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Self-Organizing Receptive Fields Using a Variational Approach

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Goals

- Kohonen Self-Organizing maps are interesting computational structures because of their original properties, including adaptive topology and competition, their biological plausibility and their successful applications to a variety of real-world applications.
- Here, this neuronal model is presented, together with its possible implementation with a variational approach. We then explain why, beyond the interest for understanding the visual cortex, this approach is also interesting for making easier and more efficient the choice of this neuronal technique for real-world applications.

Self-organizing-maps specification

Given an input sequence of retinal ``icons'':

$$x_\tau(\mathbf{p}), \text{ with } \mathbf{p} = \{0 \cdots S\{^2 \text{ and } \tau \in \{0 \cdots T\{$$

and a 2D neuronal map of linear neurons of equation

$$z_{\tau\nu} = \mathbf{x}_\tau^T \, \mathbf{w}_\nu \text{ with weights } w_\nu(\mathbf{p}) \text{ and } \nu \in \{0..N\{^2$$

we consider the unsupervised learning of the neuronal weights minimizing

$$\mathcal{L} = \int_\tau \int_\nu \int_\mathbf{p} \underbrace{\psi(|\mathbf{x}_\tau - \mathbf{w}_\nu|^2)}_{\text{input}} + \int_\nu \int_\mathbf{p} \underbrace{\phi(|\nabla_\mathbf{p} \mathbf{w}_\nu|)}_{\text{intra-neuron}} + \int_\nu \int_\mathbf{p} \underbrace{\xi(|\nabla_\nu \mathbf{w}_\nu|)}_{\text{inter-neuron}}$$

where (classical self-organizing map learning rule)

$$\psi'(u) \equiv e^{-\frac{\tau}{T} + \frac{u^\alpha}{\tau^2}}, u = d(\nu, \arg\max_\nu |\mathbf{x}_\tau - \mathbf{w}_\nu|)^2$$

while non-linear diffusion is specified via $\phi'()$ and $\xi'()$

yielding:

$$\dot{w}_\tau \equiv -\nabla_\mathbf{w} \mathcal{L} \equiv \psi'\left(|\mathbf{x}_\tau - \mathbf{w}_\nu|^2\right)(\mathbf{x}_\tau - \mathbf{w}_\nu) - \phi'(|\nabla_\mathbf{p} \mathbf{w}_\nu|)\Delta_\mathbf{p} \mathbf{w}_\nu - \xi'(|\nabla_\nu \mathbf{w}_\nu|)\Delta_\nu \mathbf{w}_\nu$$

Convergence à la Lyapounov is obvious flw e.g. Kanstein and Goser
This formulation includes non-linear SOM as proposed by Fort and Pages.

Implementing the WTA mechanism in this context.

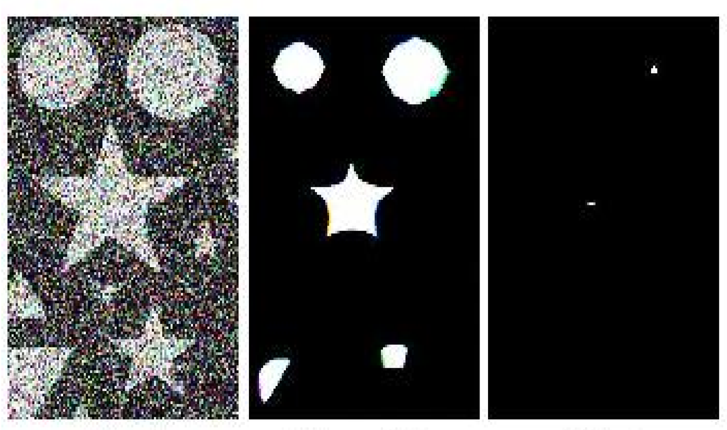
$$\bar{v} = \arg\min_v \int (w - v)^2 + \int |\nabla v|^2 + \int \psi(v) \qquad v_{t=0} = w$$

$$\psi(v) = v^{2\,t/(1-t)} (1 - v)^2 : [0, 1] \rightarrow \mathcal{R},$$

bi-modal $(\psi(0) = \psi(1) = 0, \text{ max at } t > 1/2)$

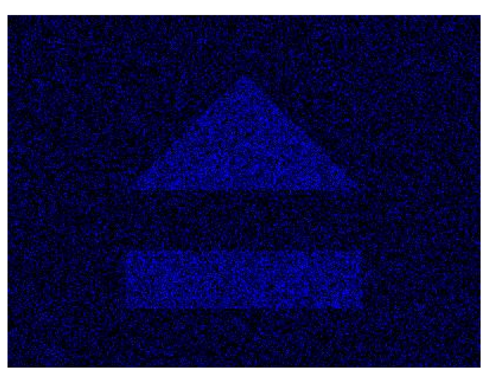
(here t increases with time to speed-up convergence)

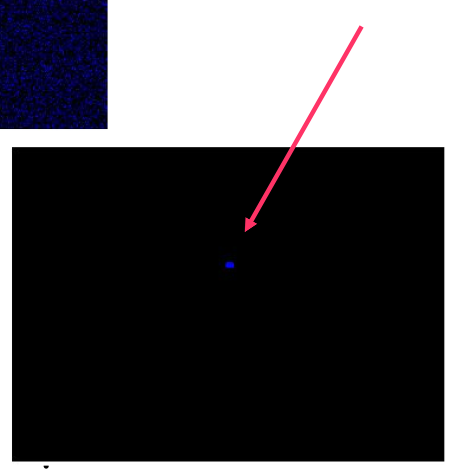
This formulation leads to a local distributed implementation
and guaranties the convergence towards a local minimum of the criterion.



Input *Intermediate* *Output*

Fig. 5. One example of result for the winner-take-all mechanism implemented using the proposed method. The very noisy (more than 80%) original image is on the left; the intermediate result shows how diffusion is combined with erosion yielding the final result, shown also with a zoom. Clearly the focus is given on the main structures of the image. We have experimented a correct behavior on many different inputs.





Winner take all in a very noisy case.

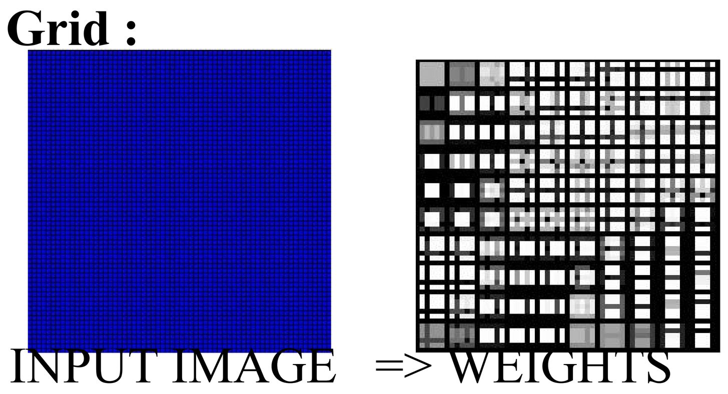
References

A non-linear Kohonen algorithm, J.C. Fort, G Pagès, ESSAN'94

Self-organizing maps based on differential equations, A Keinstein, C. Groser, ESSAN'94

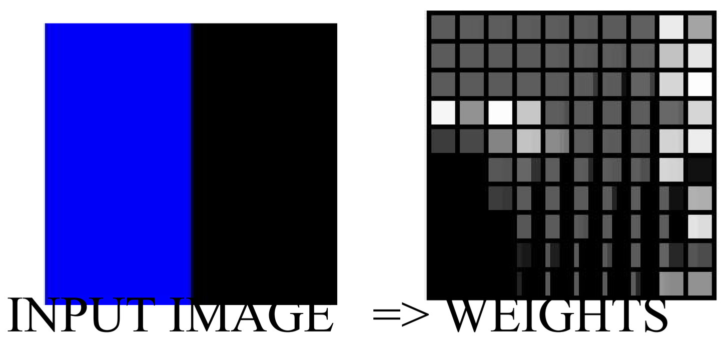
Preliminary experimental results

Grid :



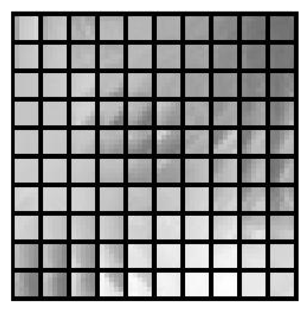
INPUT IMAGE => WEIGHTS

Black/White:



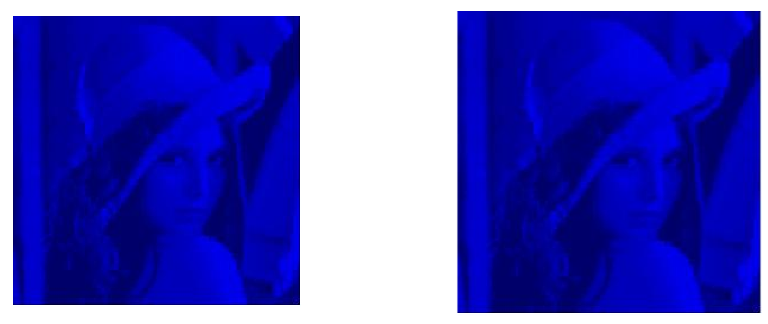
INPUT IMAGE => WEIGHTS

No input:



WEIGHTS
(internal regularization)

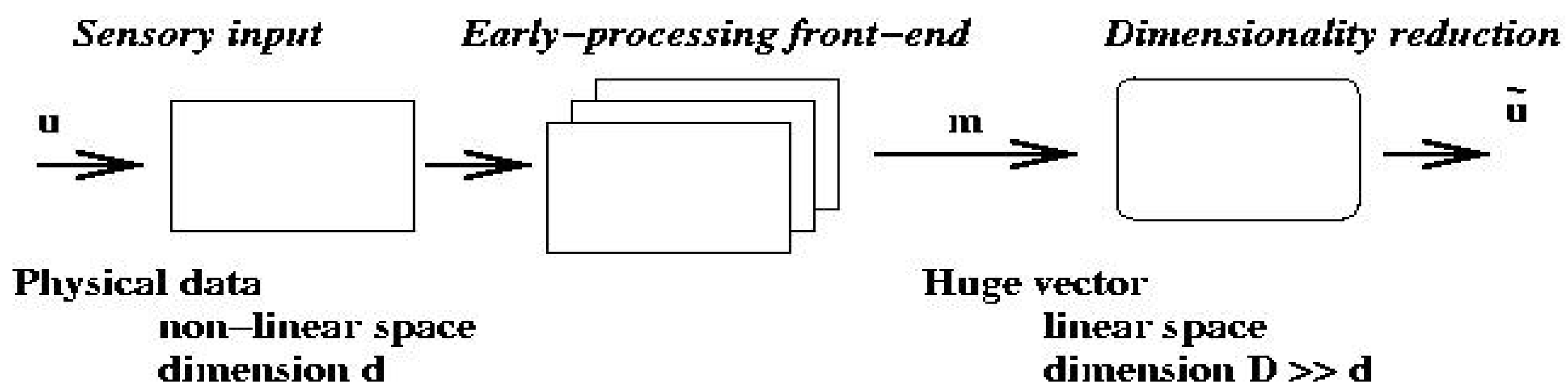
Lena:



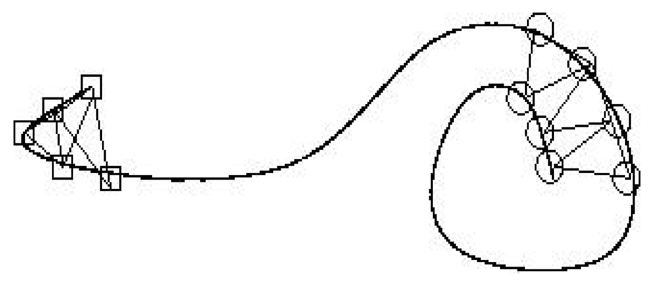
INPUT IMAGE => RECONSTRUCTION
(using weights on the left)

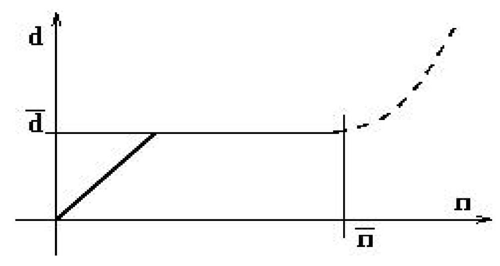
From SOM to dimensional reduction mechanisms

- Bio. plausible dimensional reduction (to be implemented in a spiking NN)

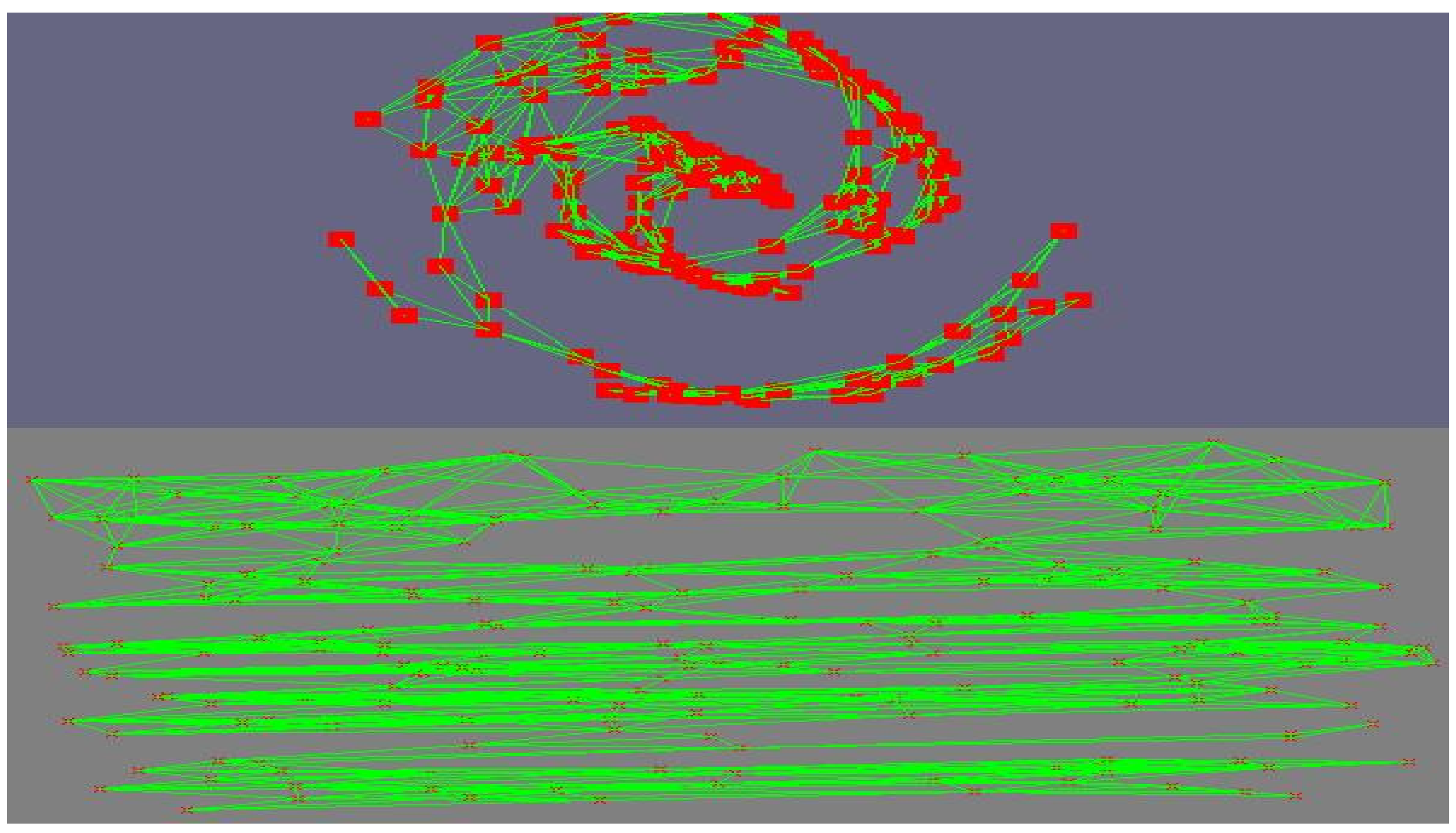


based on a local linear mechanism in a fully connected netwotk (flw LLE)





to be related to Laplacian/Hessian eigen-map (e.g. Donoho-Grimes'03)
to be related to sensori-motor learning (e.g. Philipona-O'Reagan-Nadal'03)



Conclusion / Perspective

- Using a variational approach yields
  + no-input self-organization
  + better convergence
  + local WTA feedback
- Not to be limited to 2D dim. reduction
- Build a link between Eigen-Maps and SOM/TOM
- Integrate **multi-modal | sensori-motor | temporal** aspects