Revisiting time discretisation of spiking network models

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A link is built between a biologically plausible generalized integrate and fire (GIF) neuron model with conductance-based dynamics and a discrete time neural network model with spiking neurons, for which rigorous results on the spontaneous dynamics has been obtained.

I: Main Theoretical Result:

- Occurrence of periodic orbits
  is the generic regime of activity,
  with a bounded period in the presence of plasticity (S.T.D.P.),
  and arbitrary large periods at the edge of chaos
  (indistinguishable from chaos in numerical experiments)

- The dynamics of membrane potential has a one to one correspondence
  with sequences of spikes patterns ("raster plots")
  in the asymptotic regime.

-> better insight into the possible neural coding;
-> network level understanding of the system behaviour;
-> weak form of initial conditions sensitivity
due to the presence of the sharp spiking threshold

Cessac (2007)

II: Improving the Basic Model (BMS)

$$V_k(t+1) = \gamma\, V_k(t)\ [1 - Z(V_k(t))] + \left[\sum_{j=1}^{N} W_{kj} Z(V_j(t)) + I_k^{(ext)}\right]$$

- The membrane potential V() is the state variable
- This is a time discretisation of a leaky integrate and fire model
- Here Z(x) = 0 when x < threshold and = 1 otherwise (firing)
- Only current synapses are considered

-> how can this extends to a more realistic model ?

Soula Beslon Mazet (02)

III: Reduction of a bio-physical model

- We consider a "gIF" bio-physical model with:
  - Condutance based synapses
  - Adaptive and non-linear currents
  -> and propose a functional reduction to a leaky IF model
  which parameters depends on the raster plot only

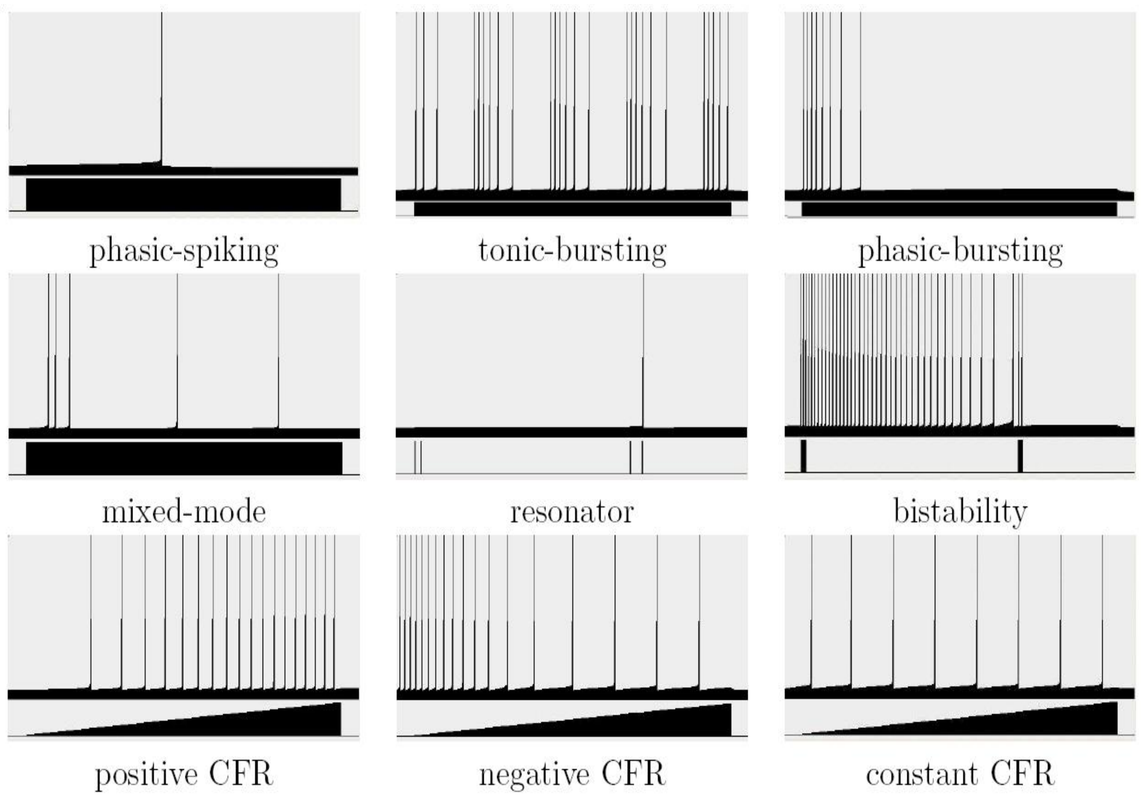
- We use an unbiased discretisation scheme
  - Calculated at the network level
  -> and obtain a reduction in terms of a gBMS
  where parameters depends on the raster plot history

- **The main theoretical results in I seems preserved !**

Cessac Viéville (2007)

IV: Performance of a gBMS model of a neuron

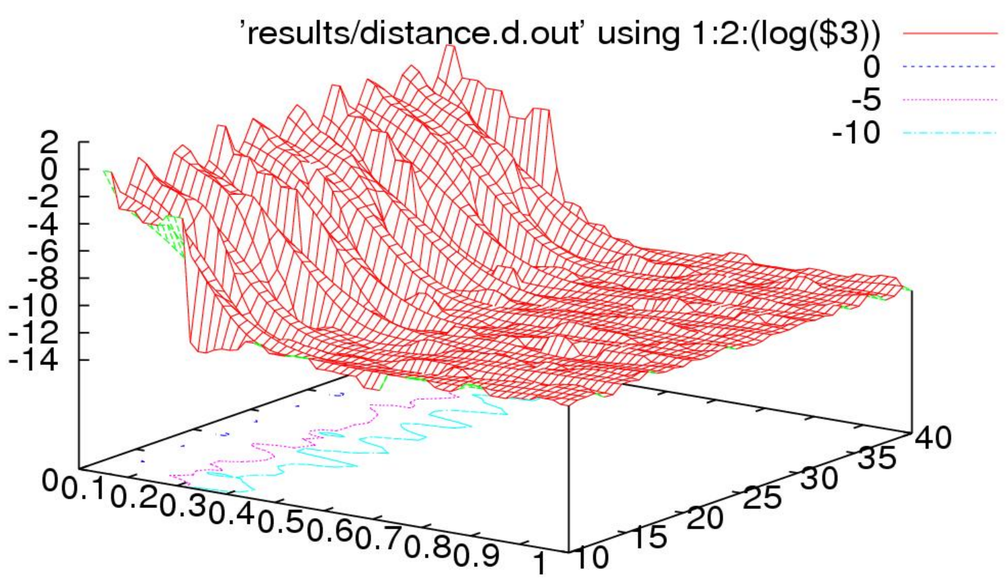
- From the bio-physical reduction the resulting "Yvette" model is
  - A raster plot dependent leaky Integrate and Fire
  - With a bi-stable raster plot dependent adaptive current
  -> yielding a very large class of event-based behaviours



Typical results showing the versatility of the reduced model for spiking, bursting and other modes, including different current-frequency-responses (CFR). For each mode, the upper trace shows the action potentials, the lower trace the input current.

V: Numerical results with a gBMS network

- In order to numerically study the Yvette network dynamics the minimal average distance to the firing threshold singularity set is measured, as proposed and discussed in Cessac 2007.
- Huge simulations have been conducted on the INRIA cluster to carefully study the network behaviour.



Typical results showing the evolution of the dynamics for a membrane leak between 10-40 ms and a 0-1 synaptic weight relative dispersion. The gBMS network behaviour is related to the BMS except for the occurrence of leak dependent periodic variation. . still in study.

A step further, constructive conditions are now derived, in order to properly implement visual functions on such networks . .

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