Multi-sorted Argumentation

Tjitze Rienstra[1] and Alan Perotti[2] and Serena Villata[3] and Dov Gabbay[4] and Leendert van der Torre[1]

[1] Computer Science and Communication, University of Luxembourg
tjitze.rienstra@uni.lu,leendert@vandertorre.com
[2] Dip. di Informatica, University of Turin
perotti@di.unito.it
[3] Edelweiss, INRIA Sophia Antipolis
serena.villata@inria.fr
[4] Dept. Computer Science, King's College London
dov.gabbay@kcl.ac.uk

Abstract. In the theory of abstract argumentation, the acceptance status of arguments is normally determined for the complete set of arguments at once, under a single semantics. However, this is not always desired. In this paper, we extend the notion of an argumentation framework to a multi-sorted argumentation framework, and we motivate this extension using an example which considers practical and epistemic arguments. In a multi-sorted argumentation framework, the arguments are partitioned into a number of *cells*, where each cell is associated with a semantics under which its arguments are evaluated. We prove the properties of the proposed framework, and we demonstrate our theory with a number of examples. Finally, we relate our theory to the theory of modal fibring of argumentation networks.

1 Introduction

Abstract argumentation frameworks [10] are used to model sets of arguments and the attacks among these arguments. Given an abstract argumentation framework, we can ask the question of which arguments are acceptable, and which arguments are not. This question is answered by what is called an *acceptability semantics*. Different modes of reasoning are possible, each giving rise to a different acceptability semantics. Well-known examples are the *grounded semantics* that minimizes the number of accepted arguments, and the *preferred semantics*, that maximizes the number of accepted arguments. The choice of which semantics is appropriate depends on the kind of arguments, and the attitude towards these arguments. A skeptical attitude, for example, can be modeled with grounded semantics, whereas a credulous attitude can be modeled using preferred semantics [8]. In most literature on acceptability semantics (see for instance [10, 3, 7, 9]), the assumption is made that all arguments of a framework are evaluated under a single semantics.

In this paper, we argue for a generalization, and we answer the research question *how to define an abstract argumentation framework where the arguments can be evaluated under different semantics?* We motivate this through an example about practical and epistemic arguments, and we introduce a system called *multi-sorted argumentation*. The motivating example, which now follows, is taken from Prakken [13], and adapted to an abstract argumentation framework.

Consider a university lecturer (let us call him John) with two conflicting desires. He wants to finish a paper on Friday, but he also promised to give a talk in a town called Faraway on the same day. There are two ways to travel to Faraway: by car and by bus. In neither case will he be able to finish the paper while traveling; he cannot work while driving, and he gets sick when working in a bus. Figure 1 shows an informal instantiation of an abstract framework for this situation: the three arguments a, b and c represent the situation described so far. Note that the arguments are forms of the practical syllogism. For example, argument b is an inference consisting of John's belief that traveling by bus (to Faraway) allows him to give the lecture in Faraway, his desire to give a lecture in Faraway, and the conclusion that he must therefore travel by bus. John has more information: his friend Bob tells him that there is a train connection to Faraway. So if John travels by train, he will be able to finish the paper (argument d, another practical syllogism). Now, John's other friend Mary warns him about a railway strike, which would defeat d (argument e). On the other hand, Bob believes there will be no strike (argument f). John has no reason to trust either of his friends more than the other. To be on the safe side, John does not want to act on the credulous belief that there will not be a train strike, and that there will be a train to Faraway on Friday.

In this example, we have four arguments that pertain to actions (arguments a, b, c, d) and two arguments expressing beliefs about the world (arguments e and f). Prakken [13] calls them *practical arguments* and *epistemic arguments*, and he argues that practical arguments should be evaluated credulously and epistemic arguments skeptically. The reason is that for practical arguments, it is rational for an agent to consider credulously all possible 'action alternatives' that have skeptical support of epistemic arguments. Since a credulous attitude can be modeled using preferred semantics, and a skeptical attitude using grounded semantics, the evaluation of this framework is a combination of preferred semantics for a, b, c and d, and grounded semantics for e and f.

This example motivates the evaluation of different parts of the same framework under both preferred and grounded semantics. We believe that the point made by this example extends to the more general case. Suppose we have a set of frameworks $\{A_1, \ldots, A_n\}$ and for each framework A_i there is an appropriate semantics s_i. Then it is also possible that there is a framework A which is a merge of the frameworks $A_1, \ldots, A_n$, where the n different parts of A may interact mutually through additional attacks. Determining the acceptance status of the arguments in A would amount to the application of the s_i semantics to the part corresponding to A_i, for all parts $1 \ldots n$.

What we need, then, is a method to apply different semantics to different parts of the same framework. To this end, we propose a system called *multi-sorted argumentation*. The system is based on two elements: a regular argumentation

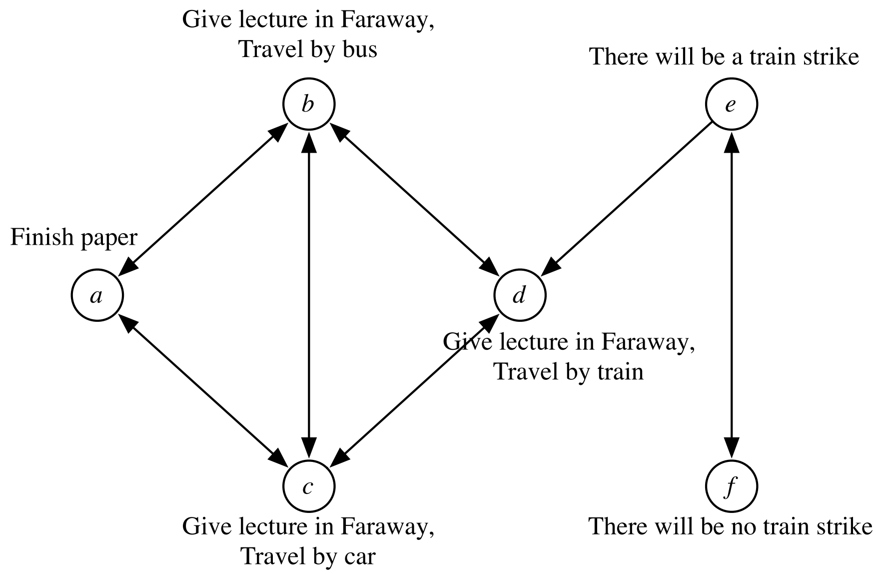

Fig. 1. An example of an argumentation framework with both practical and epistemic arguments.

framework and a sorting. A sorting supplements the argumentation framework with information on how the framework is divided into cells, and which cell in the sorting is to be evaluated under which semantics. A sorted extension is a set of arguments that are acceptable with respect to the sorting. We prove a number of desired properties of sorted extensions. For example, sorted extensions should be conflict-free. Moreover, some properties of the semantics associated with each cell are preserved, i.e., if the semantics of all cells are admissible (resp. complete), then the sorted extensions should also be admissible (resp. complete).

Finally, we show how to formalize multi-sorted argumentation using the modal fibring approach. Multi-sorted argumentation is expressed as a special case of the fibring of modal argumentation frameworks. We present this kind of multi-sorted argumentation by means of a number of examples, and we discuss the properties which hold for this method of evaluating the cells under different semantics.

The paper is organized as follows: Section 2 provides the basic concepts of argumentation theory; in Section 3 we introduce the notions of sortings and sorted extension, in Section 4 we study some properties of sorted extensions, and in Section 5 we relate our theory to the theory of modal fibring. Finally, Sections 6 and 7 present related work, conclusions and future work.

2 Preliminaries

The following definitions set forth the basics of Dung's well-known theory of abstract argumentation [10].

Definition 1 (Argumentation Framework) *We assume as given a set $\mathcal{U}$, called the* universe of arguments*. An* argumentation framework $\mathcal{AF}$ *is a pair $\langle A, R\rangle$ with finite $A \subseteq \mathcal{U}$ and a binary relation (used in infix) $R \subseteq A \times A$, called the* attack relation*.*

Definition 2 (Conflict Free) *Let $\mathcal{AF} = \langle A, R\rangle$ be an argumentation framework. A set $\mathcal{S} \subseteq A$ is* conflict free *iff there are no arguments $a, b \in \mathcal{S}$ such that aRb. If $\mathcal{S}$ is conflict free, we write $cf(\mathcal{S})$.*

We follow Baroni & Giacomin's [3] generalized approach, where the acceptability of arguments is considered with respect to a designated subset of arguments. This set, which we call the set of *qualified arguments*, contains the arguments that an extension may consist of. Intuitively, it is used to filter out arguments that do not qualify for acceptance. This is necessary when we evaluate only a subset of arguments, but at the same time we know that some arguments in this subset cannot be accepted due to attacks from outside the subset.

Definition 3 (Defense) *Let $\mathcal{AF} = \langle A, R\rangle$ be an argumentation framework. A set $\mathcal{S} \subseteq A$* defends a from A *iff $\forall b \in A$ such that bRa, $\exists c \in \mathcal{S}$ such that cRb. Let $D_Q(\mathcal{S}) = \{a \in Q \mid \mathcal{S}$ defends a from $A\}$, where $Q \subseteq A$ is called a* qualified set of arguments*.*

We will sometimes say that $\mathcal{S} \subseteq A$ defends a, without mentioning the set from which $\mathcal{S}$ defends a. In that case, we mean that $\mathcal{S}$ defends a from all arguments, i.e. from A.

Definition 4 (Acceptance Function) *An* acceptance function

$$\mathcal{E} : 2^{\mathcal{U}} \times 2^{\mathcal{U}\times\mathcal{U}} \times 2^{\mathcal{U}} \to 2^{2^{\mathcal{U}}}$$

is a partial function that associates each argumentation framework $\langle A, R\rangle$ and each set of qualified arguments $Q \subseteq A$, with sets of subsets of A, called extensions*: $\mathcal{E}(\langle A, R\rangle, Q) \subseteq 2^A$.*

Dung [10] presents several acceptability semantics which produce zero, one, or several sets of accepted arguments. These semantics are grounded on the two main concepts of conflict-freeness and defense. The following definitions are equivalent to those in Dung's original theory, if we set $Q = A$.

Definition 5 (Acceptability Semantics) *Let $\mathcal{AF} = \langle A, R\rangle$ be an argumentation framework and $Q \subseteq A$ a set of qualified arguments. Acceptance functions for conflict free ($\mathcal{E}_{cf}$), admissible ($\mathcal{E}_{ad}$), complete ($\mathcal{E}_{co}$), grounded ($\mathcal{E}_{gr}$) and preferred ($\mathcal{E}_{pr}$) extensions are defined as follows:*

- $\mathcal{S} \in \mathcal{E}_{cf}(\mathcal{AF}, Q)$ *iff* $\mathcal{S} \subseteq Q$ *and* $cf(\mathcal{S})$.
- $\mathcal{S} \in \mathcal{E}_{ad}(\mathcal{AF}, Q)$ *iff* $cf(\mathcal{S})$ *and* $\mathcal{S} \subseteq D_Q(\mathcal{S})$.
- $\mathcal{S} \in \mathcal{E}_{co}(\mathcal{AF}, Q)$ *iff* $cf(\mathcal{S})$ *and* $\mathcal{S} = D_Q(\mathcal{S})$.
- $\mathcal{S} \in \mathcal{E}_{gr}(\mathcal{AF}, Q)$ *iff* $\mathcal{S}$ *is minimal in* $\mathcal{E}_{co}(\mathcal{AF}, Q)$ *w.r.t. set inclusion.*
- $\mathcal{S} \in \mathcal{E}_{pr}(\mathcal{AF}, Q)$ *iff* $\mathcal{S}$ *is maximal in* $\mathcal{E}_{co}(\mathcal{AF}, Q)$ *w.r.t. set inclusion.*
- $\mathcal{S} \in \mathcal{E}_{st}(\mathcal{AF}, Q)$ *iff* $\mathcal{S} \in \mathcal{E}_{cf}(\mathcal{AF}, Q)$ *and* $\forall a \in \mathcal{A}, a \notin \mathcal{S} \rightarrow \exists b \in \mathcal{S}$ *s.t.* bRa.

The definitions above are reformulations of those proposed by Baroni & Giacomin [3].

Example 1 (Admissible extension). Consider the framework of Figure 1. Let $Q = \{a, b, c, d\}$. Given Q, the set $\{a\}$ is an admissible extension, i.e., it is included in $\mathcal{E}_{ad}(\mathcal{AF}, Q)$. This can be seen as follows: we have that $\{a\}$ is conflict free, and $\{a\} \subseteq D_Q(\{a\}) = \{a\}$. However, the set $\{a, d\}$ is not an admissible extension, and is not included in $\mathcal{E}_{ad}(\mathcal{AF}, Q)$. The reason is that the set does not defended itself from the attack by f: $\{a, d\} \nsubseteq D_Q(\{a, d\}) = \{a\}$.

Note, in the running example, that the arguments of an admissible set need to be defended from all arguments in A, so not only from arguments in Q.

Example 2 (Complete extension). Consider the framework of Figure 1. Let $Q = \{a, b, c, d\}$. Given Q, the set $\{b\}$ is a complete extension, i.e., it is included in $\mathcal{E}_{co}(\mathcal{AF}, Q)$. This can be seen as follows: we have that $\{b\}$ is conflict free, and $\{b\} = D_Q(\{b\}) = \{b\}$. However, the set $\{a, d\}$ is not a complete extension, namely for the same reason that it is not an admissible extension (see above). Now, let $Q = \{b, c, d, e, f\}$. The set $\{d, f\}$ is a complete extension, because $\{d, f\} = D_Q(\{d, f\})$. However, the set $\{f\}$ is not, because $\{f\} \neq D_Q(\{f\}) = \{d, f\}$.

Example 3 (Grounded, preferred extensions). Consider the framework of Figure 1. Let $Q = \{b, c, d, e, f\}$. Given Q, the complete extensions are $\{b, e\}$, $\{b, f\}$, $\{b\}$, $\{c, e\}$, $\{c, f\}$, $\{c\}$, $\{d, f\}$,$\{f\}$ and $\emptyset$. The extensions $\{b, e\}$, $\{b, f\}$, $\{c, e\}$, $\{c, f\}$ and $\{d, f\}$ are preferred extensions, since they are maximal with respect to set-inclusion. The extension $\emptyset$ is the grounded extension, since it is minimal with respect to set-inclusion.

3 Multi-sorted Argumentation

We now define the main ingredients of our system: sortings and sorted extensions. A sorting supplements the argumentation framework with information on how the framework is divided into cells, and which cell in the sorting is to be evaluated under which semantics. In the following definitions, we assume a fixed argumentation framework $\mathcal{AF} = \langle A, R \rangle$.

Definition 6 (Sorting) *A* sorting $\mathbb{S}$ *is a pair* $\langle P, T \rangle$*, where* P *is a partition of* A *and* $T : P \rightarrow \{cf, ad, co, gr, pr\}$ *a function associating each cell in* P *to a semantics.*

The following example demonstrates this representation, for the framework shown in Figure 1, and discussed in the introduction.

Example 4. The situation shown in Figure 1 is formally represented by a framework $AF = \langle\{a, b, c, d, e, f\}, R\rangle$, where aRb, bRa, aRc, cRa, bRc, cRb, bRd, dRb, cRd, dRc, eRd, eRf, fRe and sorting $\mathbb{S} = \langle\{C_1, C_2\}, T\rangle$, where $C_1 = \{a, b, c, d\}$, $C_2 = \{e, f\}$, $T(C_1) = pr$ and $T(C_2) = gr$, i.e., arguments a, b, c, d are evaluated under the preferred semantics, and arguments e, f are evaluated under the grounded semantics.

We will shortly give the condition, given a sorting, for a set of arguments to be a multi-sorted extension of an argumentation framework. Before we do so, we introduce the concepts of a subframework and of the set of qualified arguments of a subframework. These concepts define a way of evaluating the arguments in a cell, given an extension $\mathcal{S}$. The intuition behind them is as follows. Given a cell C and extension $\mathcal{S}$, we determine whether $\mathcal{S} \cap C$ is an extension for C, by first restricting C to those arguments that are not defeated by arguments outside C. This set, denoted by C', makes up the arguments of what we call the subframework for C. Next, we further restrict the arguments of C' to those that are defended by $\mathcal{S}$ from attacks outside C. This set, denoted by C'', contains the arguments in C that are qualified for acceptance.

Definition 7 (Subframework) *Let P be a partition of A, $C \in P$ a cell and $\mathcal{S} \subseteq A$ an extension. The* subframework *for C, given $\mathcal{S}$, is the argumentation framework $\langle C', R \downarrow C'\rangle$ where $C' = \{a \in C \mid \nexists b \in \mathcal{S} \setminus C, bRa\}$ and where $R \downarrow C'$ is the attack relation R restricted to the arguments in C', i.e. $R \downarrow C' = \{(a, b) \in R \mid a, b \in C'\}$.*

Definition 8 (Qualified Arguments of a Subframework) *Let $\langle C', R \downarrow C'\rangle$ be a subframework for a cell C and extension $\mathcal{S}$. The* qualified arguments *of $\langle C', R \downarrow C'\rangle$, denoted by C'', are defined as follows.*

$$C'' = \{a \in C' \mid \forall b \in A \setminus C, (bRa \rightarrow \exists c \in \mathcal{S}, cRb)\}$$

Given an extension $\mathcal{S}$, we can determine whether it is a sorted extension by checking that for each $C \in P$, we have that $C \cap \mathcal{S}$ is an extension of the subframework for C, given the qualified arguments of the subframework for C. The semantics under which the subframework for C is evaluated, is the semantics associated with C.

Definition 9 (Sorted Extension) *A set $\mathcal{S} \subseteq A$ is a* sorted extension *of $AF = \langle A, R\rangle$ and $\mathbb{S} = \langle P, T\rangle$ iff for all $C \in P$, we have*

$$C \cap \mathcal{S} \in \mathcal{E}_{T(C)}(\langle C', R \downarrow C'\rangle, C'')$$

The sorted acceptance function *$\mathcal{E}_{srt}$ is defined as follows: $\mathcal{S} \in \mathcal{E}_{srt}(AF, \mathbb{S})$ iff $\mathcal{S}$ is a sorted extension of AF and $\mathbb{S}$.*

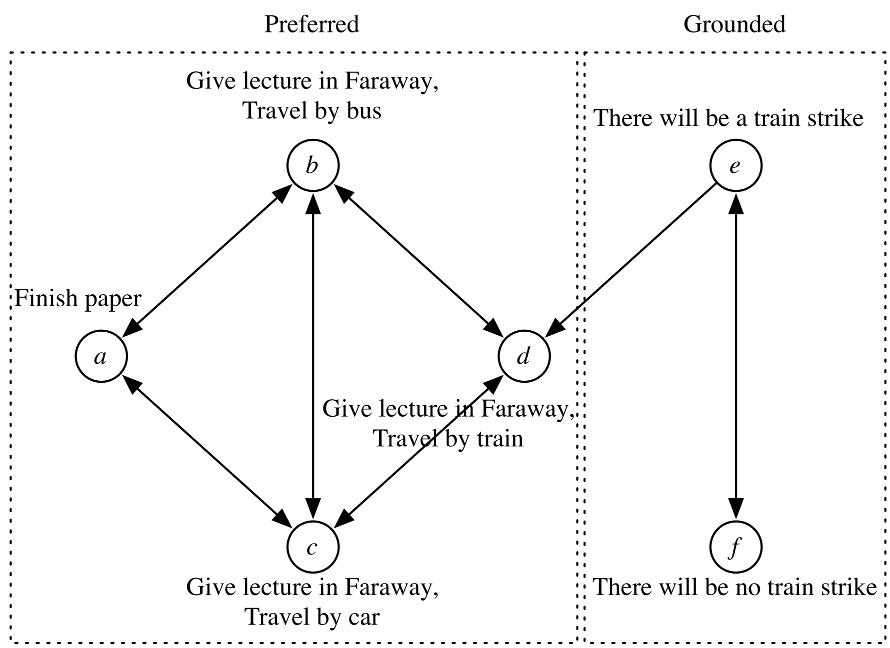

Fig. 2. The multi-sorted argumentation framework of the running example about practical and epistemic arguments.

The following example demonstrates the computation of the sorted extensions for the framework used in our running example.

Example 4 (Continued). Consider the following extensions for the framework AF shown in Figure 2: $\mathcal{S}_1 = \emptyset$, which is the grounded extension of AF; $\mathcal{S}_2 = \{a, d, f\}$, which is a preferred extension of AF; and $\mathcal{S}_3 = \{b\}$, which is neither the grounded nor a preferred extension of AF. We determine whether they are multi-sorted extensions of AF, given the sorting $\mathbb{S}$ introduced earlier. That is to say, we determine whether $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3 \in \mathcal{E}_{srt}(AF, \mathbb{S})$.

- Given $\mathcal{S}_1$, we have $C'_1 = \{a, b, c, d\}$ and $C'_2 = \{e, f\}$ (no argument in C_1 defeats an argument in C_2 or vice versa); and $C''_1 = \{a, b, c\}$ and $C''_2 = \{e, f\}$ (d is not defended from e). We have that $\mathcal{S}_1 \cap C_1 = \emptyset$ and $\emptyset \notin \mathcal{E}_{pr}(\langle C'_1, R \downarrow C'_1\rangle, C''_1)$ and $\mathcal{S}_1 \cap C_2 = \emptyset$ and $\emptyset \in \mathcal{E}_{gr}(\langle C'_2, R \downarrow C'_2\rangle, C''_2)$. While $\mathcal{S}_1 \cap C_2$ is the grounded extension of $\langle C'_2, R \downarrow C'_2\rangle$, $\mathcal{S}_1 \cap C_1$ is not a preferred extension of $\langle C'_1, R \downarrow C'_1\rangle$. It follows that $\mathcal{S}_1 \notin \mathcal{E}_{srt}(AF, \mathbb{S})$.

- Given $\mathcal{S}_2$, we have $C'_1 = \{a, b, c, d\}$ and $C'_2 = \{e, f\}$ (no argument in C_1 defeats an argument in C_2 or vice versa); and $C''_1 = \{a, b, c, d\}$ and $C''_2 = \{e, f\}$ (no argument is undefended from attacks by other cells). We have that $\mathcal{S}_2 \cap C_1 = \{a, d\}$ and $\{a, d\} \in \mathcal{E}_{pr}(\langle C'_1, R \downarrow C'_1\rangle, C''_1)$ and $\mathcal{S}_1 \cap C_2 = \{f\}$ and $\{f\} \notin \mathcal{E}_{gr}(\langle C'_2, R \downarrow C'_2\rangle, C''_2)$. While $\mathcal{S}_1 \cap C_1$ is a preferred extension of $\langle C'_1, R \downarrow C'_1\rangle$, $\mathcal{S}_1 \cap C_2$ is not the grounded extension of $\langle C'_2, R \downarrow C'_2\rangle$. It follows that $\mathcal{S}_2 \notin \mathcal{E}_{srt}(AF, \mathbb{S})$.

- Given $\mathcal{S}_3$, we have $C'_1 = \{a, b, c, d\}$ and $C'_2 = \{e, f\}$; and $C''_1 = \{a, b, c\}$ (d is not defended from e) and $C''_2 = \{e, f\}$. We have that $\mathcal{S}_3 \cap C_1 = \{b\}$ and $\{b\} \in \mathcal{E}_{pr}(\langle C'_1, R \downarrow C'_1\rangle, C''_1)$ and $\mathcal{S}_3 \cap C_2 = \emptyset$ and $\emptyset \in \mathcal{E}_{gr}(\langle C'_2, R \downarrow C'_2\rangle, C''_2)$. It follows that $\mathcal{S}_3 \in \mathcal{E}_{srt}(AF, \mathbb{S})$.

In conclusion, $\mathcal{S}_3$ is a sorted extension, and $\mathcal{S}_1$ and $\mathcal{S}_2$ are not. The only other sorted extensions are $\{a\}$ and $\{c\}$. Switching back to the language of the running example in the introduction, the three acceptable options are to either finish the paper (extension $\{a\}$), to give the lecture and travel by bus (extension $\{b\}$) or to give the lecture and travel by car (extension $\{c\}$). As desired, the option to travel by train and both give the lecture and finish the paper, which would correspond to the extension $\{a, d, f\}$, is not supported.

4 Properties

In this section, we present some desired properties of sorted extensions. In particular, we aim to show that the sorted extensions presented in Section 3 satisfy conflict-freeness, admissibility, and completeness. We say that a semantics s satisfies conflict-freeness (resp. admissibility, completeness) if, given any framework $\langle A, R\rangle$ and any $Q \subseteq A$, all extensions $\mathcal{E}_s(\langle A, R\rangle, Q)$ are conflict-free (resp. admissible, complete). We then have that all the semantics considered here satisfy conflict-freeness; that all semantics except conflict-free satisfy admissibility; and that all semantics except conflict-free and admissible satisfy completeness.

We first prove the preservation of the conflict-free, admissible and completeness properties of sorted extensions, i.e., whenever the semantics associated with all cells of the partitioning satisfy these properties, then the sorted extensions satisfy them as well.

Proposition 1. *For any AF and $\mathbb{S} = \langle P, T \rangle$, if $\forall C \in P$, $T(C)$ is a conflict-free semantics, then $\forall \mathcal{S} \in \mathcal{E}_{srt}(AF, \mathbb{S})$, $\mathcal{S}$ is conflict-free.*

Proof. Let $AF = \langle A, R \rangle$, $\mathbb{S} = \langle P, T \rangle$, and $\mathcal{S} \in \mathcal{E}_{srt}(AF, \mathbb{S})$. We know that $\forall C \in P, T(C)$ is a conflict-free semantics. Note that it follows that $\forall C \in P$, $\mathcal{S} \cap C$ is conflict free. Now suppose the contrary, i.e. there are $a, b \in \mathcal{S}$ s.t. bRa. Let $C \in P$ be the cell s.t. $a \in C$. Because $\mathcal{S} \cap C$ is conflict-free, we have $b \in \mathcal{S} \setminus C$. Then by Definition 7, $a \notin C'$. Because we have that $C'' \subseteq C' \subseteq C$ and $\mathcal{S} \cap (C \setminus C'') = \emptyset$, it follows that $a \notin \mathcal{S}$. Contradiction. $\square$

Note that, since all the semantics that we consider satisfy conflict-freeness, we have that every sorted extension is conflict-free.

Proposition 2. *For any AF and $\mathbb{S} = \langle P, T \rangle$, if $\forall C \in P$, $T(C)$ is an admissible semantics, then $\forall \mathcal{S} \in \mathcal{E}_{srt}(AF, \mathbb{S})$, $\mathcal{S}$ is admissible.*

Proof. Let $AF = \langle A, R \rangle$, $\mathbb{S} = \langle P, T \rangle$, and $\mathcal{S} \in \mathcal{E}_{srt}(AF, \mathbb{S})$. By Proposition 1 we have that $\mathcal{S}$ is conflict-free. We also know that $\forall C \in P, T(C)$ is an admissible semantics. Note that it follows that $\forall C \in P$, $\mathcal{S} \cap C$ is admissible w.r.t. the framework $\langle C', R \downarrow C' \rangle$. Suppose now that $\mathcal{S}$ is not admissible, i.e., there are $a \in \mathcal{S}$, $b \in A$, bRa and $\nexists c \in \mathcal{S}$ s.t. cRb. Because $\mathcal{S}$ is conflict free, we know that $b \notin \mathcal{S}$. Let $C \in P$ be the cell s.t. $a \in C$. Because $\mathcal{S} \cap C$ is admissible w.r.t. the framework $\langle C', R \downarrow C' \rangle$, we have that $b \notin C'$. By Definition 7 we have that $\forall b' \in (C \setminus C')$, $\exists c' \in \mathcal{S}$ s.t. $c'Rb'$. Therefore, $b \notin (C \setminus C')$ and so $b \in A \setminus C$. By Definition 8 it now follows that $a \notin C''$. But then, because we have that $C'' \subseteq C' \subseteq C$ and $\mathcal{S} \cap (C \setminus C'') = \emptyset$, it follows that $a \notin \mathcal{S}$. Contradiction. $\square$

Proposition 3. *For any AF and $\mathbb{S} = \langle P, T \rangle$, if $\forall C \in P$, $T(C)$ satisfies completeness, then $\forall \mathcal{S} \in \mathcal{E}_{srt}(AF, \mathbb{S})$, $\mathcal{S}$ is complete.*

Proof. Let $AF = \langle A, R \rangle$, $\mathbb{S} = \langle P, T \rangle$, and $\mathcal{S} \in \mathcal{E}_{srt}(AF, \mathbb{S})$. Suppose $\mathcal{S}$ defends a from A. We need to show that $a \in \mathcal{S}$ (i.e. that $\mathcal{S}$ is complete). Let $C \in P$ be the cell s.t. $a \in C$. First we show that $S \cap C'$ defends a from C'. Let $b \in C'$ be an argument from which $S \cap C'$ needs to defend a, i.e. bRa. Because $\mathcal{S}$ defends a from A, there is a $c \in \mathcal{S}$ s.t. cRb. There are two possibilities: either $c \in C'$ or $c \notin C'$. Suppose $c \notin C'$. Then by definition 9, $c \notin C$ and by definition 7, $b \notin C'$. Contradiction. It follows that $c \in C'$, and that $S \cap C'$ defends a from b. Because this holds for any $b \in C'$, we have that $S \cap C'$ defends a from C'. Finally, from definition 9, and from the assumption that $T(C)$ is complete, it follows that (for any $C'' \subseteq C'$) $a \in S$. $\square$

These properties are highly desirable in a multi-sorted argumentation framework, because they allow us to guarantee that the properties which hold for the standard Dung framework, are preserved in the multi-sorted one.

Consider now the case where the sorting associates all cells with the same semantics. We call this the *uniform* case. A natural question to ask is whether the set of sorted extensions will then be equivalent to the set of extensions of the framework evaluated under this semantics in the conventional way. We formalize this property as follows.

Definition 10 (Uniform Case Extension Equivalence) *Let $AF = \langle A, R\rangle$ and $\mathbb{S} = \langle\{C_1, \ldots, C_n\}, T\rangle$. Uniform case equivalence holds if and only if*

$$T(C_1) = \ldots = T(C_n) = s \textit{ implies } \mathcal{E}_{srt}(AF, \mathbb{S}) = \mathcal{E}_s(AF, A)$$

This property does not hold in all the cases. Consider the following example:

Example 5. Let $AF = \langle\{a, b\}, R\rangle$, where aRb, bRa; and $\mathbb{S} = \langle\{C_1, C_2\}, T\rangle$, where $C_1 = \{a\}$, $C_2 = \{b\}$ and $T(C_1) = T(C_2) = gr$. The grounded extension of AF is $\emptyset$. We now show that $\emptyset \notin \mathcal{E}_{srt}(AF, \mathbb{S})$: we have $C_1' = \{a\}$, and $C_2' = \{b\}$ (no argument is defeated); and $C_1'' = \emptyset$, and $C_2'' = \emptyset$ (both arguments are undefended). We have that $\mathcal{S} \cap C_1 = \emptyset$ and $\emptyset \notin \mathcal{E}_{gr}(\langle C_1', R \downarrow C_1'\rangle, C_1'')$ (the grounded extension of $\langle C_1', R \downarrow C_1'\rangle$ is not a subset of C_1''). It follows that $\emptyset \notin \mathcal{E}_{srt}(AF, \mathbb{S})$.

The reason why the uniform case extension equivalence does not hold is the following: every cell is evaluated separately, and the separate evaluation of a cell C under a semantics s may lead to a result that is different from the result of evaluating the complete framework under semantics s. Consider for instance to adopt multi-sorted argumentation to model the merging of the argumentation frameworks of single agents. Even if these agents adopt the same semantics, the evaluation of their single frameworks may lead to different extensions of the merged framework. However, if the multi-sorted framework is used in a context where this property is required to hold, then the equivalence can be guaranteed by replacing the cells associated with the same semantics with their union.

Finally, we underline that, given a cell associated with a certain semantics, say grounded, a sorted extension may not represent a grounded evaluation of the arguments in this cell, when we consider this cell in isolation. Consider the following example.

Example 6. The framework shown in Figure 3 is formally represented by $AF = \langle\{a, b, c, d\}, R\rangle$, where aRb, bRa, bRc, cRb, cRd and dRc; and a sorting $\langle\{C_1, C_2\}, T\rangle$, where $C_1 = \{a, b\}$, $C_2 = \{c, d\}$, and $T(C_1) = gr$, and $T(C_2) = pr$.

Consider the extension $\mathcal{S} = \{a, c\}$. Note that a is accepted, while the cell $\{a, b\}$ is associated with the grounded semantics. Let us check if $\mathcal{S}$ satisfies the conditions for being a sorted extension.

- Given $\mathcal{S}$, we have $C_1' = \{a\}$, and $C_2' = \{c, d\}$ (b is defeated by c); and $C_1'' = \{a\}$, and $C_2'' = \{c, d\}$ (no argument is undefended). We have that

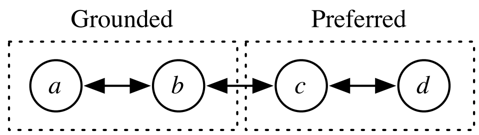

Fig. 3. A sorted argumentation framework.

$\mathcal{S} \cap C_1 = \{a\}$, and $\{a\} \in \mathcal{E}_{gr}(\langle C_1', R \downarrow C_1' \rangle, C_1'')$, and $\mathcal{S} \cap C_2 = \{c\}$, and $\{c\} \in \mathcal{E}_{pr}(\langle C_2', R \downarrow C_2' \rangle, C_2'')$. It follows that $\mathcal{S} \in \mathcal{E}_{srt}(AF, \mathbb{S})$.

In the example above, selecting c to be accepted accords with the preferred evaluation of the cell $\{c, d\}$. But given this selection, the only complete extension is $\{a, c\}$. Note also that the extension $\{a\}$ is in fact a grounded extension for the subframework $\langle C_1', R \downarrow C_1' \rangle$, where $C_1' = \{a\}$. Consider again the informal example concerning the merging of the single frameworks of the agents. We can note that the merging of the single frameworks may lead to an evaluation such that the arguments accepted under a particular semantics, are then not accepted into the merged framework in the same semantics.

If the multi-sorted framework is used in a context where this behavior needs to be avoided, then a possible way to deal with this behavior is to apply a selection criteria for extensions based on a notion of preference. For example, another sorted extension of the framework described above is $\{d\}$. If actual groundedness for the cell C_1 is important, then this extension would be preferred over $\{a, c\}$.

5 The Modal Fibring Approach

In this section we present an alternative representation for multi-sorted argumentation frameworks. Here, we represent every cell as a separate argumentation framework, of which the argument status can be evaluated independently of the arguments in the other cells. We apply the well known concept of the *possibility* modality from modal logic to express inter-cell attacks within these frameworks. The approach here is based on the idea of fibring modal argumentation frameworks presented by Barringer and Gabbay [4].

A modal argumentation framework represents a regular Dung framework extended with information on possible attacks from outside the framework, and with a semantics under which the framework is to be evaluated. The possible attacks are modeled using an additional set of arguments and attacks, called meta-arguments and meta-attacks.

Definition 11 (Modal Argumentation Framework) *A* modal argumentation framework *MAF is a tuple $\langle A, R, MA, MR, s \rangle$ where A is the set of arguments, $R \subseteq A \times A$ the attack relation, MA the set of* meta-arguments*, $MR \subseteq MA \times A$ the set of* meta-attacks *and $s \in \{cf, ad, co, gr, pr\}$.*

A cell of a sorting may be represented by a MAF (modal argumentation framework). An argument x from an outside cell attacking an argument y inside the cell is translated into a meta-argument $\Diamond x$ and a meta-attack $\Diamond x MRy$. These meta-arguments represent *possible attacks*, in that they may or may not be accepted, depending on the evaluation of arguments in other MAFs. A framework $\mathcal{AF} = \langle A, R \rangle$ and sorting $\mathbb{S} = \langle P, T \rangle$ can thus be translated into a set of MAFs.

This scheme induces a dependency relation between different MAFs: for a given MAF p containing a meta-argument $\Diamond x$, there is a MAF q in which x may be an accepted argument. This dependency relation is represented using a modal accessibility relation AR. We call a set W of MAFs, together with an accessibility relation AR over W a DAF (distributed argumentation framework). If the MAFs in W represent the cells of the sorting $\mathbb{S}$ of the framework $\mathcal{AF}$, and if AR reflects the dependency relation just described, then we say that $\langle W, AR \rangle$ is based on $\mathcal{AF}$ and $\mathbb{S}$.

Definition 12 (Distributed Argumentation Framework) *A* distributed argumentation framework *DAF is a tuple $\langle W, AR \rangle$ where W is the set of MAFs and $AR \subseteq W \times W$ the* accessibility relation. *We say that a DAF $\langle W, AR \rangle$ is* based on *the argumentation framework $\langle A, R \rangle$ and sorting $\langle P, T \rangle$ if and only if:*

- *For each $C \in P$, there is a unique $p \in W$, where $p = \langle A^p, R^p, MA^p, MR^p, s^p \rangle$ is defined as follows:*
  - $A^p = C$
  - $R^p = R \downarrow C$
  - $MA^p = \{\Diamond x \mid x \in A \setminus C, y \in C, xRy\}$
  - $MR^p = \{(\Diamond x, y) \mid x \in A \setminus C, y \in C, xRy\}$
  - $s^p = T(C)$
- $AR = \{(q, r) \mid q, r \in W, \exists \Diamond x \in MA^q, x \in A^r\}$

The evaluation of arguments in a modal argumentation framework is defined using the concepts of a modal subframework and of a set of qualified arguments. The intuition is similar to the one described in section 3: the subframework of a MAF p, given an extension $\mathcal{S}$, is the restriction of A^p to those arguments that are not defeated by a meta-argument. The set of qualified arguments further restricts this set to all the arguments that are defended by $\mathcal{S}$ from outside attacks. Additionally, to determine the subframework and qualified arguments of a MAF p, we can restrict $\mathcal{S}$ to only those arguments on which the status of the meta-arguments in p actually depends. We will call this the relevant subextension of $\mathcal{S}$ for p. In the following definitions, we assume a fixed argumentation framework $\mathcal{AF} = \langle A, R \rangle$ and sorting $\mathbb{S} = \langle P, T \rangle$, and a DAF $\langle W, AR \rangle$ based on $\mathcal{AF}$ and $\mathbb{S}$.

Definition 13 (Relevant subextension) *Let $p \in W$ and let $\mathcal{S} \subseteq A$ be an extension. The* relevant subextension *of $\mathcal{S}$ for p is the set $\bigcup_{q \in W, pARq} \mathcal{S} \cap A^q$*

Definition 14 (Modal Subframework) *Let $p \in W$. The modal subframework for p, given an extension $\mathcal{S}$, is the framework $\langle A', R' \rangle$ where $A' = \{x \in A^p \mid \nexists y \in \mathcal{S}'$ s.t. $\Diamond y \in MA^p$ and $\Diamond y MR^p x\}$ and $R' = R^p \downarrow A'$, and where $\mathcal{S}'$ is the relevant subextension of $\mathcal{S}$ for p.*

Definition 15 (Qualified arguments of a Modal Subframework) *Let $p \in W$ and let $\langle A', R' \rangle$ be the modal subframework for p, given the extension $\mathcal{S}$. The qualified arguments of $\langle A', R' \rangle$, given $\mathcal{S}$, is the set $A'' = \{x \in A' \mid \forall \Diamond y \in MA^p$ s.t. $\Diamond y MR^p x, \exists z \in \mathcal{S}'$ s.t. $zRy\}$, where $\mathcal{S}'$ is the relevant subextension of $\mathcal{S}$ for p.*

Given an extension $\mathcal{S}$, we can determine whether it is a sorted extension of a DAF by checking that for each $p \in W$, we have that $\mathcal{A}^p \cap \mathcal{S}$ is an extension of the subframework for p, given the qualified arguments of the subframework for p. The semantics under which the subframework for p is evaluated is s^p.

Definition 16 (Sorted Extension of a DAF) *Let $\mathcal{AF} = \langle A, R \rangle$, $\mathbb{S} = \langle P, T \rangle$ and let $\langle W, AR \rangle$ be the DAF based on $\mathcal{AF}$ and $\mathbb{S}$. A set $\mathcal{S} \subseteq A$ is a* sorted extension *of $\langle W, AR \rangle$ if and only if $\forall p \in W$, $A^p \cap S \in \mathcal{E}_{s^p}(\langle A', R' \rangle, A'')$, where, given $\mathcal{S}$, $\langle A', R' \rangle$ is the modal subframework for p and C'' is the set of qualified arguments of the subframework.*

Note that the definition above does not exploit the fact that the evaluation of arguments of a MAF p depends *only* on all MAFs q such that $pARq$.

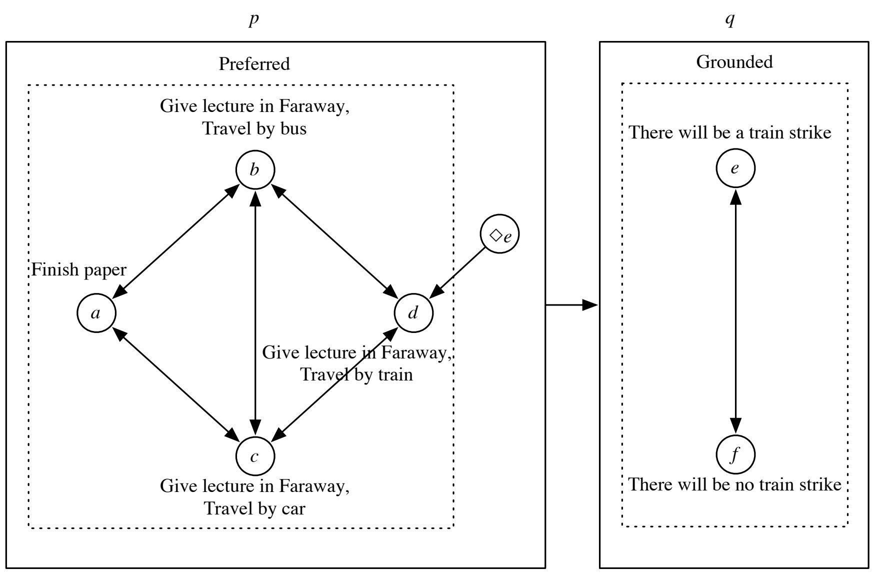

Fig. 4. The sorting-based *DAF* corresponding to the framework in Figure 1.

Example 7. Figure 4 shows the DAF based on the framework $\mathcal{AF}$ and sorting $\mathbb{S}$ used in example 4. The DAF $\langle W, AR \rangle$ consists of $W = \{p, q\}$ with

$$p = \langle \{a, b, c, d\}, \{(a, b), \ldots, (d, c)\}, \{\Diamond e\}, \{(\Diamond e, d)\}, pr \rangle$$

and

$$q = \langle \{e, f\}, \{(e, f), (f, e)\}, \emptyset, \emptyset, gr \rangle$$

and $AR = \{(p, q)\}$.

Consider the extension $\mathcal{S} = \{b\}$. Given $\mathcal{S}$, the argument d is not a qualified argument for p, because $\Diamond e$ attacks d and the argument e is not defended from all attacks. Hence, for p, we have the preferred extensions $\{a\}, \{b\}$ and $\{c\}$ (the extension $\{a, d\}$ is suppressed). For q we have the grounded extension $\emptyset$. We have that $\mathcal{S} \cap A^p = \{b\}$ and $\mathcal{S} \cap A^q = \emptyset$. The extension $\mathcal{S}$ is therefore a sorted extension of the DAF, as are the extensions $\{b\}$ and $\{c\}$.

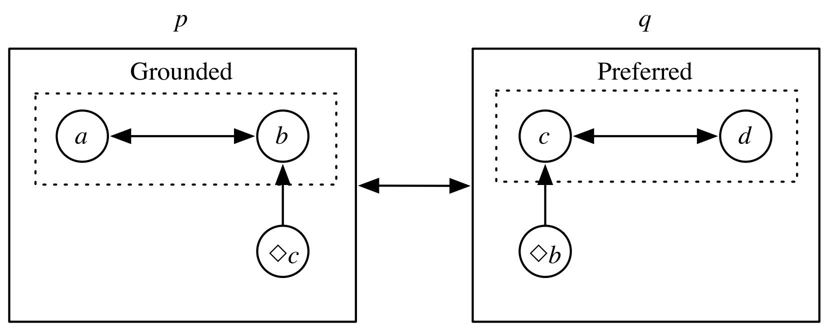

Fig. 5. The sorting-based DAF corresponding to the framework in Figure 3.

Example 8. Figure 5 shows the DAF based on the framework $\mathcal{AF}$ and sorting $\mathbb{S}$ used in example 3. The DAF $\langle W, AR \rangle$ consists of $W = \{p, q\}$ with

$$p = \langle \{a, b\}, \{(a, b), (b, a)\}, \{\Diamond c\}, \{(\Diamond c, b)\}, gr \rangle$$

and

$$q = \langle \{c, d\}, \{(c, d), (d, c)\}, \{\Diamond b\}, \{(\Diamond b, c)\}, pr \rangle$$

and $AR = \{(p, q), (q, p)\}$.

Consider the extension $\mathcal{S} = \{a, c\}$. Given $\mathcal{S}$, only the argument a in the modal subframework for p ($\Diamond c$ is activated and attacks b. Hence, for p, we have the grounded extension $\{a\}$. For q both arguments c and d are in the modal subframework and are qualified ($\Diamond b$ is deactivated). Hence we have two preferred extensions $\{c\}$ and $\{d\}$. We have that $\mathcal{S} \cap A^p = \{a\}$ and $\mathcal{S} \cap A^q = \{c\}$. The extension $\mathcal{S}$ is therefore a sorted extension of the DAF, as is the extension $\{d\}$.

In conclusion, the benefit of the modal fibring approach to multi-sorted extension lies in the modular representation. While our definitions

6 Related Work

The proposal that is most related to ours comes from Prakken [13], from which we also took our running example. He proposes an argument-based semantics that combines grounded and preferred semantics. The motivation, as we discussed in the introduction, is that reasoning about beliefs should be skeptical, while reasoning about actions should be credulous. Prakken's formalism can be seen as a special case of ours: there are just two cells: a preferred cell, containing practical arguments; and a grounded cell, containing epistemic arguments. Moreover, arguments in the preferred cell do not attack arguments in the grounded cell. This reflects the principle that no *Is* should be derived from an *Ought*. Other than that, the formalism takes an approach similar to ours: an extension of a framework $\mathcal{AF}$ is the preferred extension of the framework obtained by removing all arguments not defended by the epistemic part of the grounded extension of $\mathcal{AF}$ (of course, this includes the grounded extension of $\mathcal{AF}$ itself).

An interesting feature of Prakken's formalism is a dialectical proof procedure which is sound and complete with respect to the 2-sorted semantics. This proof procedure combines previously developed proof procedures for the skeptical grounded and credulous preferred semantics (see e.g., [12]). It would be interesting to see whether a generalized dialectical proof procedure could be developed for our semantics. This would, of course, depend on the existence of dialectical proof procedures for the semantics associated with the individual cells.

Another related formalism comes from Brewka & Eiter [6]. They propose a framework for group argumentation, which they call argument context systems. It allows a collection of abstract argument systems to interact via mediators, where a mediator consists of so called bridge rules that associate arguments from one framework with a context to another framework. A context for a framework consists of a set of expressions that determine certain properties of that framework. One framework may then decide on these properties for another framework through the acceptance status of the arguments that appear in the body of the bridge rules. Among the properties controlled by the context are values and preferences, i.e., the framework supports value based and preference based argumentation [1, 5]. Another property is the acceptance status of an argument. This is effectuated through an extra argument *def*, that may invalidate or validate an argument, by attacking it or attacking its attackers. The resulting framework allows the interaction between different frameworks in the argument context system, where one framework may decide about values, preferences and argument acceptance status of other frameworks.

Like in our work, different frameworks may be evaluated under different semantics. Moreover, the semantics under which a framework is evaluated, is also part of the framework's context. This means that, in addition to values, preferences and acceptance status, the semantics under which a framework is evaluated is also a property about which another framework may decide. Of course, this goes beyond the expressivity of our system. On the other hand, different cells of a sorting in our system are part of the same framework, and may interact through attacks. Brewka & Eiter's system does not allow different frameworks in the same argument context system to this. This may be simulated by bridge rules that validate and invalidate arguments, but only partially. Their approach does not account for the distinction between defeated and undefended arguments.

Other related work includes Amgoud & Prade [2], who introduce explanatory, rewards and threats arguments for negotiation dialogues. In practical reasoning, Rotstein et al. [14] propose different types of arguments to represent categorized domain information, like belief, goals or plans. These works, however, do not explicitly apply different semantics to the different types of arguments they define.

7 Conclusion and Future Work

We have presented a theory of multi-sorted argumentation, that generalizes Dung's theory of abstract argumentation in that it allows different parts of a framework to be evaluated under different semantics. We have proven some basic properties, namely the preservation of conflict-freeness, admissibility and completeness. Moreover, we have analyzed the behavior of the multi-sorted framework in the cases where the same semantics is used to evaluate all the cells of the framework, or where the arguments are not accepted in the framework using the same semantics applied to evaluate the cells.

We justify the introduction of a multi-sorted argumentation framework by using a running example from Prakken [13]. In this example, some arguments pertain to actions, and some others pertain to beliefs about the world. As argued by Prakken [13] practical arguments and epistemic arguments have to be evaluated in a different way. We propose to perform this evaluation using a multi-sorted framework.

The modal fibring approach adds another interesting angle to our theory. The fact that multi-sorted argumentation is expressible in modal argumentation frameworks demonstrates the generality of modal argumentation. We expect that modal argumentation will be a useful framework to investigate more sophisticated forms of multi-sorted argumentation.

There is much work still to be done, on all the aspects described above. First of all, a further generalization is possible if we make some of the assumptions that we made optional. For example, instead of a strict partitioning of the framework, we could allow overlapping subsets. This is natural, because the same argument may be put forward by different agents, each associated with a different semantics.

Secondly, we have applied our theory only to some small examples. It will be interesting to apply it to real-world examples, and to compare it with other approaches to multi-agent argumentation and reasoning about trust.

Third, we are applying our theory to different challenges in multiagent systems. One of the possible applications is bounded reasoning in multi-agent systems: dividing a framework into different sets could facilitate a stepwise evaluation of smaller parts of a larger framework. In addition, arguments that are not the focus of a particular issue, could be evaluated using a computationally cheaper semantics. For example, a 'don't care' attitude towards a set of arguments could result in only requiring conflict-freeness for this set. Another application is, as mentioned in the paper, the merging of the argumentation frameworks of single agents into a common framework, in order to allow an easier collaboration among the agents.

Forth, we aim to redefine multi-sorted argumentation in terms of argument labelling [8], instead of argument semantics. The labelling approach is widely adopted in the argumentation community, and it may allow a simpler representation of the sorted extension.

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