UBINET: Performance Evaluation of Networks

Homework 4
To be returned on 20 October 2015

4.1 Queue with rational customers

Exercise borrowed from Alain Jean-Marie
Consider the following system: customers arrive to a queue according to a Poisson process with arrival rate $\lambda > 0$. The queue is served by only one server whose service time is exponentially distributed with service rate $\mu > 0$ (i.e. the expected service time is $1/\mu$). The waiting room is assumed to be infinite. However, and as opposed to the classical M/M/1 queue, each customer counts the number of customers already in the queue to decide what to do. For a queue size i, the customer joins the queue with probability $1/(i+1)$ and with the complementary probability leaves the system.

Let $X(t)$ be the number of customers in the system at time t.

1. What is the customers' effective arrival rate to the queue when there are i customers present?
2. What is the nature of the stochastic process $\{X(t), t \geq 0\}$?
3. Draw the transition diagram.
4. Let π_i be the stationary probability that the system is in state $i = 0, 1, \ldots$ Write the balance equations satisfied by π_i, $i = 0, 1, \ldots$
5. Compute the stationary distribution and guess what the stability condition might be.
6. What is the throughput of the system?

4.2 Queue in discrete time

Exercise borrowed from Alain Jean-Marie
We observe a queue in discrete time. At each time unit, the following occurs:

- a random number of customers enter the queue (grouped arrivals); k customers arrive with probability $q_k > 0$, with $k \geq 0$ (evidently $\sum_{k=0}^{\infty} q_k = 1$);
- two customers are served (paired service) and leave the queue; if less than two customers are present in the queue, all are served and leave the queue.

Even though arrivals and departures occur simultaneously, the same customer cannot arrive, be served and leave the queue in the same time unit.

1. Define a Markov chain to describe the queue's behavior.
2. Write the state space $\mathcal{E}$.
3. Write the possible transitions and their probabilities for each possible state in the state space $\mathcal{E}$. Write then the transition probability matrix $\mathbf{P}$.
4. Is this Markov chain irreducible and aperiodic?