Directed Rounding Arithmetic Operations

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Abstract

We propose the addition of new functions to the C++0x standard library that provide floating-point operations (+, -, *, /, sqrt and fma) as well as conversion functions with directed rounding. This set of functions is necessary to provide efficient support for interval arithmetic and related computations, and they directly map to IEEE-754 specifications. These functions require special compiler support due to their constexpr nature.

Motivation and context

WG21 has chosen to postpone the consideration of the proposal to add interval arithmetic to the standard library, N21371 after C++0x. While a new standard is being developed to provide a language-independent base for interval arithmetic (IEEE-1788), we believe that progress can be made to enhance low-level support for interval arithmetic in C++0x, and make it easier to build an interval arithmetic library on top of C++0x efficiently and portably.

The current CD provides the fegetround and fesetround functions in the <cfenv> header to access and change the current rounding mode. Those functions were imported from C99, and they are at the root of every interval arithmetic implementation. There are however two problems which prevent efficient support for interval arithmetic:

- the CD does not provide the accompanying FENV_ACCESS pragma, which instructs the compiler that a block of code cares about the rounding mode. The consequence is that one must use temporary volatile variables, or any other equivalent implementation-dependent method, which is inefficient. This can be considered as an incomplete addition to C++0x compared to C++03.

- building interval constants is very useful, but it is not possible to build constexpr interval operations since the restricted form of expressions allowed by constexpr functions forbids calling fesetround. Extending a general feature such as constexpr to support this, does not seem appropriate for this particular use.

A different approach

We propose to go away from fesetround as the low-level primitive used to implement interval arithmetic. Instead of a global state rounding mode, we propose to add functions with directed rounding modes that perform floating-point addition, subtraction, multiplication, division, square root and fma, as well as conversions between floating-point and integral types, and between floating-point types themselves. These functions correspond to the functionality mandated by IEEE-754-1985 (and by IEEE-754-2008 for fma).

1http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2006/n2137.pdf
These functions, especially as they would be constexpr, require specific compiler support.

On hardwares that support a static rounding direction in the floating-point operation instructions or that support several floating-point control registers, the new functions make it easier for the compiler to generate those instructions than currently, since using the fesetround interface around a block of floating-point operations requires some (possibly inter-procedural) flow analysis.

Fundamentally, there is no reason why C++0x provides built-in operators +, -, *, / which allow to build constexpr functions with the default rounding mode, while it would not be possible with its directed rounding cousins.

Note on optimization: on hardware that needs to set a register holding the rounding mode, using fesetround in a block of code can be more efficient. In practice however, this kind of computation is typically needed for adding vectors of intervals, and it is our belief that, in our approach, a quality compiler should be able to move the rounding mode change instructions outside of the loops.

Syntax and design choices
There are 4 rounding modes specified by IEEE-754-1985: to the nearest (default), towards zero, towards plus infinity and towards minus infinity.

We propose that the new functions use a constant rounding mode. That is, it would not be possible to pass it as an argument, and even less be a run time entity.

Moreover, C++0x currently has 2 ways of referring to rounding modes:
- fesetround takes an argument of type int with possible values FE_DOWNWARD, FE_TONEAREST, FE_TOWARDZERO and FE_UPWARD.
- the floating_round_style enum which has 4 values plus a fifth round_indeterminate.

We prefer the latter alternative for type checking and consistency reasons. Moreover, we suggest to pass the rounding mode as explicit template argument, to make sure it is a constant, while still not hardcoding it in the function name like add_up or mul_down. Passing round_indeterminate is not useful, and is not allowed.

An example of use would then be:

```cpp
double d = add<round_toward_infinity>(a, b);
float pi_up = rounded_cast<round_toward_infinity, float>(3.14159265358979323);
int i = rounded_cast<round_toward_infinity, int>(134675./4247.);
```

Side note: some languages which support more mathematical symbols for operators, like Fortress\(^2\), provide a nice syntax like a △ b.

Promotion rules
Ideally, the add function should behave just like the built-in floating-point operator+, concerning the types of the operands, following the promotion rules. And similarly for the other functions. We propose to use constrained templates, auto and decltype to achieve this.

Side effects
Division by zero, or taking the square root of a negative number, can produce side effects like raising floating-point exceptions and setting errno. Obviously, this cannot work for constexpr functions at compile-time. We therefore propose that the evaluation of these functions follow the IEEE-754 “default non-stop exception handling” mode (producing NaNs and Inf, but no floating-point exception nor errno change), and with the caveat that the compile-time evaluation is done without raising the corresponding status flags.

A compiler may choose to issue or not a warning in such cases.

\(^2\)http://research.sun.com/projects/plrg/Publications/fortress1.0beta.pdf
Header file
The closest related headers are `<cmath>` and `<numeric>`. However, neither seem perfectly appropriate to provide these functions. Moreover, since these functions are compiler-supported, it makes sense to put them in a separate header file, so as to ease the work of library vendors. This header could then be provided by compiler vendors.

We therefore propose a new header file `<rounded_math>`, and a new section to describe it.

Acknowledgements
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Proposal

26.8 Directed rounding functions

26.8.1 Header `<rounded_math>` synopsis

```cpp
namespace std {
  template < float_round_style r, FloatingPointLike T, ArithmeticLike U >
  requires True<(r != round_indeterminate)>
  constexpr T rounded_cast(U u);

  template < float_round_style r, IntegralLike T, FloatingPointLike U >
  requires True<(r != round_indeterminate)>
  constexpr T rounded_cast(U u);

  template < float_round_style r, FloatingPointLike T, FloatingPointLike U >
  requires True<(r != round_indeterminate)>
  constexpr auto add(T t, U u) -> decltype(t + u);

  template < float_round_style r, FloatingPointLike T, FloatingPointLike U >
  requires True<(r != round_indeterminate)>
  constexpr auto sub(T t, U u) -> decltype(t - u);

  template < float_round_style r, FloatingPointLike T, FloatingPointLike U, FloatingPointLike V >
  requires True<(r != round_indeterminate)>
  constexpr auto fma(T t, U u, V v) -> decltype(fma(t, u, v));
}
```
The compile-time evaluation of these functions ignores any floating-point status flags raised, if any.

The run-time evaluation of these functions shall not raise any floating-point exception, nor change errno.

```cpp
template <float_round_style r, FloatingPointLike T, ArithmeticLike U>
requires True<(r != round_indeterminate)>
constexpr T rounded_cast(U u);
```

**Returns:** The conversion u to the type T rounded according to r.

```cpp
template <float_round_style r, FloatingPointLike T, FloatingPointLike U>
requires True<(r != round_indeterminate)>
constexpr auto add(T t, U u) -> decltype(t + u);
```

**Returns:** The addition of t and u rounded according to r.

```cpp
template <float_round_style r, FloatingPointLike T, FloatingPointLike U>
requires True<(r != round_indeterminate)>
constexpr auto sub(T t, U u) -> decltype(t - u);
```

**Returns:** The subtraction of t and u rounded according to r.

```cpp
template <float_round_style r, FloatingPointLike T, FloatingPointLike U>
requires True<(r != round_indeterminate)>
constexpr auto mul(T t, U u) -> decltype(t * u);
```

**Returns:** The multiplication of t and u rounded according to r.

```cpp
template <float_round_style r, FloatingPointLike T, FloatingPointLike U>
requires True<(r != round_indeterminate)>
constexpr auto div(T t, U u) -> decltype(t / u);
```

**Returns:** The division of t and u rounded according to r.

```cpp
template <float_round_style r, FloatingPointLike T>
requires True<(r != round_indeterminate)>
constexpr T sqrt(T t);
```

**Returns:** The square root of t rounded according to r.

```cpp
template <float_round_style r, FloatingPointLike T, FloatingPointLike U, FloatingPointLike V>
requires True<(r != round_indeterminate)>
constexpr auto fma(T t, U u, V v) -> decltype(fma(t, u, v));
```

**Returns:** The fused multiply-and-add of t, u, and v, rounded according to r.