UBINET: Performance Evaluation of Networks

Homework 1
To be returned on 29 September 2015

1.1 Minimum and maximum of exponential distributions

Exercise borrowed from Philippe Nain
Let X_1 and X_2 be two independent exponentially distributed random variables (rvs) with parameter $\lambda_1 > 0$ and $\lambda_2 > 0$, respectively.

Determine the cumulative distribution function of

1. $Z_1 := \min(X_1, X_2)$; (any comment on the distribution of Z_1?)
2. $Z_2 := \max(X_1, X_2)$.

1.2 Poisson process

Exercise borrowed from Philippe Nain
Consider a stream of packets arriving at a communication network. We assume that the arrival times of these packets can be modeled as a Poisson process with rate 0.3 per second. Compute the probabilities of the following events:

- Exactly three packets will arrive during a ten-second interval;
- At most twenty packets will arrive in a period of twenty seconds;
- The number of packets in an interval of duration five seconds is between three and seven;
- The time between two consecutive packet arrivals is larger than one minute

1.3 A simple DTMC

Exercise borrowed from Alain Jean-Marie
Consider the homogeneous discrete-time Markov chain with state-space $\mathcal{E} = \{1, 2, 3\}$ and transition matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 1-p & p \\ 1 & 0 & 0 \\ 0 & \alpha & 1-\alpha \end{pmatrix} .$$

1. Under which conditions is the chain irreducible and aperiodic?
2. Compute the steady state probability vector.

1.4 A computer system

Exercise borrowed from Philippe Nain

Consider a model of a *uniprogrammed* computer system with K I/O devices and a CPU. For the program currently under execution, the system will be in one of the $K+1$ states denoted by $0, 1, \ldots, K$, so that in state 0 the program is using the CPU, and in state k $(1 \leq k \leq K)$ the program is performing an I/O operation on device k. Each I/O operation lasts exactly one unit of time after which the program being executed will return to the CPU. Assume that the request for device k occurs at the end of a CPU burst with probability q_k, independent of the past history of the program. The program will finish execution at the end of a CPU burst with the probability q_0 such that $\sum_{k=0}^{K} q_k = 1$. We assume that the system is saturated so that upon completion of one program, another statistically identical program will enter the system instantaneously. We assume that $0 < q_k < 1$ for $k = 0, 1, \ldots, K$.

With these assumptions the system can be modeled as an irreducible aperiodic discrete-time Markov chain on the state-space $\{0, 1, \ldots, K\}$.

1. Draw the transition diagram.
2. Determine the transition matrix P.
3. Compute the limiting probabilities $\pi(i)$ for $i \in \{0, 1, \ldots, K\}$.