UBINET: Performance Evaluation of Networks

Homework 3
To be returned on 11 October 2016

3.1 Computers are not forever

A computer can be in any of the following states: in service (state 1), in repair (state 2), broken (state 3), or spare (state 4). Each new computer starts by being in service. It may happen that some of its hardware fails, it will then be in repair. Most of the time the hardware can be replaced and the computer will again be in service. In rare occasions, the computer cannot be repaired and according to the damage type, it will either be declared broken or it will be kept and used for its spare parts.

The technical department staff records the state of each computer every morning. It seems that a computer that was in service remains in service with probability 0.95, otherwise it becomes in repair. Most repairs are successful, a computer in repair has 80% chances to return to service, but in 15% of the cases it will remain in repair, in 1% of the cases it is unrepairable so the computer is declared broken and in the last 4% of the cases the computer is not repairable but it can still be used for its spare parts. Computers that are broken or for spare remain in that state.

Let $X(n)$ be the state of a computer at day n.

1. Describe the stochastic process $\{X(n), n > 0\}$.
   Write its state-space $\mathcal{E}$.
2. Write its transition probability matrix $\mathbf{P}$.
3. What is the expected time until a computer is declared unrepairable?
4. What is the probability that a computer breaks?
   What is the probability that a computer ends as spare parts?

3.2 A simple CTMC

Consider a Continuous-Time Markov Chain (CTMC) with state space $\mathcal{E} = \{1, 2, 3, 4\}$ and infinitesimal generator $\mathbf{Q}$ given by

$$\mathbf{Q} = \begin{bmatrix} -\alpha & \alpha & 0 & 0 \\ 0 & -\beta & \beta & 0 \\ \gamma & 0 & -(\gamma+\delta) & \delta \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1. Is this CTMC irreducible? Explain your answer.
2. Compute $T(i)$, the expected time until absorption when the CTMC starts in state $i \in \mathcal{E}$.
3. Simplify the expressions when $\gamma = 0$.
   Comment on the results.