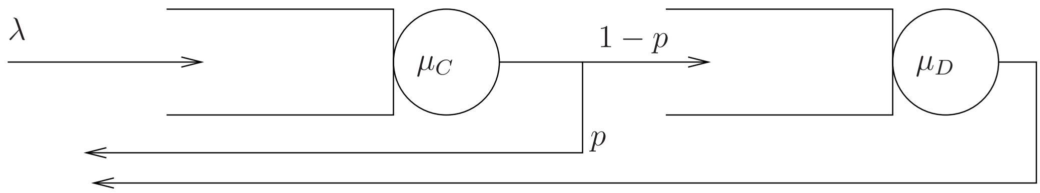
UBINET/RIF: Performance Evaluation of Networks

Homework 6
To be returned on 7 November 2017

6.1 A content server with a cache

Part A: Aggregate view of the traffic

Consider a content server which stores in its cache a portion of the set of documents it has on disk. At each request arrival the server checks whether it is in its cache. With probability p, the requested content is in the cache and it is retrieved instantly and send back to the requester. A content that is not cached must be fetched from the disk before it can be sent back to the requester. We assume that the aggregate traffic arrives to the content server according to a Poisson process with rate λ. The time to process requests (including the time to check the cache) is assumed to be exponentially distributed with rate μ_C. The time to fetch a document from disk and send it back to the requester is also exponentially distributed with rate μ_D. We naturally have $\mu_D < \mu_C$. The queueing network modeling this content server is shown below.



A.1. Compute the arrival rate at each node.

A.2. Write the stability conditions of this system.
(Find an upper bound on λ.)

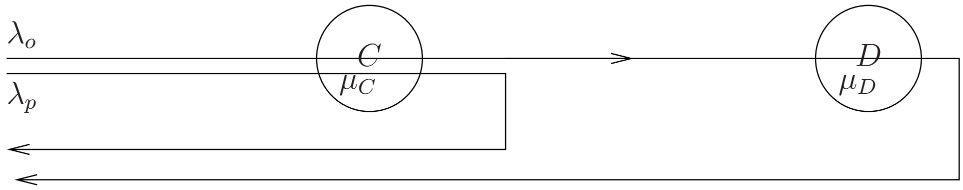
A.3. Compute as a function of λ, p, μ_C and μ_D the requests mean response time.

A.4. Numerical evaluation: let $\frac{1}{\mu_C} = 10$ ms, $\frac{1}{\mu_D} = 100$ ms, and $p = 0.5$.

- A.4.1 Where is the bottleneck of this server?
- A.4.2 What is the maximum requests serving rate?
- A.4.3 What is the requests mean response time when $\lambda = 10$ requests per second?

Part B: Two classes of traffic

Consider again the same server, but this time, we know that only popular contents are cached. The requests for popular content form a Poisson process with rate λ_p, and requests for other contents form also a Poisson process with rate λ_o. The server can be seen as below.



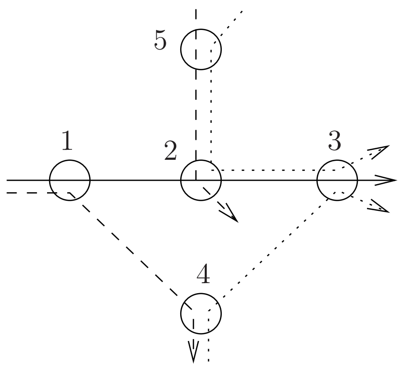
B.1. Compute the arrival rate at each node.

B.2. Write the stability conditions of this system.
Draw a graph showing the set of feasible values for the arrival rates.

B.3. Compute as a function of λ_p, λ_o, μ_C and μ_D the requests mean response time.

B.4. To map the two models, how should λ_p, λ_o, λ and p be related?

6.2 Optimal routing in a Kelly network



Consider the network shown at the right: there are five nodes and packets flow in the network according to five different routes. These are: (1,2,3), (1,4), (4,3), (5,2) and (5,2,3). In each route, the packet arrival process is Poisson. The rate of arrivals along route (1,2,3) is noted λ_{123}, and the same notation is used for the other routes. Each node of the network has one server which serves packets according to the FIFO discipline. Assume the service time at each node is exponentially distributed with parameter μ.

1. Compute the expected response time of a packet along a given route,
2. Compute the expected response time of a packet chosen randomly.

Assume packets have a length exponentially distributed with mean 1000 bytes. The speed of each node is 4 megabit/s. Consider the following values for the packets arrival rates along the different routes: $\lambda_{14} = 100$ packets/s, $\lambda_{43} = 50$, $\lambda_{52} = 200$, and $\lambda_{523} = 50$.

3. What is the value of μ in paquets/s?
4. What is the maximum possible rate of the flow along route (1,2,3)?

Consider now that packets flowing from node 1 to node 3 can take one of two possible routes: route (1,2,3) and route (1,4,3). The traffic rate from node 1 to node 3 is $\lambda_{13} = 200$ packets/s. Let p be the probability that a packet goes through route (1,2,3), so that $1 - p$ is the probability that it takes route (1,4,3).

5. What are the new total arrival rates at the five nodes?
6. Evaluate the expected response time of packets flowing from node 1 to node 3.
7. What is the probability that optimizes the routing for these packets?
8. Evaluate in ms the optimal response time of these packets.