UBINET/RIF: Performance Evaluation of Networks

Homework 1
To be returned on 3 October 2017

1.1 Collecting fridge magnets

A cereal manufacturer sells cereal boxes having each a fridge magnet. The complete set of fridge magnets represents a world map. There are a total of n distinct fridge magnets. A given family takes a random time to finish a cereal box and buy a new one. This time is exponentially distributed with parameter λ, therefore the number of cereal boxes purchased by this family is a Poisson process with rate λ. The children of this family are collecting the fridge magnets.

1. Let X be a random variable denoting the time between two purchases of cereal boxes. Write the cumulative distribution function of X and its probability density function.

2. Let $N(t)$ be the number of magnets collected at time t. What is the nature of the stochastic process $\{N(t), t > 0\}$?

3. There is in total exactly the same number of each distinct fridge magnet. What is the probability p_i that a cereal box has a fridge magnet of type i?

4. Let $N_i(t)$ be the number of magnets of type i collected at time t. What is the nature of the stochastic process $\{N_i(t), t > 0\}$?

5. Let T_i be the time needed to get the first magnet of type i. What is the distribution of T_i?

6. Let T be the time needed to collect the entire collection of fridge magnets. What is the distribution of T (hint: write T as a function of the T_i's)?

1.2 An unreliable computer system

Assume that a computer system is in one of 3 states: busy, idle, or undergoing repair, respectively, denoted by states 0, 1, and 2, respectively. Observing its state at 2pm each day, we believe that the system approximately behaves like a homogeneous discrete-time Markov chain with the transition matrix

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

Prove that the chain is irreducible and aperiodic, and determine the stationary probabilities.

1.3 A simple DTMC

Consider a discrete-time Markov chain having the following transition matrix:

$$P = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

1. What are the limiting probabilities $\pi = (\pi_1, \pi_2, \pi_3) = \lim_{t\to\infty} \pi(t)$ for this Markov chain?
2. What is the 2-step transition matrix for this Markov chain?
3. Let $\pi(0) = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. What is the transient probability distribution after two time steps?