PHASOR ESTIMATION UNDER NONSTATIONARY CONDITIONS

Praveen Pankajakshan[a]

[a] Texas A&M University, College Station, TX

Abstract. This paper describes the design and analysis of a recursive Kalman Filter for phasor estimation of nonstationary signals. To detect the transmission or distribution line disturbances (or faults), it is necessary to track the amplitude and phase of the steady state, post-fault (or post-disturbance) fundamental signal from the distorted signal. The model developed assumes a constant-frequency, rotating phasor for the sampled voltage and current signals. The filter was developed in the Simulink environment of MATLAB, and the M Programming Language was used to model it. The parameters of the filter and the model are input through the S-functions. The developed filter is an extended form of the discrete Kalman filter [3] and estimates the phasors in the presence of decaying dc offsets, odd or even harmonic distortions, and measurement noise. The model settings and parameters are customizable depending on the input signal and the noise characteristics. The filter assumes a white noise characteristic for the signal variance and the measurement noise variance. However, in a real-time system, the signal and the noise frequencies are band limited and are therefore they are not perfectly white. The model can still be applied but the estimation error will be different from the test bench created signal. The open-system solution and user-friendly nature of our interface makes it a useful educational tool for students and engineers in understanding the mathematical theory and reasoning behind power system protection and voltage restoration approaches. Performance evaluation of the algorithm on estimation is also shown in comparison with other nonrecursive techniques. The filter has a faster convergence time and the mean square difference between the output and actual values is a minimum. The results of the sensitivity study on the changes in filter parameters, signal sampling rate, and fundamental frequency drifts reveals the effectiveness of the developed scheme.
Key-words: Recursive kalman filter, phasor estimation, state-space model, noise variance, MATLAB, digital relay, voltage restorer.

1 Introduction

When a fault occurs in a power transmission line, the voltage and current signals are severely distorted. These signals may contain contain dc offsets, high frequency transients, and oscillation components. To frame the state of the power system, it is necessary to estimate the fundamental components of the steady state post fault currents and voltages from these corrupted voltage and current signals. Digital Relays use algorithms for estimating the functioning parameters of a power system and then use the selected characteristics to make right decisions to disconnect a

failed element, such as a line, transformer or a generator.
To achieve this, the transient components are superimposed on the steady state signals immediately after the fault, so the transients become the corrupting noise. The problem can thus be reduced to estimating the steady state components of the sending-end voltages and currents in the presence of transient components. However, precise calculation of these quantities is difficult in the presence of noise and harmonics produced by the power electronic equipments and arc furnaces. Therefore, it is essential to seek and develop a flexible and reliable method that can measure frequency and phasor in presence of these distorting factors.
The important classes of theoretical and statistical problems in communication and control that overlap with that experienced above in power systems are:

- prediction of random signals
- separation of these random signals from random noise
- detection of signals of known form (like pulse or sinusoid) in the presence of random noise.

Some of the algorithms that have been proposed in literature are essentially nonrecursive digital finite impulse response (FIR) filters. These filters use finite length windows, and their outputs essentially depend on the data contained within this window. The short window technique computes the voltage and current phasors from sampled data and then estimates the impedance as seen from the relay locations. Short window algorithms can be adversely affected by the presence of nonfundamental frequencies in the system voltages and currents. [8] describes the design, implementation, and testing of an adaptive distance relay that uses data windows of different lengths for calculating phasors and impedances from those phasors. The conventional methods ([9]) incur leakage errors in estimating frequency and phasor when there is deviation from the nominal frequency ($60\,Hz$). The long window algorithms, if designed properly, can adequately suppress the effects of the presence of nonfundamental frequencies in the fault waveforms. The earliest application of Kalman filter [3] to power system was in [6] for relaying. The Kalman filter is a recursive filter whose output depends on the present inputs as well as on all previous inputs. Effectively, the weight assigned to the latest input is maximum whereas the assigned weight decreases as the input becomes older. The Kalman filter differs from the other filtering algorithms in that its gain coefficients vary with time (the gains are nonstationary) [1]. The recursive form of the discrete fourier transform (DFT) on the other hand is simple and needs fewer computations, but the presence of an exponentially decaying dc component in a signal adversely affects the phasor estimates.
This paper consists of five sections. In section 2, the state space model for the voltage and current phasor is developed with the recursive kalman filter. The inclusion of the dc and harmonic components in the model and the implementation aspects of the algorithm is described in section 3. Section 4 describes the simulation test bed and sensitivity of the changes in Kalman parameters and the model to the estimated amplitude. In section 5 these results are compared with the DFT based method for changes in sampling rate f_s and the cut-off frequency f_c of the

low-pass butterworth filter. The various notations and conventions used, and their associations is listed in the appendix.

2 Proposed Digital Algorithm

The procedure to implement the technique consists of the following steps.

- Select a suitable model for representing the waveform of a signal.
- Select a sampling rate
- Select the initial Kalman parameters
- Express the model in state-space form.
- Estimate the values of the elements of the vector of unknowns.

One of the roles of digital relays for protection of power systems is to estimate voltage and current phasors from sampled data. The samples contain information, noise, and measurement errors. A filtering technique is therefore needed to condition the data before estimating the phasors. The *signal generator* is used

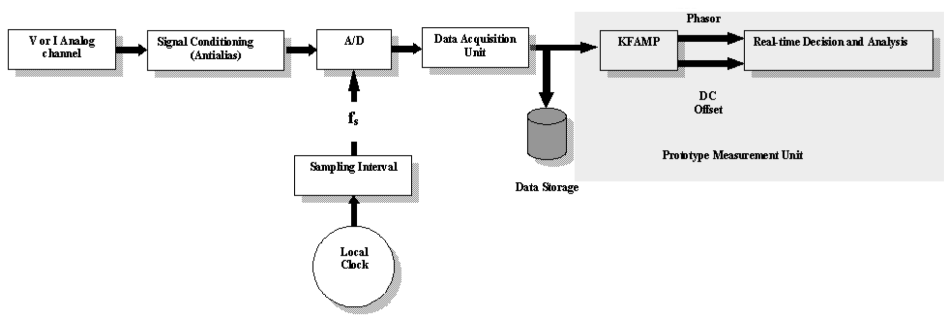


Figure 1: Phasor Measurement Process implemented in the prototype

to simulate the voltage or current signals of the desired frequency and amplitude. The input dialog box is used for entering the amplitude and the frequency of the fundamental signal and the odd and even harmonics. The data acquisition board can be simulated to include an anti-aliasing butterworth filter, vertical resolution tuner, amplifier, and an A/D converter. The order, coefficients, and the cutoff frequency for the butterworth filter can be tuned according to the need. The vertical resolution of the signal can be 8, 12, or 16 bit. But, from the nyquist criterion, the sampling frequency (f_s) of the A/D converter should be at least 2 times higher than the highest frequency (f_h) of the signal and its harmonics. That is,

$$f_s \geq 2f_h \tag{1}$$

2.1 Signal representation as Phasors

The analog signal to be measured is filtered, conditioned, and supplied to the data acquisition board, which samples the analog signal into the digital signal (forwarded as a data window of signal samples). The choices include analog filtering, A/D vertical resolution enable/disable switches; type, order and cut-off frequency of the analog filter; range and number of bits of the A/D converter; conditioning gain and the length of the output data window. The digital signal ($\mathbf{z}_s$) is input to the recursive Kalman filter block. The output of the block is a real number giving the estimated amplitude and phase of the fundamental signal (60 Hz for the voltage signal) and the DC offset (if any) in the input signal. We note the usefulness of the Kalman filter in the presence of harmonics, dc offset, and even measurement noise. Both the process and measurement noise can be assumed to be a random white noise sequence that occurs because of analog to digital converters or other sensing devices connected to the system.
The waveform of voltage or current can be considered to be comprised of components of the fundamental frequency, decaying dc, harmonic and nonharmonic frequencies, and noise. A noise-free current or voltage signal can be expressed by the real or imaginary part of

$$s(t) = Ae^{(j\omega_o t+\phi)} \tag{2}$$

where A is the amplitude and ϕ is the phase angle at $t = 0$. Consider a Rayleigh

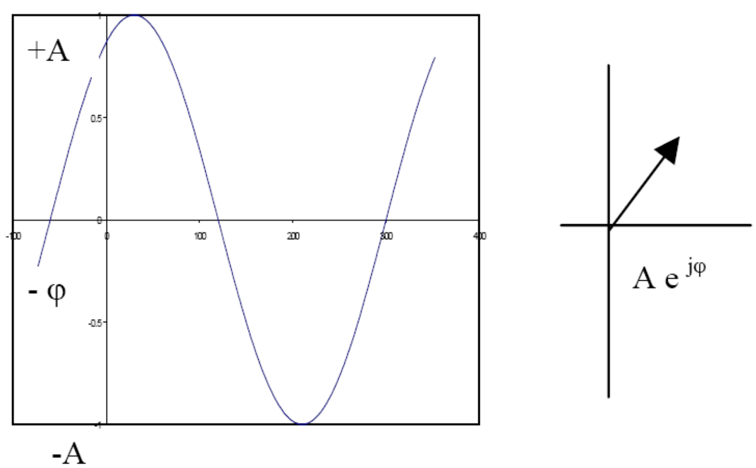


Figure 2: Signal as Phasor Representation

distribution for the amplitude and an uniform distribution for the phase angle, the noise-free current or voltage following the fault event is expressible as [2]:

$$s(t) = C_1 s_r + C_2 s_i \tag{3}$$

where s_r and s_i are statistically independent, zero mean, Gaussian random variables. The variance of s_r or s_i is chosen as σ_v^2 for the voltage signals and σ_i^2 for the current signals.

2.2 State Space Model

The Kalman filtering is applied to a state space model of the system. The model includes state transition and output equations. If the signal is sampled with a frequency of f_s then, the time between one sample and the other is ΔT the estimation of the voltage at any time $(n+1)\Delta T$ in terms of the phasors at time $n\Delta T$ are as follows:

$$\begin{bmatrix} x_r(n+1) \\ x_i(n+1) \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} x_r(n) \\ x_i(n) \end{bmatrix} + \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} \Delta x_r(n) \\ \Delta x_i(n) \end{bmatrix} \tag{4}$$

The state transition equation can also be written in vector notation as:

$$\mathbf{x}(n+1) = P\mathbf{x}(n) + Q\Delta\mathbf{x}(n) \tag{5}$$

The sampled value of the signal is related to the signal phasor. This relation is called the output equation and is as follows:

$$z_s(n) = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix} \begin{bmatrix} x_r(n+1) \\ x_i(n+1) \end{bmatrix} + b(n) \tag{6}$$

$$z_s(n) = \mathbf{c}\mathbf{x}(n) + b(n) \tag{7}$$

The definitions of the terms used and their implications are explained in the Appendix at the end of this paper.
The measurement noise $b(n)$ and the change in voltage Δv because of varying loads are included in the Kalman filter using principles of statistics. They can be considered as a white sequence with an exponentially decreasing variance [2]. It is also assumed that there does not exist any cross relationship between the two. This leads us to a covariance matrix having only diagonal terms. Since these are random variables, and they obey Gaussian distribution, it is possible to estimate reasonable values of the elements of the covariance matrices. The diagonal elements of these matrices are the variances of the matrix. The current noise signal on the other hand would appear as a random exponential process plus a white noise sequence with a decreasing variance. The value of the initial variance was found nearly equal to the mean square of the sending end current.

2.3 Recursive Kalman Filter

The recursive Kalman filter block is essentially used for estimating the amplitude of the fundamental signal. It also has Phase and DC offset estimation as added features. The design of the Kalman filter is based on the statistical properties of the signal that is to be processed. The time varying filter coefficients are calculated to minimize the square of the expected errors between the values of the actual and estimated system states. The Kalman gain can be calculated from (8) as:

$$\mathbf{k}(n) = M(n)\mathbf{c}^{'}[\mathbf{c}M(n)\mathbf{c}^{'} + B]^{-1} \tag{8}$$

The state estimation error covariance can be calculated as:

$$Z(n) = I - \mathbf{k}(n)\mathbf{c}M(n) \tag{9}$$

The recursive Kalman filter block invokes two S-functions, *KalmanParameters* and *kfamp*. The *KalmanParameters* function computes the parameters required for the Kalman filter algorithm, and the *kfamp* function estimates the phasor of the input signal, and the DC offset.

3 Modeling DC and harmonic components

In reality, the power system signal is composed of not only the fundamental component but also a decaying dc component, components of harmonic and nonharmonic frequencies, and noise. There are many algorithms to estimate the DC component parameters. Filtering (rejection) of DC component is often performed with application of band-pass or high pass filters of FIR type or mimic filters. Least error squares based in recursive or nonrecursive versions are also used to filter out the DC component that is either taken or not considered as a part of the signal model. Each frequency component can be considered as a phasor comprised of a real and imaginary component, which should be included among the system states. As the number of states increases the Kalman parameters gets augmented accordingly. Decaying dc terms can be modeled as a single state in which the values of the state are reduced to $e^{-\lambda\Delta T}$ times the previous value. Three specific models are introduced in this section.

3.1 Three State Model

To include a dc term decaying according to $e^{-\lambda\Delta T}$, the P and Q matrix should be augmented as in Table 1. where, U_{33} is the variance of the unexpected changes in

$P = Q$	$\begin{bmatrix} cos(\omega\Delta T) & -sin(\omega\Delta T) & 0 \\ sin(\omega\Delta T) & cos(\omega\Delta T) & 0 \\ 0 & 0 & e^{-\lambda\Delta T} \end{bmatrix}$
$\mathbf{c}$	$[1\,0\,1]$
$\Delta I(n)$	$[\Delta I_r(n) \Delta I_i(n) \Delta I_{dc}(n)]$
U	$\begin{bmatrix} U & 0 \\ 0 & U_{33} \end{bmatrix}$
$M(n)$	Initial covariance includes an added term

Table 1: Kalman Parameters for a three-state model

the decaying dc component.

3.2 Four State Model

This section discusses the inclusion of a second frequency in the model. Assume that the pre-fault currents are of the fundamental. The parameters are given in Table 2. U_{33} and U_{44} is the variance of the unexpected changes in the in-phase and quadrature components.

$P = Q$	$\begin{bmatrix} cos(\omega\Delta T) & -sin(\omega\Delta T) & 0 & 0 \\ sin(\omega\Delta T) & cos(\omega\Delta T) & 0 & 0 \\ 0 & 0 & cos(2\omega\Delta T) & -sin(2\omega\Delta T) \\ 0 & 0 & sin(2\omega\Delta T) & cos(2\omega\Delta T) \end{bmatrix}$
$\mathbf{c}$	$[1\,0\,1\,0]$
$\Delta I(n)$	$[\Delta I_{r,1}(n)\ \Delta I_{i,1}(n)\ \Delta I_{r,2}(n)\ \Delta I_{i,2}(n)]$
U	$\begin{bmatrix} U & 0 & 0 \\ 0 & U_{33} & 0 \\ 0 & 0 & U_{44} \end{bmatrix}$
$M(n)$	Initial covariance includes an additional term

Table 2: Kalman Parameters for a four-state model

3.3 Higher State Model

The highest frequency of a signal that can be recovered from a sampled signal should be at least one half of the sampling frequency. The antialiasing filter removes components of higher frequencies substantially lesser than the sampling frequency. The filter model considered in this paper assumes a 11 state model. The 10 states correspond to the in-phase and quadrature components of the fundamental and the higher order frequencies. The last state models the decaying dc component.

3.4 Implementation

The developed system is tested for some typical Kalman parameters and then changes were made to this to check the sensitivity. We assume the input signal to have a fundamental frequency of 60 Hz and the sampling frequency used is around 720 Hz.

3.4.1 State transition and driving function

From Appendix B it is clear the state transition matrix that rotates the phasor by an angle 30^o corresponding to one sample interval is:

$$P = Q = \begin{bmatrix} 0.8660 & -0.5000 \\ 0.5000 & 0.8660 \end{bmatrix} \tag{10}$$

3.4.2 Noise and Voltage Covariance

As explained in Appendix B, the covariance matrix for noise and voltage can be written as:

$$B = \begin{bmatrix} \sigma_n^2 & 0 \\ 0 & \sigma_n^2 \end{bmatrix} \tag{11}$$

where, $\sigma_n = 0.01$p.u.

$$U = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \tag{12}$$

where, $\sigma_v = 0.005$p.u.

3.4.3 Single step transition errors covariance matrix

The matrix $M(n)$ is comprised of the variance of the single step prediction errors. These are the estimated errors from propagating incorrect approximations of the states. The inverse of the initial covariance matrix reflects the degree of knowledge of how close the initial estimate is to the unknown states to be estimated. The matrix is initialized to the covariance matrix of the transitions from the prefault to the postfault states. As the Kalman gain equations are recursively solved, the numerical values of the diagonal terms converge to the steady state values.
Without further knowledge, a reasonable estimate for the ratio of the mean postfault magnitude to the prefault magnitude would be 0.7. As well, the standard deviation of the transition may be assumed to be 0.24 of the prefault value, so the estimate for the covariance matrix $M(1)$ would be:

$$M(1) = \begin{bmatrix} 0.24^2 & 0 \\ 0 & 0.24^2 \end{bmatrix} \tag{13}$$

3.4.4 DC Offset Parameter λ

If the DC offset is negligible, the DC parameter (λ) should have a very low value ($\lambda < 0.1$). A high variance would give the best result for a signal with DC offset. Similar, rules apply for process noise and measurement noise.

4 Test Setup and Results

These simulation tools are used by the manufacturer during development to assess the reliability and soundness of their design to voltage and current fault waveforms obtained after real-time data acquisition or generated using softwares like the EMTP/ATP. Once the development process is successful, the simulation software is converted to the machine language of the relay's microprocessor.
The prototype kalman filter block is shown in Fig. 3. This was then used to test various conditions like fault detection (Fig. 7), and phasor measurement in the presence of dc offsets, harmonics (Fig. 9) and nonstationary disturbances (Fig. 8).

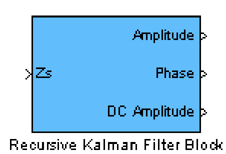


Figure 3: Recursive Kalman Filter Block

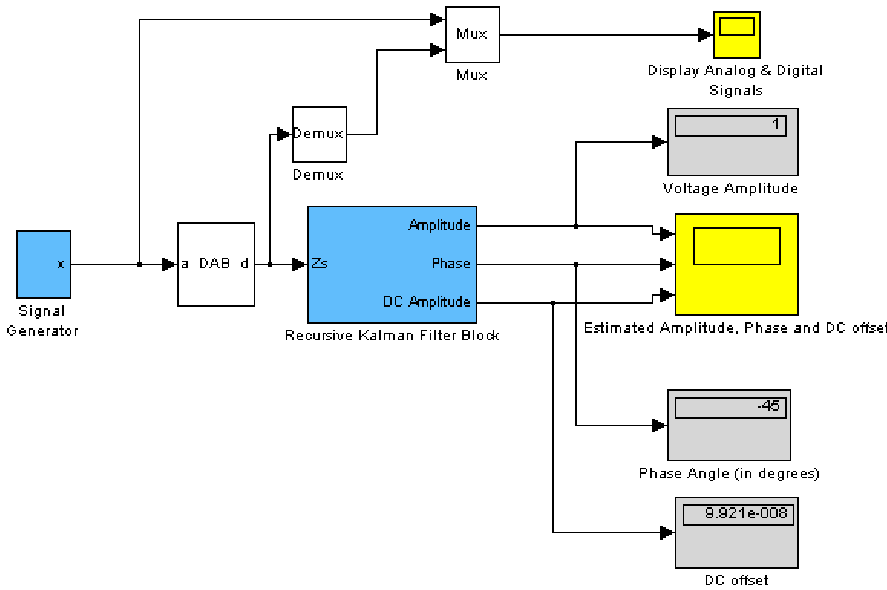


Figure 4: Phasor Measurement

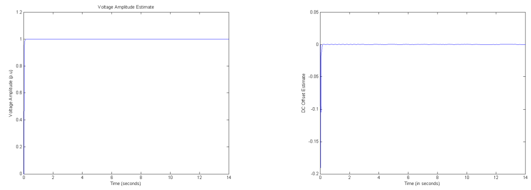
4.1 Kalman Parameters

The parameters that are used for measuring the phasors during single phase fault (Fig. 6) and nonstationary conditions (Fig. 8) are as shown in the Table 3. $\omega =$

P=Q	$\begin{bmatrix} \cos(\omega\Delta T) & -\sin(\omega\Delta T) & 0 \\ \sin(\omega\Delta T) & \cos(\omega\Delta T) & 0 \\ 0 & 0 & e^{-\lambda\Delta T} \end{bmatrix}$
$\mathbf{x}_0$	$[0\,0\,0]^T$
$\mathbf{c}$	$[1\,0\,1]$
σ_n	0.01
σ_v	0.005
M	$\begin{bmatrix} 0.24^2 & 0 & 0 \\ 0 & 0.24^2 & 0 \\ 0 & 0 & 0.24^2 \end{bmatrix}$
λ	1000

Table 3: Parameters for the recursive kalman filter

$2\Pi f$, $\Delta T = f_s$.



(a) (b)

Figure 5: (a) Amplitude Estimate (Error ϵ: 0.00) and (b) DC Offset for the generated signal (Error ϵ: 9.921×10^{-8})

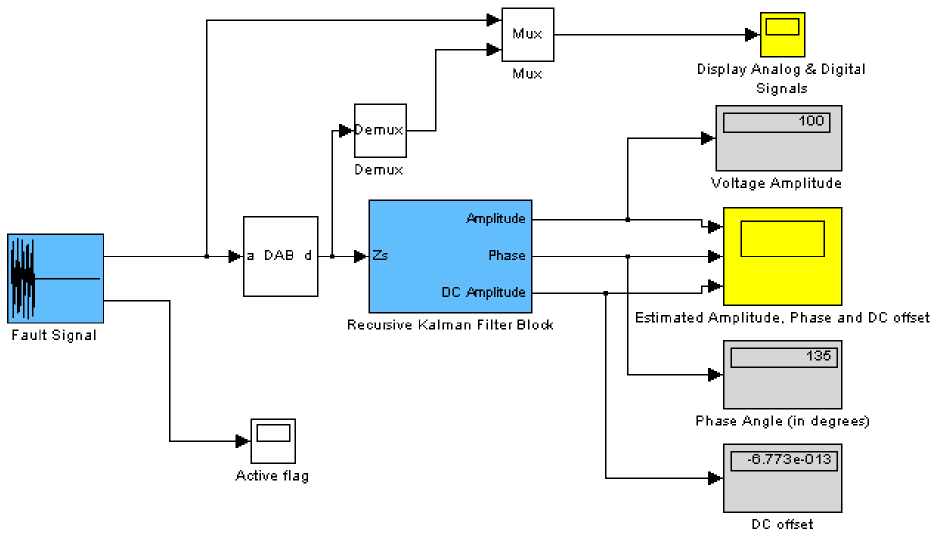
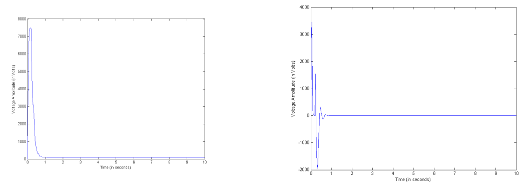


Figure 6: Test Set up for phasor measurements of fault signals

4.2 Sensitivity Studies

The sensitivity test is carried out by varying the various model parameters and noting the effects of these on the estimated value ([4]). In the two-state Kalman filter, the initial covariance, the initial value of the noise variance and the noise variance rate of decrease were changed by more than $\pm 50\%$. The magnitude of the voltage estimate was got using the various parameters and compared to the exact value. In addition, the effect of deleting the third state on the three-state Kalman filter was examined. Finally the sampling rate was changed from 8 samples per cycle to 128 samples per cycle and then examined. The three-state Kalman filter model is not sensitive to $\pm 50\%$ changes in the noise variance parameters, or, $\pm 50\%$ change in the initial covariance matrix. Deleting the third state in the three-state Kalman filter model reduced the rate of convergence and increased the steady state error. In the three-state Kalman filter, changing the sampling rate from 64



(a) (b)

Figure 7: (a) Fault Voltage Amplitude Estimate (Error ϵ: 0.00) and (b) Fault DC Offset Estimate (Error ϵ: -6.773×10^{-13})

to 32 samples per cycle did not change the estimated values noticeably. However a change was noticed at a sampling rate of 8 samples per cycle.

5 Performance Comparison

The performance of the algorithm is compared with the conventional DFT-based algorithm. The inputs to both the blocks were the same, and the outputs were recorded. The sensitivity of the algorithm was studied for:

1. same input
2. changes in sampling rate
3. changes in low-pass filter cut-off frequency

5.1 Same Input

The results that are given in Table 4, compare the Kalman filter output with the Fourier method output. All the three test examples that was explained in the earlier section was used for the purpose.

Measured Parameter(s)	**Actual Value** (in p.u.)	**Prototype output**	**DFT Output**
Fundamental Amplitude (no DC offset)	1.00	1.00	0.77
Fundamental with harmonics	1.00	1.00	0.73
Fundamental with DC offset	1.00	1.00	1.00

Table 4: Performance comparison against the Fourier algorithm

5.2 Changes in Sampling Rate

Any decrease in the sampling rate would reduce the efficiency of the filter in estimating the actual amplitude. This test was carried out for sampling rates that are 8, 16, 32, 64, and 128 times the dominant frequency in the signal. This is listed in Table 5.

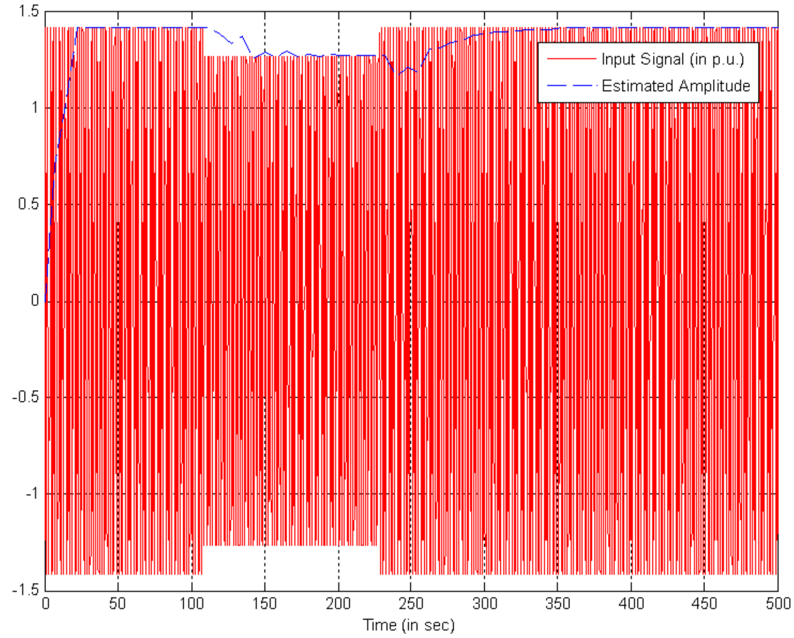


Figure 8: Estimation under nonstationary conditions

Sampling Frequency f_s (samples per second)	Actual Value (in p.u.)	Prototype output	DFT Output
480	1.00	1.00	0.77
960	1.00	0.97	0.94
1920	1.00	1.00	0.77
3840	1.00	1.00	0.23
7680	1.00	1.00	0.21

Table 5: Performance comparison for changes in sampling frequency

5.3 Changes in cut-off frequency

A *Butterworth* low pass filter is used as an antialiasing filter. The sampling frequency (f_s) was kept constant at 960 Hz and the testing was carried out for various cut-off frequencies. The results are as shown in Table 6.

6 Conclusion

A method to estimate the voltage and current phasors from the sampled signal based on Kalman filter was developed. A method for including decaying dc and harmonic components in the filter was described. For a sinusoidal input, a two-state model reaches the correct estimates within two sample periods. However, this model does not provide acceptable results if nonfundamental frequency com-

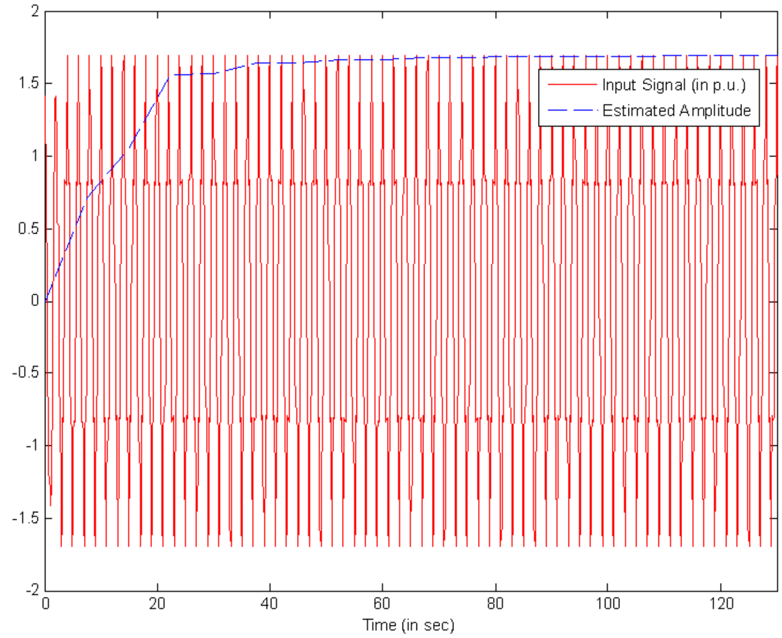


Figure 9: Estimate in the presence of harmonics

Cut-off Frequency $f_c\,(Hz)$	**Actual Value** (in p.u.)	**Prototype output**	**DFT Output**
70	1.00	0.83	0.68
120	1.00	0.97	0.96
240	1.00	0.97	1.00
450	1.00	0.97	0.98

Table 6: Performance comparison for changes in cut-off frequency

ponents are present in the input. The Kalman filter theory requires that the $\Delta\mathbf{v}(n)$ and $\Delta\mathbf{b}(n)$ components are white sequence. For inputs that satisfy the assumption, the Kalman gain will give the ideal result. However, in real systems the noise are band limited and are therefore not perfectly white. However, the Kalman filter can still be applied but the design will not be ideal. To minimize the phasor calculation errors when the system operates at off-nominal frequencies, the sampling should be carried out at multiples of the power system operating frequency. The system should calculate the power system operating frequency and uses this frequency information to obtain the sampling frequency (f_s) as a multiple of the system operating frequency. The remaining error depends only on frequency estimation error, usually lesser than 0.01 Hz. The three state model reaches the correct estimates within 0.42 of a cycle. If the signal is corrupted by some random white noise, the estimate provided by the Kalman filter is more

Error Cause	Error in Degrees	Error in μs
Instrument Transformers	± 0.3	± 14
Phasor Estimation Device	± 0.1	± 5

Table 7: Errors in Phasor Estimation and typical corresponding values

accurate (0.9907 p.u.) than that of DFT based method (0.7702 p.u.). The DC offset can also be measured with high accuracy. Estimation of the postfault voltage and currents was used for fault identification and classification ([7]). The Kalman filter is also suitable for estimating several variables other than voltage or current. Apart from protecting the power system against overcurrent faults, the more recent applications include voltage restorers for compensating against disturbances, power quality, and stability analysis ([5]).

A Appendix A

A.1 State Space variables

The variables used in (5) are defined as:

- $\mathbf{x}(n)$: State vector composed of the real and imaginary components of the signal phasor at the instant $n\Delta T$
- $\mathbf{x}_r(n)$: Real component of the signal phasor.
- $\mathbf{x}_i(n)$: Imaginary component of the signal phasor.
- P: State transition matrix
- Q: Driving function matrix that advances the changes $\Delta\mathbf{x}(n)$ by one sampling interval

The notations used in the output equation 7, the Kalman gain 8 and covariance 9 calculations are:

- $\mathbf{z}_s(n)$: Measured signal at the time instant $n\Delta T$
- C: Defines the relationship between instantaneous sampled voltage and the phasor representation
- $\mathbf{b}(n)$: Measurement noise modeled as a random variable
- B: Covariance of the noise model
- U: The covariance matrix for $\Delta\mathbf{x}(n)$ inputs
- $\mathbf{k}(n)$: Kalman gain vector
- $Z(n)$: State estimation error covariance
- $M(n)$: Single step transition errors covariance matrix
- I: Identity matrix

A.2 Covariance matrix

The covariance matrix D of $[d] = [d_{11} d_{21}]^T$ is defined as:

$$D = \begin{bmatrix} E[d_{11}^2] & E[d_{11} d_{21}] \\ E[d_{21} d_{11}] & E[d_{21}^2] \end{bmatrix} \tag{14}$$

where, E denotes the expectation operator, and if the expected value $E[d_{11} d_{12}] = 0$, then d_{11} and d_{12} are orthogonal to each other (uncorrelated).

B Appendix B

B.1 Signal Characteristics

- Fundamental frequency (f): 60 Hz
- Sampling Frequency (f_s): 720 samples/sec
- Highest Frequency (f_h): < 360 Hz
- Time between samples (ΔT): ≈ 1.389 ms
- Variation of voltage to loads (σ_v): 0.005 p.u.

The voltage variation is statistically considered to be white sequence, uncorrelated, with zero mean and Gaussian distribution.

B.2 Noise Characteristics

We assume a model similar to the voltage variation for the measurement noise. These may be random noise or transducer nonlinearity errors, truncations and other considerations. The standard deviation for the noise is (σ_n) 0.01 p.u.

References

References

[1] R. G. Brown and P. Y. Hwang, *Introduction to Random Signals and Applied Kalman Filtering*, 2nd ed. New York, NY, USA: John Wiley & Sons, 1992.

[2] A. A. Girgis, "A new kalman filtering based digital distance relay," *Power Appar. and Sys., IEEE Trans. on*, vol. PAS 101, no. 9, pp. 3471–3480, Sept. 1982.

[3] R. E. Kalman, "A new Approach to Linear Filtering and Prediction Problems," *Transactions ASME, Journal of basic engineering*, no. 82, pp. 34–45, March 1960.

[4] M. Kezunovic, J. T. Cain, and B. Perunicic, “Digital protective relaying algorithm sensitivity study and solutions,” *Power Delivery, IEEE Trans. on*, vol. 3, no. 3, p. 347113480, July 1988.

[5] Y. Ota, H. Ukai, K. Nakamura, and H. Fujita, “Evaluation of stability and electric power quality in power system by using phasor measurements,” in *Power System Technology, 2000. Proceedings. PowerCon 2000. International Conference on*, vol. 3, Dec. 2000, pp. 1335–1340.

[6] M. S. Sachdev, H. C. Wood, and N. G. Johnson, “Kalman filtering applied to power system measurements for relaying,” *IEEE Power Engineering Review*, vol. PAS-104, pp. 3565–3573, 1985.

[7] M. Sachdev, T. Sidhu, I. Uttamchandani, and W. Kennedy, “Design of phase and amplitude comparators for transmission line protection,” in *WESCANEX 97: Communications, Power and Computing. Conference Proceedings., IEEE*, May 1997.

[8] T. Sidhu, D. Ghotra, and M. Sachdev, “An adaptive distance relay and its performance comparison with a fixed data window distance relay,” *Power Delivery, IEEE Transactions on*, vol. 17, no. 3, pp. 691–697, Jul. 2002.

[9] J. Z. Yang and C. W. Liu, “A new family of measurement technique for tracking voltage phasor,local system frequency, harmonics and dc offset,” in *Power Engineering Society Summer Meeting, 2000. IEEE*, vol. 3, July 2000, pp. 1327–1332.

Praveen Pankajakshan: Department of Electrical Engineering, Texas A&M University,
601 Cross Street, Apt#42, College Station, TX, USA.
E-mail: praveenpankaj@ieee.org