Multiscale Fairness and its Networking Applications

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Abstract: In this paper, we introduce *T-scale* and *multiscale* fairness. This new concept allows one to distribute the network resources fairly among different classes of traffic.

Keywords: Resource allocation, α-fairness, T-scale fairness, Multiscale fairness, Wireless networks.

1. Introduction

Different applications require averaging over different time scales. Consider two users receiving service: User 1 is engaged in a conversation, while, user 2 is downloading a file. User 1's voice call has a play-out buffer which can store up to 100 msec of voice samples. User 2 would be happy if he can download the file within 10 seconds. Thus user 1 is interested in a time average utility over 100 msec, while for user 2, its over 10 sec. Let's assume that the scheduler has to assign the channel to either user 1 or user 2 every 10 msec. Then, the objective of the scheduler is to ensure that the the play-out buffer is never empty for user 1 and user 2 can download his file within 10 seconds.

Thus resources are to be allocated so as to fair share the average utilities that corresponds to the assignments. But the exact definition of average share depends on application! Different applications require averaging over different time periods or time scales (Ex. real time, elastic, streaming, etc.,). Hence fairness needs to be defined across mixed timescales.

2. Resource Sharing model and different fairness definitions

Consider n mobiles located at points $x_1, x_2, ..., x_n$, respectively[1]. We assume that the utility U_i of mobile i depends on its location (or condition) x_i and on the amount of resources s_i it gets.

Let $\mathbf{S}$ be the set of assignments; an assignment $s \in \mathbf{S}$ is a function from the vector x to a point in the n-dimensional simplex. Its i-th component, $s_i(x)$ is the fraction of resource assigned to mobile i.

Definition 1 *(α-fair assignment) An assignment s is α-fair if it is a solution of:*
$Z(x,s,\alpha) := \max_s \sum_i Z_i(x_i, s_i, \alpha)$, *such that,*
$\sum_i s_i = 1,\ s_i \geq 0 \ \ \forall i = 1,...,n$, *where,*
$Z_i(x_i, s_i, \alpha) := \frac{(U_i(x_i,s_i))^{1-\alpha}}{1-\alpha}$ *for* $\alpha \neq 1$ *and*
$Z_i(x_i, s_i, \alpha) := \log(U_i(x_i, s_i))$ *for* $\alpha = 1$.

[1] Refer [1] for references, further details and discussions.

Definition 2 *(Mo and Walrand) We call $Z_i(x, s_i, \alpha)$ the fairness utility of mobile i under s_i, and we call $Z(x, s, \alpha)$ the instantaneous degree of α-fairness under s.*

The utility of an user can depend on its state in which case, we replace x in the above by a random variable X.

Definition 3 *We call $E[Z(s, X, \alpha)]$ the expected instantaneous degree of α-fairness under s.*

Next we consider the case where $x_i(t)$, $i = 1, ..., n$, may change in time.

Definition 4 *We define an assignment to be instantaneous α-fair if at each time t each mobile is assigned a resource so as to be α-fair at that instant.*

Remark 1 *We note that if the state process is stationary and ergodic, the expected instantaneous fairness criterion regards assignments at different time slots of the same player as if it were a different player at each time slot!*

Definition 5 *Assume that the state process $X(t)$ is stationary ergodic. Let λ_i be the stationary probability measure of $X(0)$. The long term α-fairness index of an assignment $s \in \mathbf{S}$ of a stationary process $X(t)$ is defined as*
$Z_\lambda(s) := \sum_{i=1}^n Z_\lambda^i(s)$, *where,*
$Z_\lambda^i(s) = \frac{\left(E_\lambda[U_i(X_i(0), s_i(X(0)))]\right)^{1-\alpha}}{1-\alpha}$. *An assignment s is long-term α-fair if it maximizes $Z_\lambda(s)$ over $s \in \mathbf{S}$.*

Definition 6 *The T-scale α-fairness index of an assignment $s \in \mathbf{S}$ is defined as*
$Z_T(s) := \sum_{i=1}^n Z_T^i(s)$, *where,*
$Z_T^i = \frac{\left[\frac{1}{T}\int_0^T U_i(X_i(t), s_i(X(t)))dt\right]^{1-\alpha}}{1-\alpha}$. *The expected T-scale α-fairness index is its expectation. An assignment s is T-scale α-fair if it maximizes $Z_T(s)$ over $s \in \mathbf{S}$.*

Definition 7 *The multiscale α-fairness index of an assignment $s \in \mathbf{S}$ is defined as*
$Z_{T_1,...,T_n}(s) := \sum_{i=1}^n Z_{T_i}^i(s)$, *where,*

$Z^i_{T_i} = \frac{\left[\frac{1}{T_i}\int_0^{T_i} U_i(X_i(t), s_i(X(t)))dt\right]^{1-\alpha}}{1-\alpha}$. *The expected multiscale α-fairness index is its expectation. An assignment s is multiscale α-fair if it maximizes $Z_{T_1,\ldots,T_n}(s)$ over $s \in \mathbf{S}$. We also say that multiscale α-fair assignment is $(T_1, \ldots, T_n)$-scale fair assignment.*

3. Applications

Spectrum allocation in random fading channels.

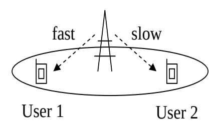

Figure 1: Spectrum allocation in random fading channels

We consider two users: fast-changing user and slowly-changing user (Fig 1). The users' channels are modeled by the Gilbert model. The users can be either in a good or in a bad state. The dynamics of the users is described by a Markov chain $\{Y_i(t)\}_{t=0,1,\ldots}$ with the transition matrix $P_i = \begin{bmatrix} 1-\epsilon_i\alpha_i & \alpha_i \\ \beta_i & 1-\epsilon_i\beta_i \end{bmatrix}$. Its stationary distribution is given by $\pi_i = \begin{bmatrix} \frac{\beta_i}{\alpha_i+\beta_i} & \frac{\alpha_i}{\alpha_i+\beta_i} \end{bmatrix}$. Let $\epsilon_1 = 1$ and $\epsilon_2 = \epsilon$. Note that the parameter ϵ does not have an effect on the stationary distribution, but, it influences for how long the slowly-changing user stays in some state. The smaller ϵ, the more seldom the user changes the states.

We assume that state 1 is a bad state and state 2 is a good state. Let h_{ij} represent the channel gain coefficient of user i in channel state j. The utility (achievable throughput via Shannon capacity) of user i in state j is given by $U_{ij} = s_{ij}\log_2(1 + \frac{|hij|^2 p_i}{\sigma^2})$, where s_{ij}, p_i is the resource allocation and power that corresponding to the user i.

We formulate and analyze the T-scale and multiscale fairness criterion (Refer [1] for more details). We provide an example case study here, where, the slowly-changing user is more often in the good channel state. We compute expected throughput and coefficient of variation for the different fairness measures considered (Fig 2).

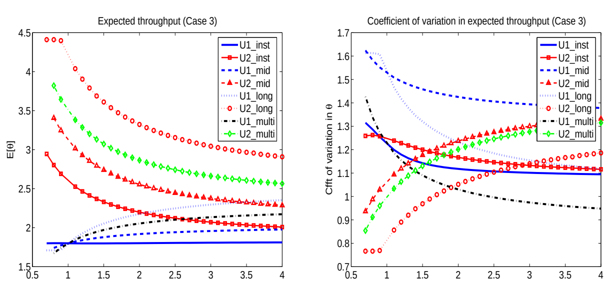

Figure 2: (a) Throughput θ and (b) Coefficient of variation $\Gamma_i = \frac{\sqrt{\mathbf{E}[\theta_i^2]}}{\mathbf{E}[\theta_i]}$ as a function of α for instantaneous, mid-term, long-term and (1,∞)-scale fairness criteria (Case 3). Refer [1] for other cases

Remark 2 *In the case of multiscale fairness, we observe that the coefficient of variation decreases for short-term fairness oriented user.*

Indoor-outdoor scenario.

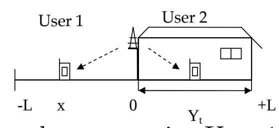

Figure 3: Indoor-outdoor scenario: User 1 located at x, user 2 located indoor at Y_t

Consider an indoor-outdoor partition (Fig 3). The outdoor user is located at x, while the indoor user's location is random Y_t over $(0, L)$. Further assume that the mobility pattern of the indoor user is uniform over $(0, L)$. We consider allocation of the fraction of time between the two mobiles (Refer [1] for more details).

In the first case, we compute the scheduler and expected throughput so as to achieve instantaneous fairness (Fig 4a,b). In the second case, we characterize the scheduler to achieve long-term fairness (Fig 4c).

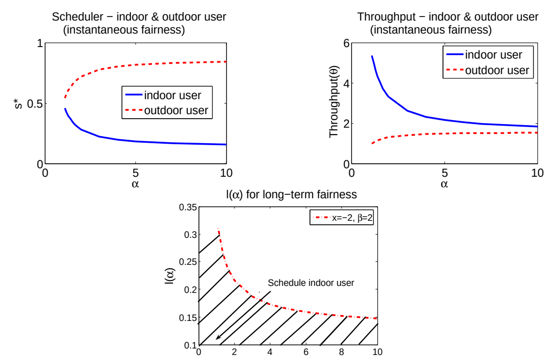

Figure 4: (a) Scheduler s^* (b) Throughput θ for the indoor and outdoor user with instantaneous fairness as a function of α ($\alpha > 1$). Wall attenuation 6 dB, path-loss $\beta = 3$, position of outdoor user $x = -3$. (c) $l(\alpha)$ for long-term fairness as a function of α ($\alpha > 1$), $\beta = 2$, $x = -2$ and wall attenuation 6 dB.

Theorem 1 *The long term α-fair policy is given by $s_2(x, y) = 1$ for $y \le l(\alpha)$ and is zero otherwise.*

References

[1] *Multiscale Fairness and its Application to Dynamic Resource Allocation in Wireless Networks*, INRIA research report number RR-7783 at http://hal.inria.fr/inria-00515430/en/