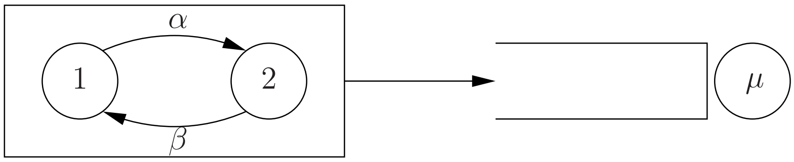
UBINET/RIF: Performance Evaluation of Networks

Homework 5
To be returned on 31 October 2017

5.1 An $E_2/M/1$ queue

Consider a single server queue having an infinite waiting room whose service time is exponentially distributed with rate μ. Customers arrive to this queue according to a stochastic process whose interval follows an Erlang distribution: the inter-arrival time is the sum of two exponential random variables, the first one with parameter α and the second one with parameter β. In other words, the customers arrival process is governed by a two-state continuous time Markov chain such that a customer is generated when the Markov chain leaves state 2, as illustrated in the figure below.



1. What is the customers' arrival rate λ to the queue?
2. Define the state of the $E_2/M/1$ queue such that it is a CTMC.
   Write its state space $\mathcal{E}$.
3. Draw the transition rate diagram.
4. Write the balance equations.
5. Define appropriately the z-transforms $f(z)$ and $g(z)$.
6. Rewrite the balance equations using $f(z)$ and $g(z)$. (You should obtain two equations at the end.)
7. Derive $f(1)$ and $g(1)$.
8. What is the probability p_0 that the system is empty?
9. Derive p_0 without explicitly deriving the z-transforms $f(z)$ and $g(z)$.
10. Derive the system utilization ρ.
    Comment on the expression provided and say why it makes sense.
11. Write the expected queue size $\overline{N}$ in terms of the z-transforms (without explicitly deriving them).