UBINET: Performance Evaluation of Networks

Homework 5
To be returned on 27 October 2015

5.1 Slotted Aloha

Exercise borrowed from Philippe Nain
Consider 2 stations sharing the same communication channel. Time is slotted such as the length of a slot is a packet transmission time over the shared channel. A station can store at most one packet: as long as this packet has not been transmitted successfully the station cannot transmit another packet.

The protocol works as follows:

- if *only one* station transmits a packet in a slot then the transmission is successful and the station will move to the *empty* state at the end of slot;
- if *two* stations transmit a packet in the same slot, then there is a collision; packets will need to be retransmitted and each of the stations will enter the *blocked* state at the end of the slot.

Transitions between states are as follows:

- for a station in the *empty* state at the end of the kth slot ($k = 1, 2, \ldots$), either a new packet arrives with probability $0 < a < 1$ and it will be transmitted in the $(k + 1)$st slot, or no packet arrives with the complementary probability $\overline{a} = 1 - a$ and the station remains hence in the *empty* state until the end of the $(k + 1)$st slot;
- for a station in the *blocked* state at the end of the kth slot ($k = 1, 2, \ldots$), with probability $0 < p < 1$ the packet is retransmitted in the $(k + 1)$st slot and with the complementary probability $\overline{p} = 1 - p$ it will not and the station remains in the *blocked* state until the end of the $(k + 1)$st slot.

Let $N(k) \in \{0, 1, 2\}$ be the number of blocked stations at the end of the kth slot.

1. What is the nature of the stochastic process $\{N(k), k = 1, 2, \ldots\}$? Draw the transition diagram and write the probability transition matrix $\mathbf{P}$ in terms of parameters a and p.
2. Compute the stationary distribution $\pi := (\pi_0, \pi_1, \pi_2)$ of the process $\{N(k), k = 1, 2, \ldots\}$ in terms of parameters a and p.
3. Compute T, the throughput of the system.
4. Let $a = p$. What is the maximum throughput of the system?

5.2 A computer virus

Exercise inspired by Alain Jean-Marie
A computer virus has been released and it is spreading from one computer to another. The computer that released the virus is never repaired, however all other computers will eventually be repaired. The infection process never dies due to the presence of the original computer that released the virus. It is assumed that

- *each* infected computer infects a non-infected computer after a random time that is exponentially distributed with rate $\lambda > 0$;
- *each* infected computer, except for the one that has released the virus, is repaired after a random time that is exponentially distributed with rate μ with $\mu > 0$.

All repair and infection durations introduced above are assumed to be mutually independent. Let $\rho = \lambda/\mu$.

1. Discuss briefly why the number of infected computers at time t (including the original one) is a continuous time Markov chain.
   Draw the state transition diagram and indicate what are the transition rates (justify).

2. Compute θ, the average infection rate.
   Compute δ, the average repair rate.
   Conclude that the average number of infected computers in steady-state $\overline{N}$ satisfies (do not evaluate the steady-state probabilities yet):

$$\overline{N} = \frac{1}{1-\rho} \ .$$

3. Use Little's law to compute $\overline{T}$, the average computer infection duration.
   How should this value compare to $1/\mu$, and why?

4. Solve for the steady-state probabilities of the number of infected computers.
   Give the conditions under which these probabilities exist.
   Check by direct summation the value of $\overline{N}$.