This text is an oral presentation given in May 2024 by Irene Kaltenmark at CIRM during the conference : *Geometric Sciences in Action : from geometric statistics to shape analysis.*

1 IA vs. humans : what shape space for the mind ?

1.1 Slide : title

I will present a brief historical and philosophical excursion leading to the concept of individuation.

Over the past few months, I've been troubled by a question : what would I answer to a student asking me whether AI would soon reproduce human thought or become smarter than humans. I discussed this issue with a few colleagues and my discomfort seemed to be shared. So I'd like to give you some insights from my current investigation into this problem.

1.2 Slide : Anaximander's representation of the cosmos

I will start with a kind of case study : Anaximander. Anaximander lived during the early 6th century BCE and he is the father of the geocentric model finalized by Aristotle two centuries later.

All known representations of the cosmos before Anaximander are organized along a vertical axis, the sky being always above the earth. Physicist Carlo Rovelli [1] emphasizes that Anaximander's genius lay in his explanation of how the Earth could be suspended in the air without falling. Aristotle reports that according to Anaximander, I quote "the Earth remains at rest due to its equilibrium. That which is established at the center and balanced with respect to the extremities cannot move more towards the top, the bottom, or the sides" (On the Heavens, Aristotle). Thus, Anaximander completely reinterprets the notion of gravity. Objects fall only towards the Earth.

It raises a naive question : if an AI is trained on the knowledge available to Anaximander, can it produce a geocentric representation of the cosmos ? (and few years ago, I thought my case ended there). But it also raises another question : how did Anaximander come up with the idea ?

Anaximander uses metaphors involving social concept such as Justice and technical objects. In his model, the Sun is a circle similar to a chariot wheel, filled with fire and projecting fire through a small aperture comparable to the nozzle of a blacksmith's bellows.

1.3 Slide : Vernant

For historians and Helenists, starting with Jean-Pierre Vernant [2], Anaximander might have found support in an increasingly spatialized conception of social and political practices, where notions of circles and centers are predominant. Hestia, goddess of the hearth, represents the center of domestic life and embodies the stability of the family. In greek cities, particularly the new colonies, the very important public place, the agora, is located in the center of the city. Assemblies, initially of the military aristocracy and later of the citizens, form a circular space where speakers take turns occupying the center.

1. Carlo Rovelli, *Anaximandre de Milet, ou la naissance de la pensée scientifique*, Dunod, 2009
2. Jean-Pierre Vernant. *Mythe et pensée chez les Grecs : études de psychologie historique.* Maspero, « Les textes à l'appui », 1965

1.4 Slide : Thales

Thales lived during the same period and in the same city as Anaximander. As a mathematician, I was moved by Thales' obsession with triangles inscribed in a circle. An anecdote recounts that if one were to look for Thales, they would find him drawing such figures on the ground in a temple.

1.5 Slide : Anaximandre's sources of inspiration

Additionally, it could be noted that the arts were significantly influenced by geometry, particularly in ceramics. And tops, which challenge the concept of gravity, have been common toys for centuries before Anaximander.

These hypotheses exemplify an initial facet of the concept of individuation. Cognitive functions evolve in tandem with language, technology, artistic endeavors, social and cultural practices, as well as through encounters with the world. Language and technical artifacts serve as memory supports and anchorage points for consciousness, enabling us to organize our thoughts and construct new representations through reinterpretations.

Now I have a new question : would I get a major grant if I promised within 5 years to embed this Anaximander's dataset in a shape space able to extract an invariant leading to a geocentric representation of the cosmos (maybe via some metamorphoses) ?

Of Anaximander, little is known. It allows, with a handful of speculations and audacity, to reduce his mental universe to a shape space. If the exercise is enjoyable, it nevertheless hides certain difficulties. I will try to highlight the narrowing of perspective and understanding that results from such a formulation of the problem and more generally from the eventually implicit use of mathematical concepts.

1.6 Slide : From Leibniz to Gödel

In the 17th century, Leibniz sought a symbolic language, that he named *characteristica universalis*, able of representing all knowledge in a systematic and precise manner. He wrote If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pens in their hands, to sit down to their desks, and to say to each other (...), 'Let us calculate.'

In the 19th century, the field of mathematics underwent significant transformations with the introduction of non-Euclidean geometries and the formalization of various types of infinities. This led to a rigorous inquiry into the foundations of mathematics. After decades of effort, Leibniz's dream appears to materialize, at least within mathematics. Advances in the field of logical axiomatic systems led Hilbert to declare on the radio in 1930 : "There is no ignorabimus, and in my opinion none whatever in natural science. We must know – we will know".

One year later Gödel tempered this optimism with his incompleteness theorems. But in their proof, he brought to its zenith the reduction of ambiguity in logical language, paving the way for the development of computers. Indeed, Gödel devised an algorithm to encode propositions in a logical system into natural numbers. What could be less open to interpretation than an isolated naturel number (an arbitrary index within an imaginary list) ? I believe that the predominance of quantitative data in data science tends to blur the distinction between data, digital data and their symbolic representations (words or numbers). This confusion leads to the conception of data as invariant to interpretation.

1.7 Slide : Bergson

In 1889, the French philosopher Bergson [3] highlighted how reducing time to its mathematical conception hinders our understanding of how human beings experience time. For Bergson, the time of physicists, represented by a straight line and thus assimilated to space, for which two different instants are as identical as two geometric points and can never touch each other, - this time - differs from the time experienced by consciousness. I'll only give you one example that Bergson uses to illustrate this : the perception of a melody. The notes don't just resonate, they penetrate and affect each other. Some notes call for the following. Similarly, thoughts do not follow one another as discrete and distinct states of mind. They have the corporeality of a melody.

When thoughts are viewed as rigid data, one loses sight of these metamorphoses that occur between consecutive thoughts as they cohesively organize together. Furthermore, a thought does not vanish abruptly. It likely fades gradually into saliences, prominences or impressions, which can condition and be shaped by subsequent thoughts. This malleability of thoughts likely contributes to the possibility of fruitful and unexpected associations of ideas.

Let's open a brief mathematical parenthesis - while slightly betraying Bergson. Let us follow the musical metaphor and suppose that we can segment elements of the stream of consciousness like the notes of a melody. Each note is then a trajectory of shapes, attached to increasing scales. However, this trajectory depends on the context in which the note was played, so it would rather be a set of trajectories of shapes. Consequently, two thoughts, represented by such sets of trajectories, might only be related at the level of a subset of trajectories, potentially across different scales.

This attempt to pretend that this talk has a shape analysis content actually serves another purpose : to invite you to think about quantifying similarity and to introduce a slight complication.

It was while reading Giuseppe Longo [4] that I thought of Bergson, as he presented information in computer science as the mathematical invariant derived from the equivalence of the various definitions of an algorithm, proposed by logicians at the beginning of the 20th century. However, the association I did with Bergson did not arise immediately from a similarity in meaning but from the recognition of the emotion felt when reading Bergson. For contrasting a mathematical concept with an associated real phenomenon highlights both the elegance of expressive simplicity and the marvelous complexity of reality.

1.8 Slide : Individuation, retentions, interpretation

As we've just seen, Bergson's critique of the reified time of mathematics also applies to data reduced to their symbolic representation. We will now extend this critique by delving into the long term.

To think that similarity functions exist a priori is to believe in the immediacy of meaning identification and to overlook the fact that the development of new concepts and practices evolves through gradual progression and requires the reliance on memory.

The content of consciousness, with now all the richness of Bergsonian duration, is what Husserl refers to as primary retentions. Consciousness selects and retains perceptions from the flow it embodies. It synthesizes and organizes these perceptions to generate meaning. This process is individualized. For instance, each of you is currently attributing a slightly different meaning to my words, appropriating them according to your personal language and memory. Secondary retentions are the primary retentions preserved by memory. According to philosopher Bernard Stiegler, primary and secondary retentions retroact upon each other. He also introduces, with Simondon,

3. Henri Bergson, *Essai sur les données immédiates de la conscience*, Félix Alcan, 1889
4. Longo, Giuseppe. *Le cauchemar de Prométhée : les sciences et leurs limites.* PUF, 2023

a third type of retention : tertiary retentions. These are collective memories engrammed in practices, books, and technical objects. This is what I showed you with Anaximander. The spinning top retains a memory of the observation, even if not formalized, of the effects of centrifugal and centripetal forces.

The term 'individuation' characterizes the individual as a process rather than a state, as the ongoing formation of the individual immersed in his environment. If the brain employs elaborate similarity functions, these are not pre-given ; rather, they seem to be constituted through a metamorphosis of the individual in the quest for meaning, both during the development of new concepts and their reception. This is exemplified, for instance, by the variable duration of familiarization required to fully apprehend the work of colleagues.

So, unlike the supercomputers at our disposal today, these metamorphic processes require allowing time and efforts to reinterpret phenomena in a non-superficial manner. Stiegler emphasizes that mechanisms of attention capture and desire anticipation deprive individuals of their capacity to individuate. Nietzsche may have been the first to express concern, as early as his own time, about a weakening of the capacity for individuation.

1.9 Slide : Nietzsche : le vivant et la confrontation au monde

To provide another perspective on the concept of individuation, I will conclude this first presentation with some highlights of the work of philosopher Barbara Stiegler [5] on Nietzsche's biology.

Nietzsche zealously studies the scientific literature of his time in biology. He concludes that living beings are characterized by their ability to be irritated and altered by their environment. He introduces the notion of incorporation, which generalizes the concept of nutrition to encompass the totality of organic exchanges between living beings and their environment. Nietzsche writes, I quote : *The power of the spirit to appropriate foreign elements reveals itself in a strong tendency to assimilate the new to the old, to simplify the manifold, to overlook or repudiate the absolutely contradictory (...) (and in fact "the spirit" resembles a stomach more than anything else).*

Similar to digestion, the organism strives to transform otherness into the familiar. However, there always remains an irreducible part that is then repressed into memory. Memory thus becomes the capacity to integrate the other within oneself and to retain its trace. The human being is then perceived as struggling to construct their own identity through this incorporation process. When the living innovates and transforms, it is this past and repressed otherness that resurfaces as a dominant and becomes part of the identity of this living being.

Nietzsche identifies the most complex organisms as the most sensitive to their environmental and the most capable at incorporation. He writes that the human body is the wonder of wonders. Its capacity for incorporation is the condition for evolution, and its internal struggle defies determinism. However, Nietzsche observes that the environment, always a continuous flow of monstrous diversity, sees its rhythm accelerate after the Industrial Revolution and the advent of the telegraph. Yet again, individuation, similar to digestion, requires time. According to Nietzsche, people who let themselves be exposed to too much excitement can no longer digest. They close themselves off from the world by becoming hypersensitive, allowing themselves to be affected only superficially and within the immediacy of the present.

In our age of constant technological evolution, Bernard Stiegler calls, regarding the use of technologies, for the development of social processes that aim to enhance our capacities for individuation. He advocates, much like the evolution of living organisms, for local experiments that allow time for reflection and deliberation.

5. Stiegler, Barbara. *Nietzsche et la vie. Une nouvelle histoire de la philosophie.* Gallimard, 2021

1.10 Slide : conclusion, motivations

Few words of conclusion. French higher education establishments are currently aiming to integrate a course on the environmental crisis into all their courses. This question about AI is a pretext to think more broadly on integrating a philosophical perspective into these educational projects on the environment. Additionally, facing the contrast between the enthusiasm for technological achievements and the indifference towards the mass extinction of species, I try to think how to incorporate into my mathematics teaching a broader perspective on its language and models to reemphasize the real world and its biodiversity, and ideally to invite renewed wonder at the complexity of life.