UBINET: Performance Evaluation of Networks

Homework 1
To be returned on 27 September 2016

1.1 Minimum and maximum of exponential distributions: Application to network reliability

Let X_1 and X_2 be two independent exponentially distributed random variables (rvs) with parameters $\lambda_1 > 0$ and $\lambda_2 > 0$, respectively.

1. Determine the cumulative distribution function of $Z_1 := \min(X_1, X_2)$.
   Any comment on the distribution of Z_1?

2. Determine the cumulative distribution function of $Z_2 := \max(X_1, X_2)$.

Application to the study of network reliability. Consider the series and parallel networks depicted in Figure 1. We assume that each node of any of the networks has an exponentially distributed lifetime with parameter μ. Let X_i be the lifetime of node i (so $X_i \sim \mathrm{Exp}(\mu)$) and let T_s and T_p denote the lifetime of the series and parallel networks respectively.

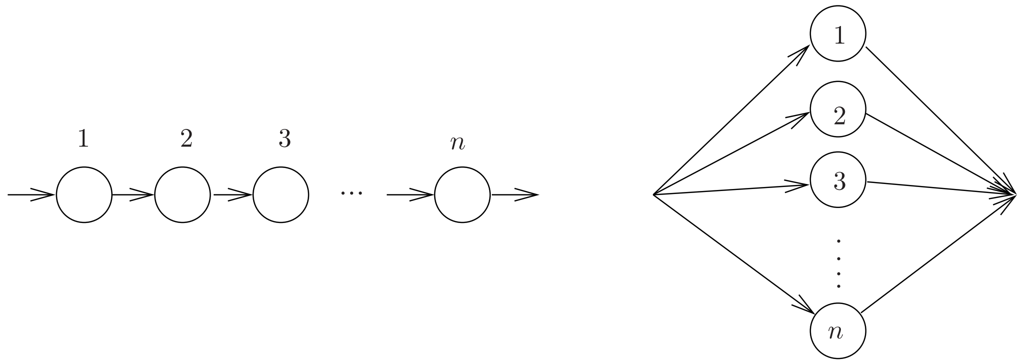

Figure 1: Series (left) and parallel (right) networks

3. When would the series network fail? The answer to this question will let you express T_s as a function of the X_i's. Compute then the cumulative distribution function of T_s and its expectation $\mathrm{E}[T_s]$. (Hint: use previous answers of this exercize.)

4. When would the parallel network fail? The answer to this question will let you express T_p as a function of the X_i's. Compute then the cumulative distribution function of T_p. (Hint: use previous answers of this exercize.)

5. By noticing that T_p is actually the sum of the times between successive node failures (it is the time until the first failure plus the additional time until the second failure plus the additional time until the third failure plus ... plus the additional time until the last failure), compute the expectation $\mathrm{E}[T_p]$. (Hint: use the memoryless property of the exponential distribution to find the expectation of the additional times.)

6. As $n \to \infty$ which network has higher reliability? (Compare $\mathrm{E}[T_s]$ and $\mathrm{E}[T_p]$.)

1.2 An unreliable computer

We observe a computer at time steps $n = 0, 1, 2, 3, \dots$ At any step n, this computer can be in one the following three states: inactive (state 1), active (state 2) or in repair (state 3).

We assume that the time-evolution of the state of the computer is modeled as a homogeneous Discrete-Time Markov Chain (DTMC) with state-space $\mathcal{E} = \{1, 2, 3\}$ and transition matrix P given by

$$P = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.8 & 0.0 & 0.2 \end{bmatrix}$$

1. Draw the transition diagram.
2. Is this DTMC irreducible and aperiodic? Explain your answer.
3. Write the stationary equations.
4. Compute its stationary probabilities (denoted as π_1, π_2, π_3). Justify your calculation. *Use fractions to express the probabilities.*