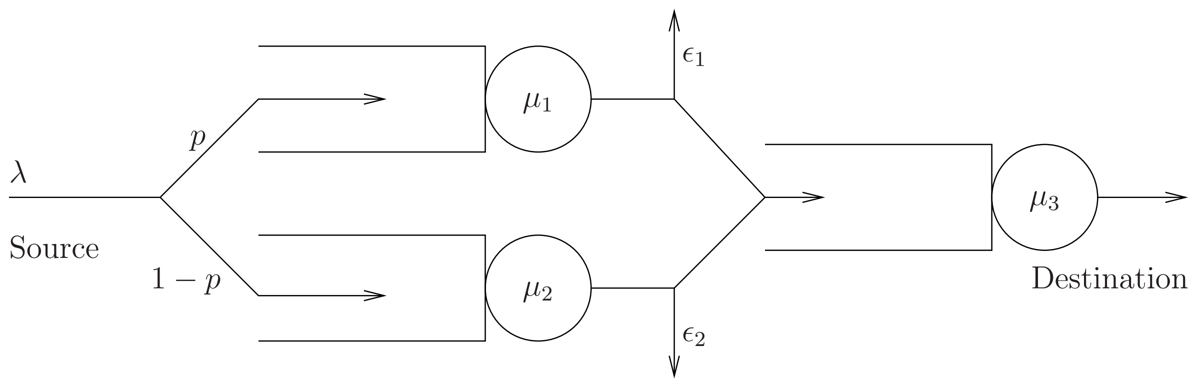
UBINET: Performance Evaluation of Networks

Homework 6

To be returned on 8 November 2016

6.1 A routing problem with lossy paths

Consider a Poisson traffic flowing from a source S to a destination D. The traffic intensity is λ. The traffic may flow through two distinct routes as depicted in the figure below. The two routes join and share a common path before reaching the destination. Each portion of each route is modeled as a queueing system with infinite waiting room and an exponentially distributed service time. The service rate along the upper path is μ_1 and that along the lower path is μ_2, assumed to be less than μ_1. The service rate along the common path is μ_3. Both paths are error prone, the loss probability in the upper path is denoted ϵ_1 and that in the lower path ϵ_2. Assume that packets flow through the upper route with a probability p, with $0 \leq p \leq 1$, and with the complementary probability, they flow through the lower route.



1. Write the traffic equations.
2. Express the stability condition by finding an upper bound on λ.
3. Find the interval of values that p can take such that the stability condition is satisfied.
4. Find the mean number of customers in the system.
5. Compute $\overline{T}(p)$, the expected sojourn time in the network of the customers reaching the destination, in terms of the traffic intensity λ, the service rates $\{\mu_i\}_{i=1,2,3}$, the loss probabilities $\{\epsilon_i\}_{i=1,2}$ and the routing probability p.
6. Find an instance in terms of λ, $\{\mu_i\}_{i=1,2,3}$ and $\{\epsilon_i\}_{i=1,2}$ for which it is better to route most of the traffic through the upper path.
7. Find an instance in terms of λ, $\{\mu_i\}_{i=1,2,3}$ and $\{\epsilon_i\}_{i=1,2}$ for which it is better to route most of the traffic through the lower path.

6.2 The principle of capacity reduction

Part 1: A Kelly network

Consider an M/M/1 FIFO queue with two classes of customers. Customers from each class arrive to the queue according to a Poisson process. The arrival rates are λ_1 and λ_2 respectively. The server takes a time that is exponentially distributed with parameter μ regardless of the class.

1. Use the notation seen in class to write the arrival rate of each class in the queue and the total arrival rate in the queue.
2. Give the pairs (λ_1, λ_2) for which the system is stable.
3. What is the expected number of customers from each class in the queue?
4. Find the expected sojourn time of customers of class 1 in the queue.

Part 2: A single class queue

Consider now an M/M/1 FIFO queue with customers arrival rate λ_1 and service rate $\mu - \lambda_2$.

1. What is the stability condition?
2. What is the expected number of customers in the queue?
3. Find the expected sojourn time of customers in the queue.
4. What can you conclude from this problem?