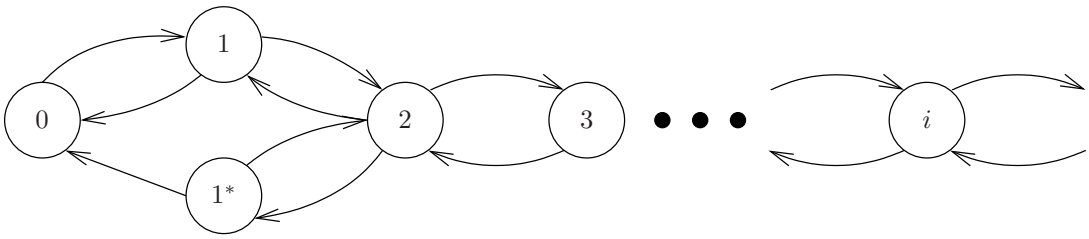
UBINET: Performance Evaluation of Networks

Homework 5
To be returned on 25 October 2016

5.1 The M/M/2 queue with heterogeneous servers

After a merger, a company starts offering a much wider range of services. To cope with the expected increase in the jobs arrival rate at the company's computer center, a second server with larger processing speed is purchased and added to the system that originally had only one server. Jobs are scheduled in priority on the faster server, such that the slower server is used only when the other one is busy. The jobs arrival process is Poisson with rate λ, the slow server has speed μ while the fast one has speed $\alpha\mu$, with $\alpha > 1$. The service time of each of the servers is exponentially distributed. Let $X(t)$ denote the number of customers in the system at time t.

1. Explain why the process $\{X(t), t > 0\}$ is **not** a Markov chain.
2. How would you define the utilization of the computer center ρ?
3. We assume the computer center has infinite capacity. What is the stability condition?
4. Consider the transition diagram below.



   Guess what are the transition rates of this Markov chain.
   What would state 1^* refer to?
5. Write the balance equations.
6. Express all stationary probabilities in terms of π_2.
7. Use the normalizing condition to find the stationary distribution.
8. Is this Markov chain time-reversible?
9. Find that the mean number of jobs in the computer center is

$$\overline{N} = \mathrm{E}[X] = \frac{1}{A(1-\rho)^2} \qquad \text{where} \quad A = \frac{\alpha\mu^2(1+2\rho)}{\lambda(\lambda+\mu)} + \frac{1}{1-\rho} \ .$$

10. Some customers complained that they experienced a very bad system response time and asked to shutdown the slow server. Find an instance in terms of λ, μ and α for which the response time of the fast M/M/1 is smaller than the response time of the heterogeneous M/M/2.
11. Explain why would this be happening.