UBINET: Performance Evaluation of Networks

Homework 4
To be returned on 18 October 2016

4.1 A touristic bus

An open air bus can take up to N tourists. For economic reasons, a company proposing bus tours enforces that its unique bus does not start the trip unless full. Tourists arrive to the bus departure point according to a Poisson process with rate λ. Once the bus is full, the driver starts the tour which lasts for a random time that is exponentially distributed with parameter μ. Tourists that arrive to the bus departure point while the bus is away do not wait and look for other activities. Once the tour is over, the bus can welcome again any tourist wishing to make the tour. The bus can be seen as a finite capacity queueing system with grouped services.

1. Define a Markov chain that describes the evolution of the queueing system.
   Write its state-space $\mathcal{E}$.
   Draw its transition rate diagram.

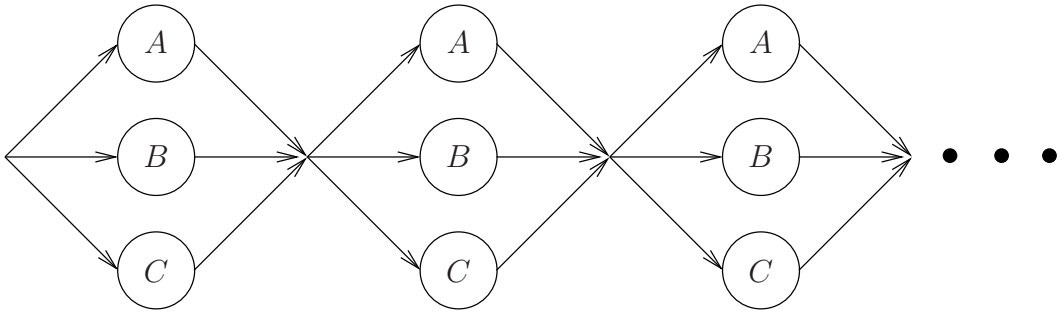
2. Compute the steady-state distribution of the number of tourists in the bus (the queue size).

3. What is the expected number of tourists in the bus?

4. What is the effective service rate θ of the company in terms of tourists served per unit of time?

4.2 Execution of a parallel program

A parallel program consists of three cyclic tasks, denoted A, B and C, running in parallel. The three tasks run on three independent processors that are dedicated to this program. The execution times of the tasks are exponentially distributed random variables with parameters λ_A, λ_B and λ_C respectively. A cycle is defined as the time needed for the execution of the three tasks. A new cycle of execution starts immediately upon completion of the previous cycle as depicted in the figure below.



1. Identify a CTMC describing this system.
   Write its state space $\mathcal{E}$.
   Draw the transition rate diagram.

2. Let θ be the system throughput, in other words, the cycle execution rate.
   Find two expressions for θ in terms of the stationary solution π and the parameters λ_A, λ_B and λ_C (do not compute π).