UBINET: Performance Evaluation of Networks

Homework 3

To be returned on 13 October 2015

3.1 Peer-to-peer storage network

We store a document on a peer-to-peer system. To mitigate the churn of peers, the document is replicated over K peers. We assume requests to retrieve the document may occur only at the beginning of every minute. Let $X(n)$ be the number of document copies that are available just before the nth minute. As peers connect and disconnect from the system, the number $X(n)$ varies from minute to minute. If at some minute no copy is available, the document cannot be retrieved and the request fails. If this happens, we will let $X(n) = F$ to express this failure. Clearly the state-space is $\mathcal{E} = \{1, \dots, K, F\}$. The transition probability matrix is

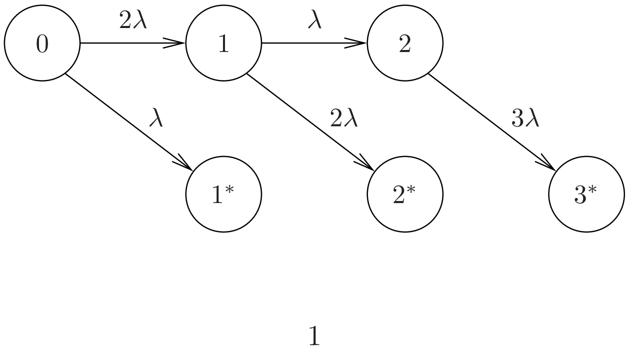
$$\mathbf{P} = \begin{bmatrix} \frac{1}{K+1} & \frac{1}{K+1} & \cdots & \frac{1}{K+1} & \frac{1}{K+1} \\ \vdots & \vdots & & \vdots & \vdots \\ \frac{1}{K+1} & \frac{1}{K+1} & \cdots & \frac{1}{K+1} & \frac{1}{K+1} \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}.$$

1. Describe the stochastic process $\{X(n), n > 0\}$.
2. What is the limiting probability that a request fails?
3. Let $T(i)$ be the expected time before absorption when the process is initially in state i. Calculate $T(1)$, $T(2)$ and $T(3)$ when $K = 3$.

3.2 Two-hop routing in delay-tolerant networks (DTN)

Consider the two-hop routing in a DTN: the source of a message transmits a copy of its message to any node it meets in the network; such a node becomes a relay. Relays can only transmist messages to their respective destinations. We assume that the meeting process is Poisson with rate λ, i.e. the inter-meeting times between any two nodes are independent and exponentially distributed. We assume that at each meeting all possible transmissions are made instantly. Let $X(t)$ be the number of times a given message has been transmitted in the interval $[0, t[$ ($t = 0$ is seen as the message generation instant at the source). We have $X(0) = 0$.

We consider a network with only 4 nodes. Consequently, the state-space is $\mathcal{E} = \{0, 1, 2, 1^*, 2^*, 3^*\}$, where the asterix denotes the fact that the message has reached its destination. For instance if the message has been transmitted i times and has reached its destination at time t then $X(t) = i^*$. The transition diagram is



We are interested in computing $T(0)$ the expected time to deliver a message given that $X(0) = 0$. The expected number of transmissions (in steady-state) $\mathrm{E}[X] := \sum_{j=1}^{3} j b_{0,j^*}$ is also a metric of interest as it gives the cost of the two-hop routing in terms of transmissions. Here, b_{0,j^*} is the probability that j transmissions are needed for the message to be delivered given that $X(0) = 0$.

1. Say why $\{X(t), t > 0\}$ is an absorbing CTMC over $\mathcal{E}$. Mention specifically the transient/absorbing states.
2. Write the infinitesimal generators $\mathbf{Q}$.
3. Derive the transition matrix $\mathbf{P}$ of the *embedded* Markov chain.
4. Use appropriately $\mathbf{Q}$ to compute $T(0)$.
5. Use appropriately $\mathbf{P}$ to compute $\mathrm{E}[X]$.
6. How would $T(0)$ vary should the number of nodes in the network increase? Justify your answer.
7. How would $\mathrm{E}[X]$ vary should the number of nodes in the network increase? Justify your answer.