CORTEX INRIA NANCY

Let us adjust CNFT parameters not [only] by chance ...

On practical neural field parameters adjustment

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(*Partially supported by the ANR MAPS*)

Methods

Linear continuous neural maps

$$\mathbf{u} : \mathcal{R}^n \rightarrow \mathcal{R}^m$$

$$\begin{cases} \tau\, \dot{\mathbf{u}}(\mathbf{p},t) = -\mathbf{u}(\mathbf{p},t) + \int_{\mathbf{q}} \mathbf{W}_\theta(\mathbf{p}-\mathbf{q})\, \mathbf{u}(\mathbf{q},t) + \mathbf{i}(\mathbf{p}) \\ \mathbf{u}(\mathbf{p},0) = \mathbf{i}(\mathbf{p}), \end{cases}$$

input **i(p)** output **u(p)** interactions **W(q)**: *neural field* (NF).

- vectorial computational maps (filtering, selection, tracking, ..)
- biologically plausible model of cortical columns maps

Excitatory/inhibitory inhibition, e.g.: $\theta = (\mathbf{A}_+, \mathbf{A}_-, \sigma_+, \sigma_-)$.

$$\mathbf{W}_\theta(\mathbf{q}) = \mathbf{A}_+\, e^{-\frac{|\mathbf{q}^2|}{\sigma_+^2}} - \mathbf{A}_-\, e^{-\frac{|\mathbf{q}^2|}{\sigma_-^2}},$$

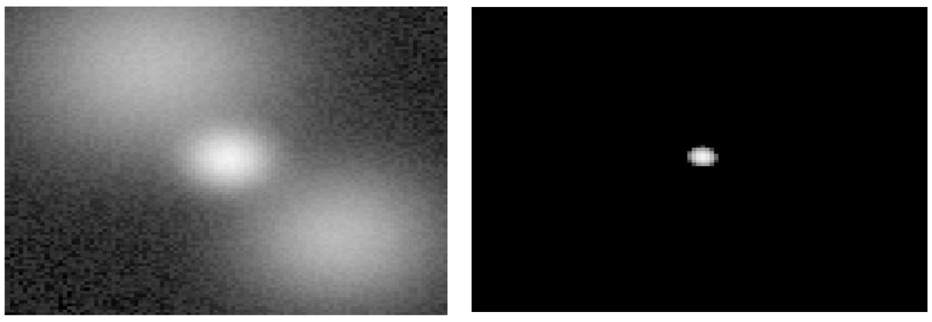
Synchronous non-linear discretized maps

$$\mathbf{p} \in \{0..N\{^n$$

$$\mathbf{u}(\mathbf{p},t+1) = \rho\left(\mathbf{u}(\mathbf{p},t) + \delta\left(-\mathbf{u}(\mathbf{p},t) + \sum_{\mathbf{q}} \mathbf{W}_\theta(\mathbf{p}-\mathbf{q})\, \mathbf{u}(\mathbf{q},t) + \mathbf{i}(\mathbf{p})\right)\right)$$

for $0 < \delta < 1$ while $\rho(u) = u > 0$? $u : 0$ stands for *rectification*.

Experimental result: $\rho()$ *allows to implement all NF functionalities.*



Example: noise filtering + target selection from input (left) to output (right).

Bumps as required output: a formal definition

- radial symmetric
- positive & decreasing

$$\mathbf{b}(\mathbf{p}) = \beta(|\mathbf{p}|^2),$$

→ Using *Gaussian series.* $\mathbf{b}(\mathbf{p}) = \sum_s \mathbf{b}^*(s)\, e^{-s\,|\mathbf{p}|^2},$

thus allowing to ``parameterize´´ bumps.

Results: about convergence

Writing: $\bar{\mathbf{u}}(t+1) = \mathbf{F}(\bar{\mathbf{u}}(t)) \doteq \rho\left(\bar{\mathbf{K}}\, \bar{\mathbf{u}}(t) + \delta\, \bar{\mathbf{i}}\right),$

$$\bar{\mathrm{K}} = 1 - \delta\left(1 - \bar{\mathrm{W}}_\theta\right)$$

convergence is related to *contractive mapping*:

$$\left|\frac{\partial \mathbf{F}}{\partial \mathbf{u}}\right| = \left|\Sigma'\, \bar{\mathbf{K}}\right| < 1$$

$$\Sigma' = \mathrm{diag}(\cdots, \rho'_i, \cdots) \quad \rho'_i = \rho'\left(\bar{\mathrm{K}}\, \bar{\mathrm{u}}(t) + \delta\, \bar{\mathrm{i}}\right) \in \{0,1\}$$

thus in the linear case:

$$|\bar{\mathrm{K}}| < 1 \Leftrightarrow |\bar{\mathrm{W}}_\theta| < 1 \text{ as soon as } 0 < \delta < 1,$$

while in the non-linear case it is sufficient to bound the excitatory weights:

$$|\bar{\mathbf{W}}_\theta \vee 0| < 1$$

- calculation of the highest matrix eigen-value of the positive weights
  - → *straightforward (power method),*
  - → *effective: given* Θ, the related stable parameters are just scaled.
- sufficient condition; weakest conditions iff the input is constrained.

It translates ``compact operator´´ conditions to something . . usable.

Implemented in http://enas.gforge.inria.fr

Results: about bump's shape

Gaussian series approximate bumps

$$\beta(r) = \sum_s \mathbf{b}^*(s)\, e^{-s\, r} = \mathcal{L}(\mathbf{b}^*)(r)$$

since, it is no more than the β radial function Laplace transform,
yielding all related properties (positivity, convergence, asymptotic, ..).

Gaussian series are linear maps solutions

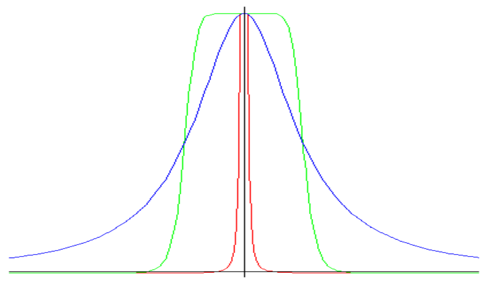
If **i(p)** and **W(q)** are Gaussian series, so is **u(p)**:

$$-u^*(\nu) + \pi \sum_{r>\nu, s>\nu, \frac{r\,s}{r+s}=\nu} \frac{W(r)\, u^*(s)}{r+s} + i(\nu) = 0,$$

defined by a linear equation.

→ If **i(p)** and **W(q)** are radial symmetric, so is **u(p)**.

Gaussian series yield ``nice´´ bumps



- well-defined
- malleable (sharp, flat, ..)
- closed-form solution

$$\beta_1(r) = 1/(1+|r|^{n/2})$$

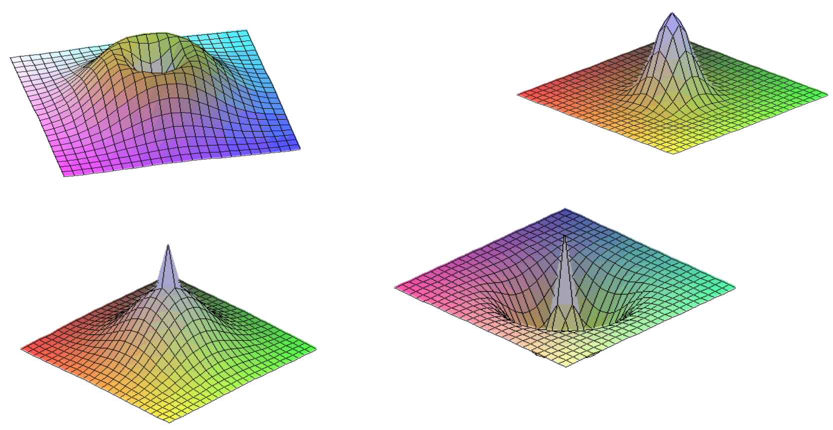
Example: rational functions

$$\begin{aligned} b_1(s)^* &= \frac{1}{n} \sum_{i=1}^n e^{-s\, \sin(\alpha_n^i)} \sin\left(s\, \sin\left(\beta_n^i\right) + \alpha_n^i\right) \\ \alpha_n^i &= \frac{\pi}{n}\left(2\,i + \frac{n}{2} - 1\right) \\ \beta_n^i &= \frac{\pi}{n}\left(1 - 2\,i\right) \end{aligned}$$

Numerical experiment: input control

What are the input profiles yielding a Gaussian output with linear maps ?

→ obvious to answer in this framework



Examples: visualization of four input profiles yielding a Gaussian output depending on the Θ parameters. It illustrates the miscellaneous profiles described by Gaussian series.

Perspectives:

- Bump control with non-linear discrete map
- Effective parameter tuning algorithms (as for convergence)

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