UBINET/RIF: Performance Evaluation of Networks

Homework 3
To be returned on 17 October 2017

3.1 Global balance equation

Consider a Markov chain in continuous-time. Its discrete state space is $\mathcal{E}$ and its infinitesimal generator is $\mathbf{Q}$. Let $\mathcal{S}$ be a subset of $\mathcal{E}$ and $\overline{\mathcal{S}}$ be its complementary subset in $\mathcal{E}$. In other words $\overline{\mathcal{S}} \cap \mathcal{S} = \varnothing$ and $\overline{\mathcal{S}} \cup \mathcal{S} = \mathcal{E}$. Let $\pi = (\pi_i, i \in \mathcal{E})$ denote the stationary distribution of the Markov chain. The *global balance equation* with respect to subset $\mathcal{S}$ is

$$\sum_{i \in \mathcal{S}} \sum_{j \in \overline{\mathcal{S}}} \pi_i q_{i,j} \;=\; \sum_{i \in \overline{\mathcal{S}}} \sum_{j \in \mathcal{S}} \pi_i q_{i,j} \;. \tag{1}$$

Prove this equation.

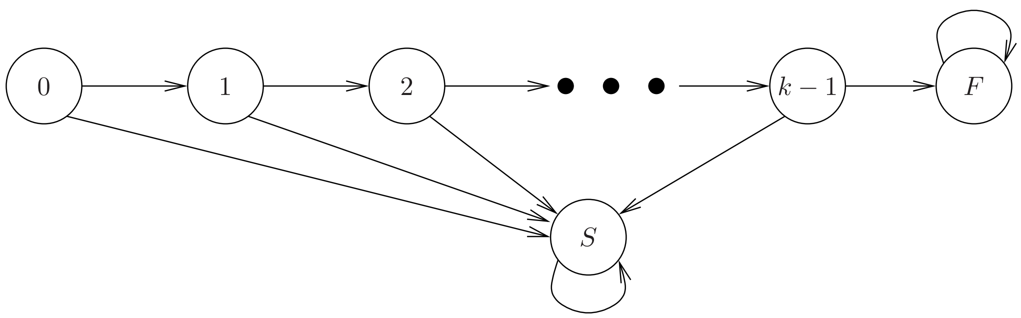
Application Consider a birth-and-death process on the state-space $\mathcal{E} = \{1, 2, 3, 4, 5\}$ with birth rate λ and death rate μ. Write the global balance equation with respect to subset $\mathcal{S} = \{1, 2, 3\}$.

3.2 Transmission over an erasure channel

A transmitter replicates k times each bit it transmits over an erasure channel to increase the probability of it being successfully decoded at the receiver. A bit is successfully transmitted if at least one of the k bits is decoded by the receiver. If all k bits are erased by the channel, the transmission of the bit fails. The state of a bit's transmission can be described by a stochastic process $\{X(n), n \geq 0\}$ that takes value in the state-space $\mathcal{E} = \{0, 1, \ldots, k-1, F, S\}$; $X(n) = S$ indicates that the bit has been successfully transmitted (i.e. at least one replica is successfully decoded), $X(n) = F$ indicates that the transmission has failed (i.e. all replicas were erased by the channel), all other values of $X(n)$ indicate the number of replicas that has been transmitted at step n. We have $X(0) = 0$.

Let p be the probability that the channel erases one bit.

1. Does $\{X(n), n \geq 0\}$ satisfy the Markov property?
   If so, write the transition probabilities of this Markov chain on the transition diagram below.



2. Write the transition probability matrix $\mathbf{P}$.
3. Is this an absorbing DTMC?
   Which states are transient? Which states are absorbing?
4. Let $k = 3$. What is the probability that the bit is transmitted successfully?