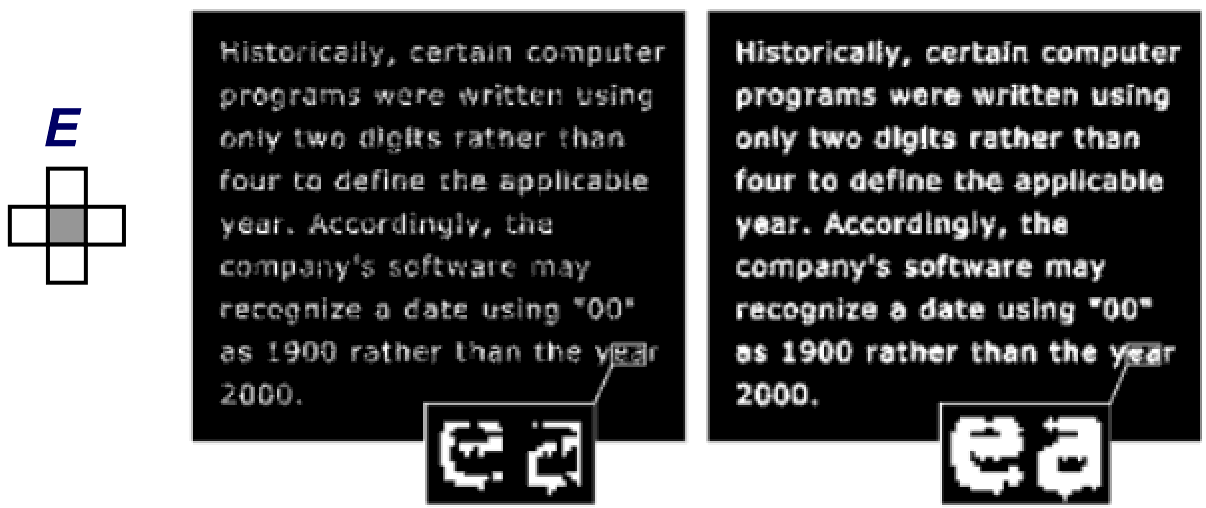



result of dilation

# Dilation: properties and use

- Basic property: $X \subseteq \delta_E(X)$
- Consequences:
  - Fill in the holes smaller than *E*
  - Enlarge capes
  - Connect two close shapes
- Example of application: bridging gaps

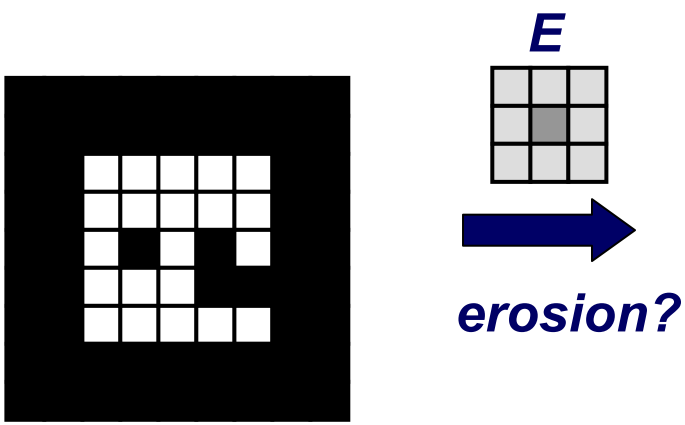



*R. C. Gonzalez, R. E. Woods, Digital Image Processing. Prentice Hall, 2002.*

# Erosion for binary images

- "Does the SE ***fit*** the set?"
- ***Erosion*** of a set $X$ by a structuring element $E$ is defined as the locus of points **x** such that $E$ is ***included*** in $X$ when its origin coincides with **x**:

$$\varepsilon_E(X) = \{\mathbf{x} \in R^d \mid E_{\mathbf{x}} \subseteq X\}$$



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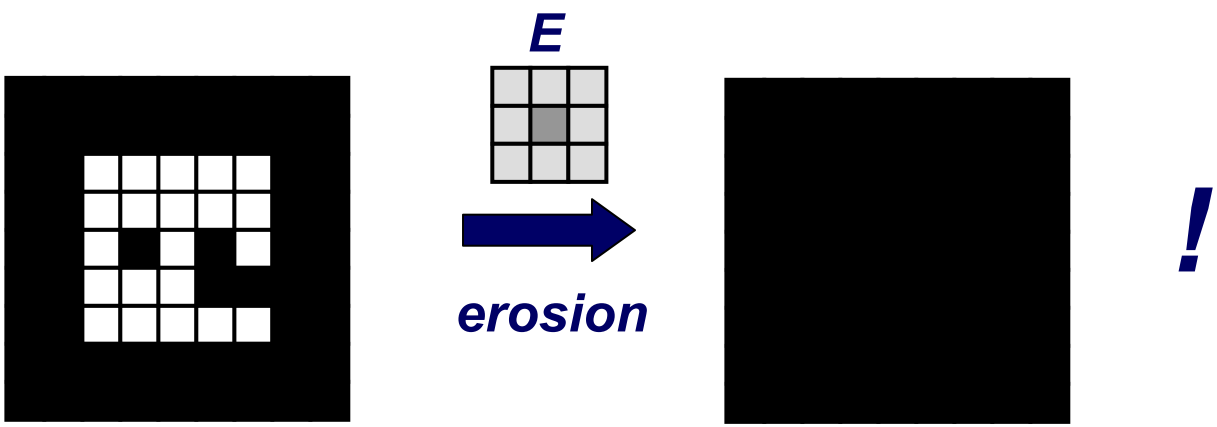

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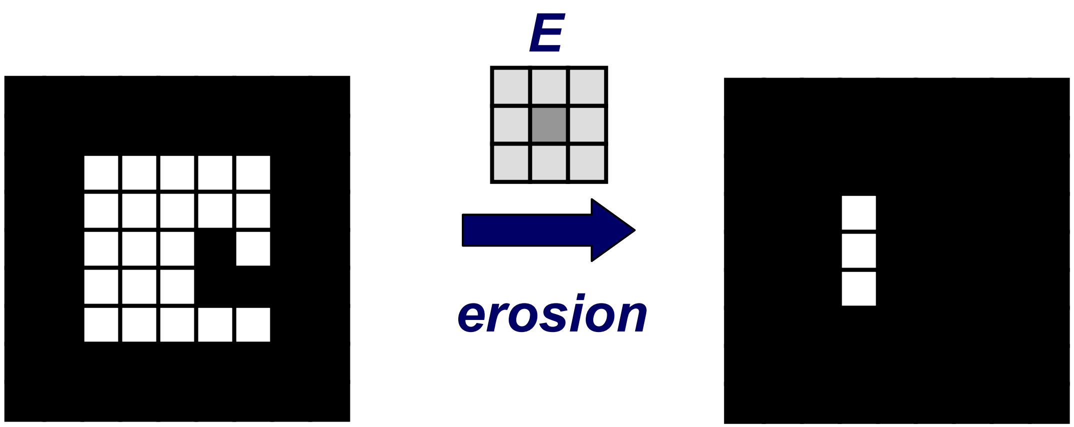

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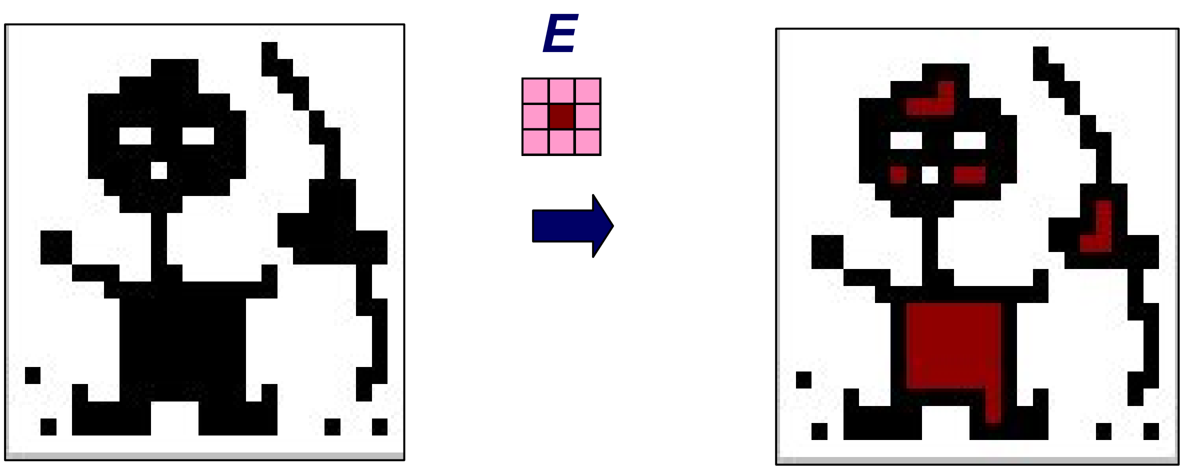
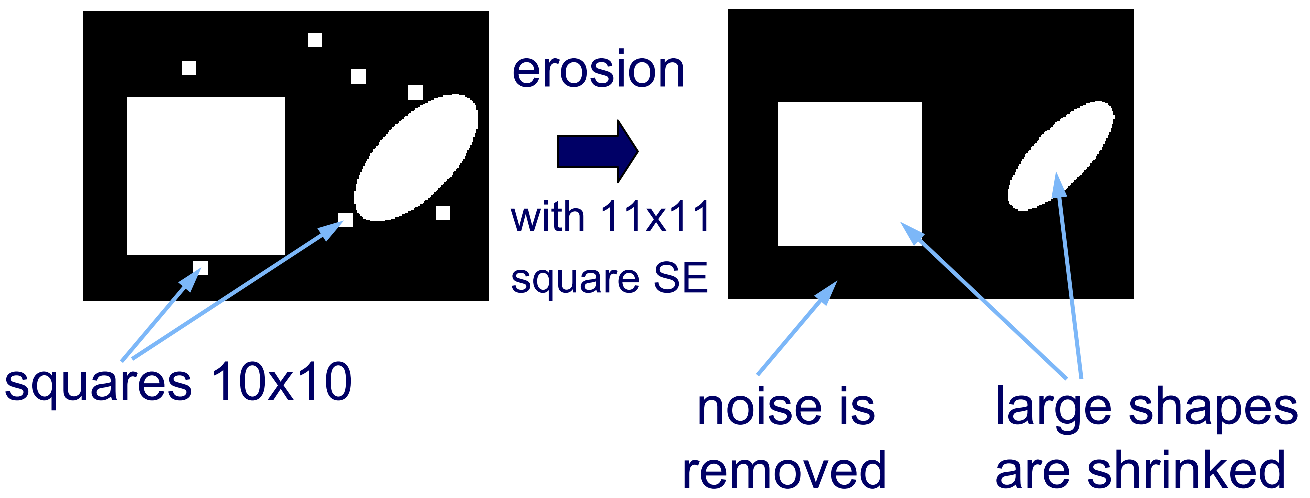



result of erosion

# Erosion: properties and use

- Basic property: $\varepsilon_E(X) \subseteq X \subseteq \delta_E(X)$
- Consequences:
  - Eliminate connected components smaller than *E*
  - Eliminate narrow capes
  - Enlarge holes
- Example of application: eliminating irrelevant details (noise)



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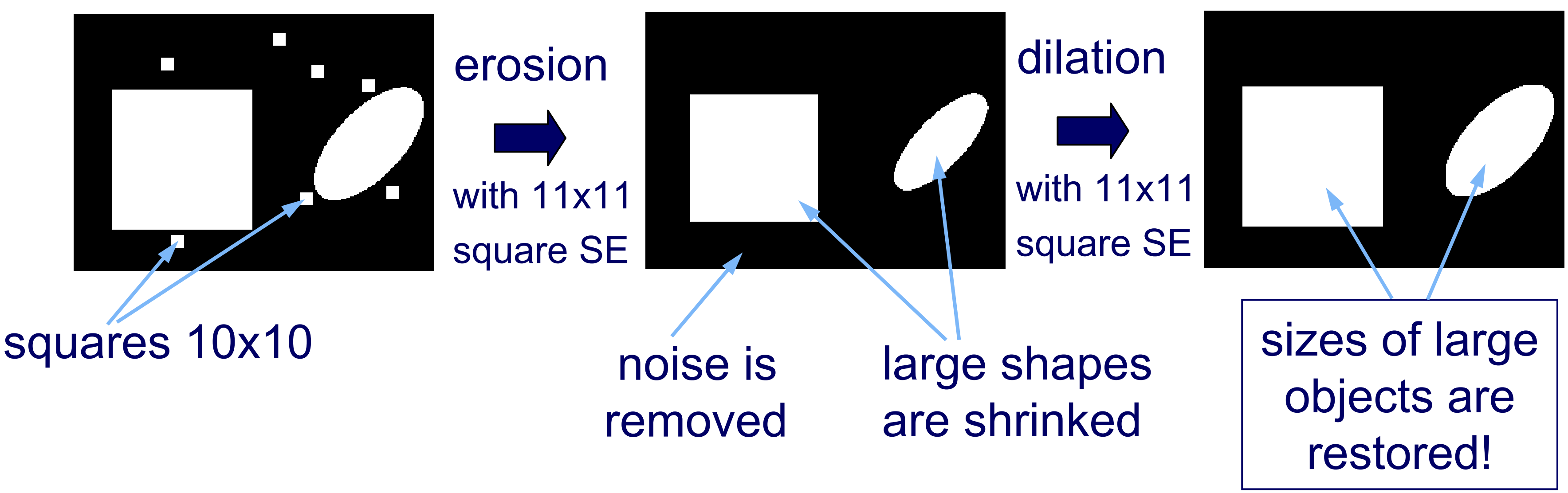
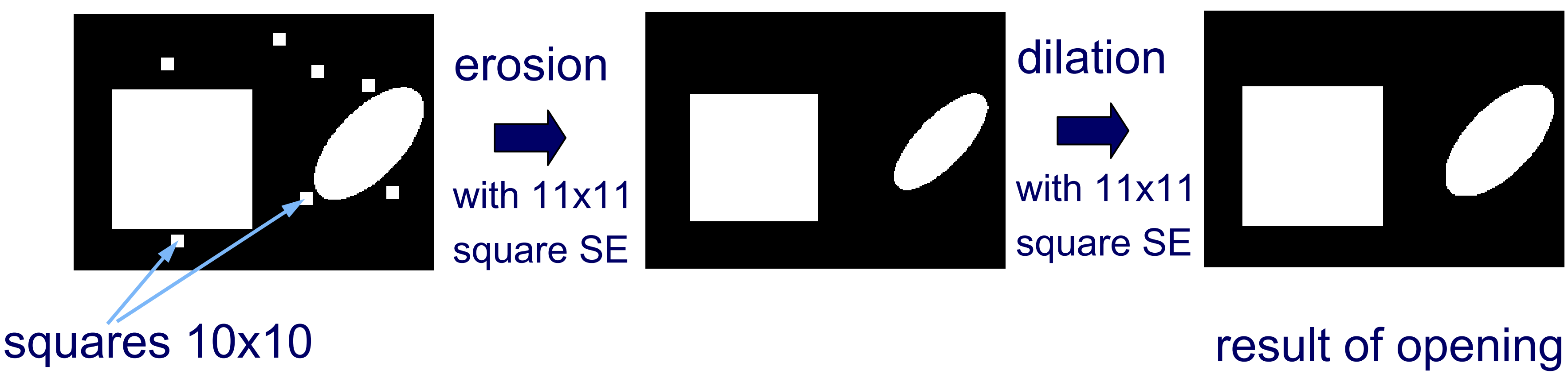


# Opening for binary images

- ***Opening*** of a set $X$ by a structuring element $E$ is defined as the erosion of $X$ by $E$ followed by the dilation with the reflected (symmetric with respect to the origin) SE $\tilde{E}$:

$$\gamma_E(X) = \delta_{\tilde{E}}[\varepsilon_E(X)]$$

- Consequences:
  - Objects smaller than $E$ disappear
  - Other objects remain "unchanged"



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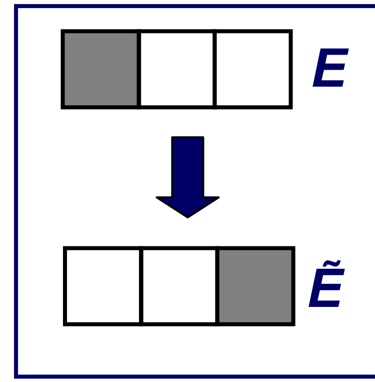



squares 10x10

result of opening

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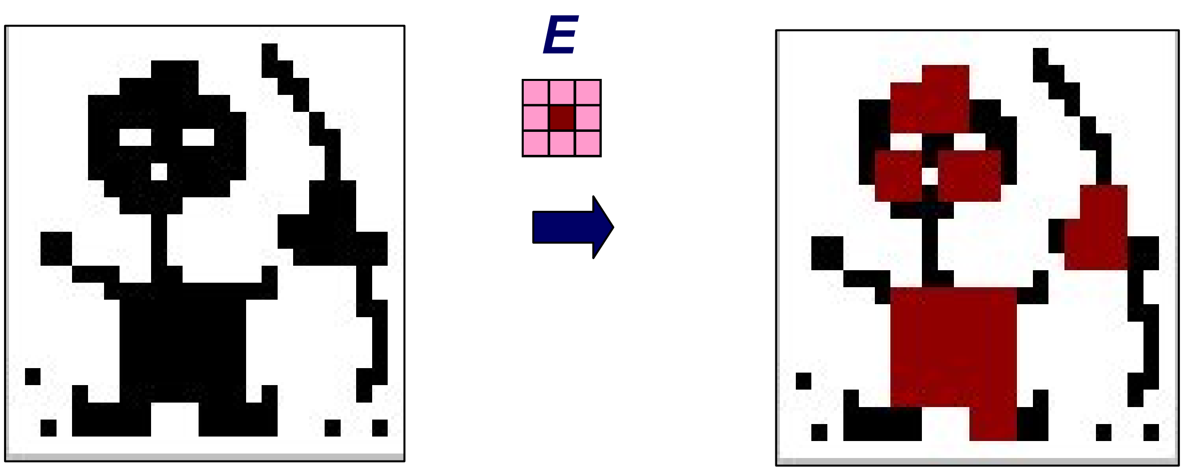


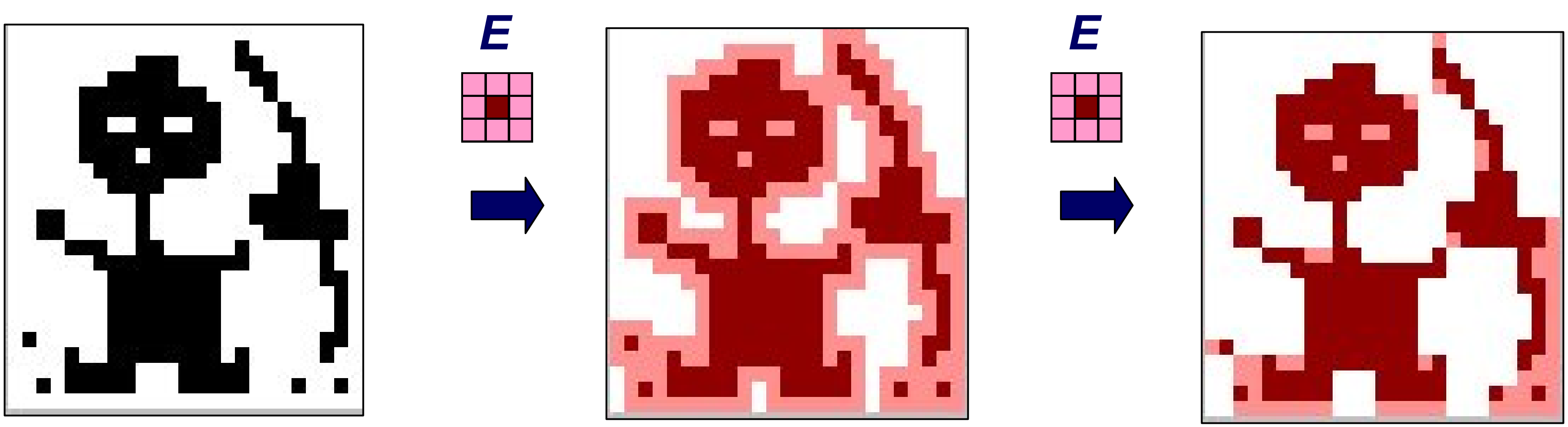

result of opening

# Closing for binary images

- ***Closing*** of a set $X$ by a structuring element $E$ is defined as the dilation of $X$ by $E$ followed by the erosion with the reflected SE $\tilde{E}$:

$$\varphi_E(X) = \varepsilon_{\tilde{E}}[\delta_E(X)]$$

- Order: $\gamma_E(X) \subseteq X \subseteq \varphi_E(X)$
- Consequences:
  - Holes smaller than $E$ are eliminated
  - Other objects remain “unchanged”




result of dilation

result of closing

# Other MM operators: Top-hat

- ***Top-hat*** transformation of an image *X* is defined as the difference between the original image *X* and its opening γ:

$$TH(X) = X - \gamma\,(X)$$

- Consequences:
  - Objects smaller than *E* are extracted
  - Other objects disappear

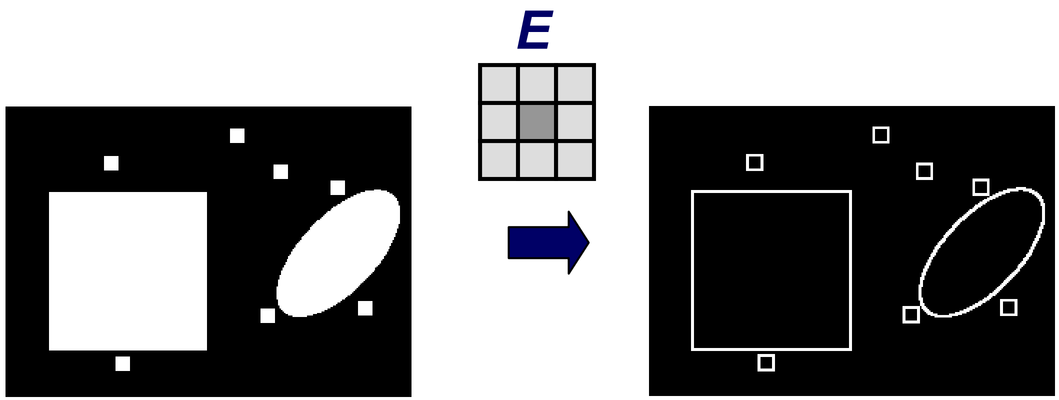
$X$ $\quad$ $\gamma_{11x11}(X)$ $\quad$ TH($X$)

# Morphological gradient

- The basic ***morphological gradient*** of an image *X* is defined as the arithmetic difference between the dilation and the erosion of *X* by the elementary SE *E* :

$$\rho_E(X) = \delta_E(X) - \varepsilon_E(X)$$

- Only symmetric SEs are considered
- Tends to depend less on edge directionality (when compared to first derivatives)




*X*

morphological gradient of *X*

# Summary: Basic concepts of MM

- Mathematical morphology can be defined as a theory for the analysis of spatial structures

- Basic idea of MM: locally compare structures within the image with a reference shape called the Structuring Element

- Dilation and erosion are two fundamental MM operators
  ➔ All other operators are expressed in terms of dilations and erosions

    - Dilation, erosion, opening, closing, top-hat and combinations of these operators are often used for image filtering