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Cournot Oligopoly: A Discrete Time Sticky-Prices Paradox

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Abstract
This article studies the issue of sticky prices in the context of a dynamic Cournot oligopoly model in discrete time with n asymmetric firms, and with costs and demand linear. We recover the somewhat surprising fact of the related continuous time literature that the asymptotic price is lower than the price of the repeated game. But contrary to the continuous time case, in discrete time we find (1) that the limit at vanishing viscosity coincides with the non-sticky case, and, more surprisingly (2) that the equlibrium price trajectory oscillates around the asymptotic price.

Keywords Sticky price · Cournot oligopoly · Dynamic game · Discrete time

JEL Classification C61 · C72

1 Introduction

Pricing is clearly at the heart of economic analysis. For a long time analyzed from the viewpoint of price theory, which we now call microeconomics [14], the desire and need to better microfound macroeconomic models has also led macroeconomists, at least since Keynes, to question this issue [2, 8]. As [11] and [12] point out, the discussions and oppositions between macroeconomists concerning money, inflation and economic fluctuations are largely centred on the question of price determination by agents.

For some macroeconomists, all nominal prices are perfectly flexible (i.e., all prices correspond to their market equilibrium value), while for others, nominal prices are often sticky (i.e., adjustment is gradual, so there is a difference between the observed price and its theoretical value resulting from market equilibrium). This generally leads the former group to consider that money is neutral in the short term (i.e., there is a dichotomy between the real and monetary spheres), which means that an increase or decrease in the quantity of money

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has no impact on real economic activity; whereas for the latter group, money is not neutral (i.e., money affects the real economy in the short term).

Although these debates continue vigorously on the theoretical level, it is noteworthy that today, most macroeconomic models of the dynamic stochastic general equilibrium (DSGE) type are based upon the assumption that firms change their prices only infrequently (see [5, 16, 18]). And these models form the academically dominant modeling of the *new synthesis* [7] and are used daily, as a complement to traditional macroeconomic modeling, by governments, central banks and international agencies [1].

On the empirical side, beyond older studies (see [9] and [15], for a review of the literature), recent access to numerous and vast microeconomic databases makes it increasingly possible to analyze the issue of price stickiness [10]. By way of illustration, a recent European Central Bank study of eleven eurozone countries [6] concluded that: (1) on average, only 12.3% of prices change each month (8.5% if we exclude sales periods), (2) differences in terms of price rigidity are limited when comparing countries, and are much greater across sectors, (3) the median upward price variation over the period 2000–2019 is 9, 6% and 13% downwards (6.7% and 8.7% respectively if sales periods are excluded), and (4) the distribution of price changes is highly dispersed (14% of price changes are less than 2%, and 10% of price changes are greater than 20%). Generally speaking, it should also be noted that the empirical literature distinguishes between countries with high inflation rates and those with very moderate inflation, the average duration of a price being much shorter in the former than in the latter.

Despite our own limitations in this field, and the fact that it is not our aim here to provide an overview of the issue, we feel that these brief elements are sufficient to pursuade the reader of the theoretical, empirical and political value of studying price stickyness.

The aim of our paper is to contribute to the investigation of the effect of non-continuous price adjustment in a dynamic Cournot oligopoly, with homogeneous good, n heterogeneous firms and discrete time. We consider an affine inverse demand function, whose parameters (a_0 and b_i) remain constant. There are no demand or supply shocks. We modelize viscosity via the following mechanism[1]: in each period t, a part θ of each firm's output (fixed in time and common to all firms) is sold at the price of the previous period $t-1$, while the other part of output $(1-\theta)$ is sold at the price of period t. This formalization of stickyness seems to us to be one of the simplest imaginable, although exogenous and trivial compared to the literature. In fact, it is like considering that each producer has two warehouses of different sizes (or of the same size), and that at each new period the first warehouse will label the products and sell them at the price of the previous period, while the second warehouse will label the products and sell them at the price of the current period. Alternatively, this assumption could represent the joint time required by each producer to price and notify consumers of the new price, since the period t price applies to both the $(1-\theta)$ portion of output in period t and the θ portion of output in period $t+1$. Thus, in our model there is a synchronized price adjustment and all firms have the same price duration.

At least three articles are close to our own in the continuous-time literature where prices evolve continuously. In each case, they analyze the dynamic Cournot–Nash equilibrium, first with open-loop strategies, then with state feedback, the state being the current price. Our paper recovers their common conclusion that the stationary asymptotic price is lower than the Cournot repeated game price but, unlike in ours, in these papers the limit at evanescent stickyness is not the stickiness-free equilibrium. The paper by [4], restricted to the duopoly,

[1] We offer in the development another interpretation of the same equations.

offers a study of the asymptotic regime in a framework where dynamics with first-order viscosity makes the current price the state variable, with a quadratic term in the production cost, necessary to avoid a singularity in the continuous time problem. The article by [3] is also restricted to the duopoly case and the asymptotic regime, but in their framework what leads to a continuous evolution of prices lies in the fact that the players have for (costly) control the speed of variation of their production rate, and not the rate itself. Thus, their production rate becomes a continuously evolving state variable. The inverse affine law of demand thus produces a continuous variation in price. Finally, the article by [17] considers n players, and is interested in the time trajectory—and not just the asymptotic regime—of prices and production. The dynamics and criteria are the same as in the paper by [4], but they offer a very detailed analysis of trajectories, as well as dependencies in the various parameters.

One work at least considers a discrete time model,[2] in [13], pp. 200–204. Its stepwise profit is the same as ours in terms of previous price and current production. But its model of price evolution is the natural discrete-time equivalent of the classical continuous-time model, involving an infinite impulse response, i.e. an infinite memory of past productions, and of the initial price. In constrast, we have chosen a model with one step memory only, which the discrete time lets us do, leading to a significantly simpler mathematical analysis. Whether one model is more realistic than the other one is debatable. Beyond, [13] considers a purely quadratic production cost, while we have a linear one, and is restricted to two identical producers, while our simpler dynamic model allows us to consider n different producers. Finally, its analysis uses a regularity assumption on the equilibrium Markov strategies allowing it to use a variational approach, leading to a deep analysis of the informational nonuniqueness of the equilibrium strategies, while we consider only the pure state feeback time-consistent subgame-perfect strategies, obtained via dynamic programming and the Carathéodory–Isaacs–Bellman sufficient conditions. The somewhat paradoxical results we emphasize below are not investigated in [13].

Our article is organized as follows. In Sect. 2 we state our dynamic Cournot oligopoly problem with sticky price in discrete time, and explain our assumptions. In Sect. 3 we present the complete solution to our problem, and discuss the dynamics, exhibiting the oscillatory nature of the equilibrium solution and the asymptotic regime. In Sect. 4 we propose a numerical analysis to compare the sticky price case with the Cournot repeated game. Section 5 analyzes three special cases: the absence of stickiness, monopoly, and Cournot oligopoly when the number n of producers goes to infinity. Section 6 concludes.

2 The Problem

2.1 Cournot Dynamic Oligopoly with Sticky Prices

We consider a typical Cournot n firms oligopoly with an affine inverse demand function. Let n be the number of producers, producer i's production be q_i, the inverse demand function be characterized by a price a_0 and coefficients b_i giving a price P:

$$P = a_0 - \sum_{i=1}^{n} b_i q_i$$

Each producer i has a linear production cost $c_i q_i$.

[2] We thank an anonymous reviewer for mentionning it.

The producers will make an infinite sequence of production decisions $q_i(t)$, $t \in \mathbb{N}$. But the specificity of this market is that a proportion θ of their production $q_i(t)$ will be sold at the previous price $P(t-1)$, while the rest, a proportion $(1-\theta)$ will be sold at the clearing price $P(t)$ given by the inverse demand function. Therefore, given the appropriate discount factor ρ, player i's profit Π_i will be:

$$\Pi_i = \sum_{t=1}^{\infty} \rho^{t-1} \left[\theta P(t-1) + (1-\theta) \left(a_0 - \sum_{k=1}^{n} b_k q_k(t) \right) - c_i \right] q_i(t) \,. \tag{1}$$

We seek a Cournot–Nash (dynamic) equilibrium.

The following hypotheses hold on the parameters of the problem:

$$a_0 > 0, \qquad \forall i,\ c_i < (1-\theta) a_0$$

so that the a_i defined thereafter, are positive. And as in any Cournot model with an affine inverse demand function, we assume that the b_i are "sufficiently small" so that realistic productions q_i keep $P \geq 0$.

Furthermore, we will restrict our analysis to the case $\theta \leq 1/2$. Two reasons lead to this restriction:

1. On the one hand, we have a slightly different interpretation of the same mathematical problem: if the production q_i is made at a constant rate q_i over the time interval of length 1 between t and $t+1$, and the price evolves linearly from $P(t)$ to $P(t+1)$ during that period, reaching $P(t+1)$ at a time $t+\tau < t+1$ and stays there until the end of the period, i.e. time $t+1$, then we have the same profit as expressed by Eq. (1) with $\theta = \tau/2$, as in our discrete time problem. Therefore, in this equivalent continuous-time model, $\theta \leq 1/2$.
2. On the other hand, and more importantly, if θ is too large, the problem may have no solution. To understand this, let us consider the monopoly problem ($n = 1$) with $\theta = 1$. Then the monopolist may produce a large quantity Q every odd numbered periods (say, years), yielding on even numbered periods a negative[3] price $P(t-1)$ which applies for that period when it produces zero, and hence a price a_0 on odd periods. Clearly, its profit will be $\Pi = (a_0 - c)Q/(1-\rho^2)$, hence arbitrarily large. The monopoly problem has therefore no solution in that case. Similar strategies are possible for the n producer model.

Notice also that our formulas will only hold if $\rho\theta^2/(1-\theta)^2 < 1$, which is ensured (and beyond) by the restriction $\theta \leq 1/2$.

2.2 Notation and Preliminary Analysis

We will use the following notation:

$$b_i q_i = r_i, \qquad \delta := \frac{\theta}{1-\theta}, \qquad a_i := a_0 - \frac{c_i}{1-\theta}.$$

Notice that $(1-\theta)(1+\delta) = 1$, so that, e.g., $a_i = a_0 - (1+\delta)c_i$. The parameter $\delta \in [0, 1]$ is an alternative measure of the stickyness, convenient in the calculations, if difficult to interpret in economic terms.

[3] Or null if we agree that $P = \max\{0, a_0 - \sum_i b_i q_i\}$.

To stress the fact that $P(t-1)$ is the state of the problem at time t, we let $P(t-1) = x(t)$. We may then notice that:

$$\frac{b_i}{1-\theta}\Pi_i = \sum_{t=1}^{\infty} \rho^{t-1} \left(\delta x(t) + a_i - \sum_{k=1}^{n} r_k(t) \right) r_i(t) \,. \tag{2}$$

and the state dynamics are very simple:

$$x(t+1) = a_0 - \sum_{k=1}^{n} r_k(t) \,. \tag{3}$$

We have a dynamic game problem with affine dynamics and quadratic payoff. No surprise that we will find a quadratic Isaacs Value function $V_i(x)$. We will let

$$\rho W_i(x) := \rho \frac{b_i}{1-\theta} V_i(x) = \alpha x^2 + \beta_i x + \gamma_i \,. \tag{4}$$

The fact that α be independent of i will result from the fact that we will succeed in finding such Value functions that satisfy Isaacs' equation.

Further notation used will be:

$$A = \sum_{k=1}^{n} a_k, \qquad D = \frac{1}{1-\theta}\sum_{k=1}^{n} c_k, \quad \text{and therefore } A = na_0 - D,$$

$$\Delta = n + 1 - 2n\alpha, \qquad R = \sum_{k=1}^{n} r_k^\star, \qquad \eta_i = a_i - \beta_i, \qquad H = \sum_{k=1}^{n} \eta_k.$$

3 Complete Solution

As stated above, we seek an equilibrium with payoffs as in (2) and Value functions as in (4). Isaacs' equation reads as follows:

$$\begin{aligned} W_i(x) = \frac{\alpha}{\rho}x^2 + \frac{\beta_i}{\rho}x + \frac{\gamma_i}{\rho} = \max_{r_i} \Bigg\{ \left(\delta x + a_i - \sum_{k=1}^{n} r_k \right) r_i \\ + \alpha \left(a_0 - \sum_{k=1}^{n} r_k \right)^2 + \beta_i \left(a_0 - \sum_{k=1}^{n} r_k \right) + \gamma_i \Bigg\} \,. \end{aligned} \tag{5}$$

Our mathematical analysis hereafter ignores the constraints $q_i \geq 0$ and $P \geq 0$. We take them for granted, and will only accept the (candidate) solution exhibited if it satisfies them, which will be left to numerical verification. Accepting these constraints, it is clear that W above is bounded over the positive real line.

The right hand side of (5) is a concave function of r_i. Differentiating and equating to zero, we obtain the equilibrium production $r_i^\star$ as:

$$r_i^\star = \delta x - (1-2\alpha)R - 2\alpha a_0 + \eta_i \,.$$

(This expression is still implicit, since R contains $r_i^\star$.) Summing over the i yields, after an elementary calculation:

$$R = \frac{1}{\Delta}[n(\delta x - 2\alpha a_0) + H] \,, \tag{6}$$

and

$$r_i^\star = \frac{1}{\Delta}[\delta x - 2\alpha a_0 - (1-2\alpha)H] + \eta_i\,. \tag{7}$$

There remains to place this back into equation (5) and identify like powers of x.

3.1 Investigation of α and the Closed-Loop Dynamics

3.1.1 Determination of α

Begining with terms in x^2, we find:

$$\frac{1}{\rho}\alpha = \frac{\delta^2}{\Delta^2}(n^2\alpha - 2n\alpha + 1)\,. \tag{8}$$

We write this equation as:

$$\text{if } \delta = 0, \quad \alpha = 0, \qquad \text{if } \delta \neq 0, \quad \frac{1}{\rho\delta^2}\alpha = f_n(\alpha)$$

with

$$f_n(\alpha) = \frac{n^2\alpha - 2n\alpha + 1}{(n+1-2n\alpha)^2}.$$

We observe that $1/\rho\delta^2 > 1$, and furthermore that:

$$f_n'(\alpha) = n\frac{2n(n-2)\alpha + n^2 - n + 2}{(n+1-2n\alpha)^3}\,,$$
$$f"_n(\alpha) = 8n^2\frac{n(n-2)\alpha + n^2 - n + 1}{(n+1-2n\alpha)^4}\,,$$

so that

$$f_n(0) = \frac{1}{(n+1)^2}, \qquad f_n\left(\frac{1}{2n}\right) = \frac{1}{2n}, \qquad f_n'\left(\frac{1}{2n}\right) = 1,$$

while

$$\forall\alpha \in \left(0, \frac{1}{2n} + \frac{1}{2}\right), \qquad f_n'(\alpha) > 0, \qquad f_n''(\alpha) > 0.$$

Therefore, in an (y, z) plane, α may be identified as the abscissa of the intersection point of the line $z = (1/\rho\delta^2)y$ and the curve $z = f_n(y)$. See Fig. 1. We know that the slope of the line is larger than one, while the curve $z = f_n(y)$ is convex, tangent to the first diagonal at $y = 1/2n$. There exists therefore one intersection for $y < 1/(2n)$, which is the limit of the recursion $y(t)/(\rho\delta^2) = f_n(y(t+1))$ as $t \to -\infty$ starting from $y = 0$, i.e. the solution that we seek. (Fig. 1 easily illustrates the two solutions for $y < (n+1)/(2n)$, a third solution is on the decreasing branch of the graph of $f_n(\cdot)$ at $y > (n+1)/2n$.)

A consequence of this graphical representation is that α increases from zero to $1/2n$ as $\rho\delta^2$ increases from zero to one, and that for a given $\rho\delta^2$, α decreases when n increases. Actually, we can even show that $2n\alpha$ goes to zero as n goes to infinity. (See appendix)

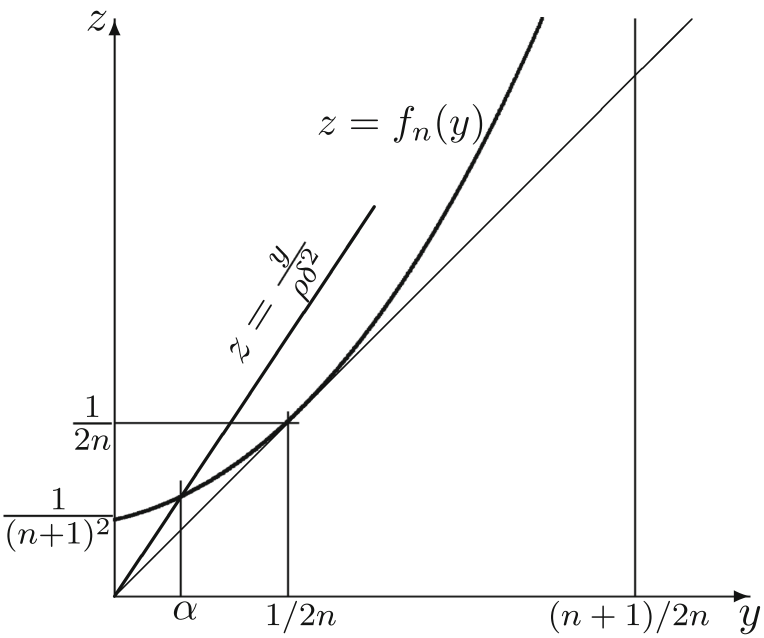

Fig. 1 Determination of α (drawing for $n = 2$, $\rho\delta^2 = 2/3$)

3.1.2 Qualitative Behavior of the Dynamics

It follows from Eqs. (3) and (6) that the price dynamics under the Cournot–Nash equilibrium strategies are:

$$x(t+1) = -\frac{n\delta}{\Delta}x(t) + \frac{(n+1)a_0 - H}{\Delta}\,. \tag{9}$$

It follows from the fact that $\alpha < 1/2n$ that $\Delta > n$. Therefore, $n\delta/\Delta < 1$. Hence, for almost all initial conditions, these dynamics oscillate around a long time, asymptotic value $\bar{x}$, which is also the asymptotic price $\bar{P}$:

$$\bar{P} = \bar{x} = \frac{(n+1)a_0 - H}{\Delta + n\delta} \tag{10}$$

that we will characterize further later on, when we have calculated H.

Two non-intuitive consequences result from this analysis.

- On the one hand, we insist that this oscillating behaviour is *not* the result of a trial-and-error process *à la* Cournot iteration. The actual Cournot–Nash equilibrium strategy yields an oscillation. Our analysis of the extreme case $\theta = 1$ gives an indication of why this may be so.
- On the other hand, although once the prices and productions have reached constant values the stickiness seems to play no role, yet these long term repeated values are *not* the repetition of the equilibrum values in a game with no stickiness, i.e. $x = (a_0 + \sum_k c_k)/(n+1)$.

It may be noticed that, as shown below in Sect. 5.2, these somewhat paradoxical facts hold even in the simple case of a one-player game, i.e. a monopoly.

3.2 Investigation of β_i and γ_i

3.2.1 Coefficients β_i and Asymptotic Price

It is usefull, for more legibility, to introduce yet another short hand notation:

$$\Gamma = n^2 - 2n\alpha + 1 = \Delta - n(n-1).$$

Identifying terms in x in Eq. (4) with the r_i as in (7) yields:

$$\frac{\beta_i}{\rho} = \frac{\delta}{\Delta^2(1+\delta\rho)}\left\{-2\alpha\Gamma a_0 + 2\Delta(1-n\alpha)a_i - 2[2n\alpha^2 - (2n+1)\alpha + 1]H\right\}. \tag{11}$$

Summing in i, recalling that $H = A - \sum_i \beta_i$, expanding Δ^2 where it appears without the coefficient $\delta\rho$, and regrouping terms, we obtain:

$$\left(\frac{\Delta^2}{\delta\rho} + \Gamma\right)\sum_i \beta_i = -2n\alpha a_0(n^2 - 2n\alpha + 1) + 2(n^2\alpha - 2n\alpha + 1)A.$$

Using again $H = A - \sum_i \beta_i$, and recognizing a term Δ which appears in the coefficient of A, it follows that

$$H = \frac{[\Delta + \delta\rho(n-1)]\Delta A + 2\delta\rho\Gamma n\alpha a_0}{\Delta^2 + \delta\rho\Gamma}. \tag{12}$$

Here, the right hand side contains only data of the problem and α that we know how to compute, at least numerically. This is an explicit, although unappealing, formula. It can be placed back into Eq. (11) to get β_i and hence $\eta_i = a_i - \beta_i$ and place this in Eq. (7) to get the equilibrium strategies. The formulas thus obtained are exceedingly complex and of little interest. If one wants numerical values, the best is to compute H and β_i numerically from their respective formulas above.

Asymptotic equilibrium price At this point, we claim that we are able to compute α from formula (8), and we have an explicit formula for H (in terms of α). Therefore, we may compute $\bar{x}$ with formula (10). We obtain:

$$\bar{P} = \bar{x} = \frac{\Delta^2(1+\delta\rho)a_0 + [\Delta + (n-1)\delta\rho]\Delta D}{\Delta^2 + \delta\rho\Gamma}.$$

See Sect. 3.2 for some numerival values.

The dynamics may be written

$$x(t+1) - \bar{x} = -\frac{n\delta}{\Delta}(x(t) - \bar{x}).$$

3.2.2 Coefficient γ_i and Equilibrium Profits

To compute equilibrium profits, we still need to evaluate the coefficients γ_i. This is obtained by equating terms without x in Eq. (4) with the r_i as in (7). An explicit expression can be found, but again of little help. As expected, γ_i goes to infinity as ρ approaches one.

$$\begin{aligned}\frac{\gamma_i}{\rho} =& \frac{1}{\Delta^2(1-\rho)}\Big\{-(2n\alpha a_0 + \Delta a_i - H)[2\alpha a_0 - \Delta(a_i - \beta_i) + (1-2\alpha)H] \\ &+ \alpha[(n+1)a_0 - H]^2 + \Delta\beta_i[(n+1)a_0 - H]\Big\}.\end{aligned}$$

Table 1 $\bar{P}$ and Π as a function of n and θ for $a_0 = 10$, $c_i = 1$, and $\rho = .95$

	θ	n 1	2	3	4	5	10	100	∞
$\bar{P}$	.5	5.443	3.297	2.536	2.151	1.920	1.457	1.045	1
$\bar{P}$	.4	5.454	3.567	2.772	2.346	2.083	1.544	1.054	1
$\bar{P}$	.3	5.467	3.716	2.924	2.484	2.205	1.619	1.063	1
$\bar{P}$	.2	5.477	3.829	3.049	2.602	2.313	1.689	1.072	1
$\bar{P}$	.1	5.489	3.921	3.156	2.706	2.411	1.755	1.080	1
P_C	0	5.5	4	3.25	2.8	2.5	1.818	1.089	1
Π_i	.5	810.13	308.96	153.35	90.618	59.613	15.657	0.1618	0
Π_i	.4	675.07	275.74	142.59	86.009	57.250	15.355	0.16131	0
Π_i	.3	578.60	244.04	129.81	79.746	53.741	14.828	0.16088	0
Π_i	.2	506.26	218.29	118.74	74.100	50.498	14.317	0.16	0
Π_i	.1	450.00	197.32	109.31	69.144	47.593	13.836	0.15942	0
Π_C	0	405.00	180.00	101.25	64.8	45	13.388	0.15881	0

It finishes to prove that indeed, a Value function of the form (4) can be found that satisfies Isaacs'equation, and therefore that the strategies (7) form a set of a dynamical Cournot–Nash equilibrium strategies.

Starting from a market price P_0, set $x(1) = P_0$, the dynamics (9) gives the sequence of prices under the equilibrium strategies. The total discounted profit of each player is then given by:

$$\Pi_i = \frac{1-\theta}{\rho b_i}(\alpha P_0^2 + \beta_i P_0 + \gamma_i)\,. \tag{13}$$

The dependence of the final price and profits on θ (or δ) is difficult to assert from these formulas. Table 1 of numerical values shows that stickyness decreases the price and increases the producers' profits. This is coherent with the rest of the literature on sticky prices.

4 Some Numerical Results

We propose here some numerical values aiming to show the effect of viscosity and compare with the repeated Cournot game. In Table 1, we show on the top part the asymptotic price $\bar{P}$ and on the bottom part the profit Π, for decreasing values of the stickiness, down to the Cournot price P_C and profit Π_C corresponding to the non sticky market. Since the profit Π_i in the sticky case depends on the initial price at time zero, we take it as the Cournot price. The values in the table are for $a_0 = 10$, $c_i = 1$, and $\rho = .95$.

We give in Fig. 2 three price trajectories with the same parameters a_0 and c_i, $n = 3$, and different values of ρ and θ.

We also show in Table 2 a nonintuitive phenomenon at very small discount rate. While we expect that the higher the discount rate $1-\rho$, the more difference we have with the non sticky case, this is not quite so for δ and ρ sufficiently close to one, as Table 2 shows. We show the asymptotic price $\bar{P}$. We have set $a_0 = 10$ and $c_i = 1$ as in Fig. 1, and $n = 3$. We have labeled the columns with ρ and the lines with θ:

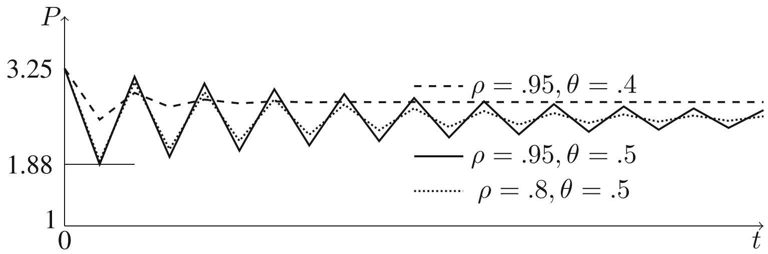

Fig. 2 Three price trajectories starting from the Cournot price for $n = 3$, $a_0 = 10$, $c_i = 1$, and different ρ and θ

Table 2 $\bar{x}$ for values of ρ and δ close to one with $n = 3$

	ρ				
θ	.8	.85	.9	.95	.99
.48	2.5878	2.5948	2.6002	2.6042	2.6060
.485	2.5764	2.5825	2.5871	2.5898	2.5903
.49	2.5644	2.5997	2.5731	2.5742	2.5725
.495	2.5519	2.5562	2.5581	2.5567	2.5513
.5	2.5389	2.5417	2.5415	2.5358	2.5196

5 Particular Cases

5.1 No Stickyness

With no stickyness, we have $\theta = \delta = 0$, and consequently, according to our formulas

$$\alpha = 0, \quad \Delta = n + 1, \quad \beta_i = 0, \quad H = A, \quad \eta_i = a_i = a_0 - c_i.$$

We recover the formulas of the classical Cournot-Nash equilibrium with affine inverse demand function. We write them using the shorthand notation $\sum_{k=1}^{n} c_k = C$ and therefore $A = na_0 - C$, as:

$$P = \frac{a_0 + C}{n+1}, \qquad r_i^\star = \frac{a_0 + C}{n+1} - c_i \qquad b_i \Pi_i = \frac{1}{1-\rho}\left(\frac{a_0 + C}{n+1} - c_i\right)^2.$$

In the case where all the production costs coefficients c_i are equal, this yields

$$\Pi_i = \frac{1}{b_i(1-\rho)}\left(\frac{a_i}{n+1}\right)^2.$$

5.2 Monopoly

We now deal with the case $n = 1$. In that case, α can be calculated exactly. We have $\Delta = 2(1 - \alpha)$, and Eq. (8) becomes

$$4\alpha(1-\alpha) - \rho\delta^2 = 0.$$

We remember that $\rho\delta^2$ is less than one, and let $\varepsilon = \sqrt{1 - \rho\delta^2}$.

Then, the smallest root of the above equation yields:

$$\alpha = \frac{1}{2}(1-\varepsilon), \qquad \Delta = 2(1-\alpha) = 1+\varepsilon, \qquad \beta = \rho\delta\frac{a_1-(1-\varepsilon)a_0}{\rho\delta+1+\varepsilon}.$$

The equilibrium dynamics are now

$$x(t+1) = -\frac{\delta}{1+\varepsilon}x(t) + \frac{(1+\rho\delta)a_0+(1+\delta)c_1}{\rho\delta+1+\varepsilon}.$$

This oscillates around the long term, asymptotic equilibrium

$$\bar{x} = \frac{(1+\varepsilon)}{(1+\varepsilon+\delta)(1+\varepsilon+\rho\delta)}[(1+\rho\delta)a_0+(1+\delta)c_1].$$

And finally, the monopoly profit is given by:

$$\Pi = \frac{1}{1-\rho}\frac{4a_0[\alpha(1+\delta)c_1+\beta]+(a_1-\beta)^2}{4b(1-\alpha)}.$$

Numerical values are given in Table 1 in the column $n = 1$.

5.3 Large Number of Producers

We may investigate what these formulas say as n goes to infinity. To make things simple, we concentrate on the symmetric case where $c_i = c$ for all i, hence $A = n(a_0 - (1+\delta)c)$, and also $b_i = b$. We have already noticed that $n\alpha \to 0$, therefore $\Delta \sim n$. It follows that

$$H \sim A = n(a_0 - (1+\delta)c), \text{ hence } n\beta \to 0.$$

Therefore, $R \sim \delta x + a$ and, given that the price x remains bounded, as in the standard Cournot case, all productions go to zero. It is also a simple matter to check that $n\gamma \to 0$. Hence the producers' profits vanish as well as the cumulative profit of all of them. And finally, the behavior of the asymptotic price is also as in the repeated Cournot case:

$$\bar{P} = \bar{x} \sim \frac{a_0 + n(1+\delta)c}{(1+\delta)n} \to c.$$

6 Conclusion

Our paper presents a simple case of sticky price in a dynamic discrete-time Cournot oligopoly. In this framework, we find the well-known result in the literature that the long-run price is lower than the Cournot repeated game price. On the other hand, in comparison with the continuous case, we establish two new results: (1) with zero stickiness, the long-run price coincides with the Cournot repeated game price, and (2) when there is stickiness, the equilibrium solution has an oscillating character. We also show that with a variation in the number of producers (n) or in the share of production sold at the previous price (θ) over time, the optimal solution starts to oscillate again.

It seems clear to us that the discounting of future profits plays an important role in this oscillating character of price trajectories, favoring an increase in profit at time t at the cost of a decrease at time $t+1$. Numerical simulations confirm that the discount rate also plays a role in the gap between the asymptotic price and the Cournot price, a gap that increases with discounting, i.e. as ρ decreases. However, we consider it an open problem that this monotonic growth is no longer true when ρ is very close to 1 and θ close to $1/2$.

A Investigation of $2n\alpha$

We prove the following theorem:

Theorem 1 *For a fixed positive $\rho\delta^2$ smaller than one, there exists a unique solution α less than $1/2n$ of Eq. (8), and $2n\alpha$ goes to zero as n goes to infinity.*

For the sake of clarity, let $\omega := 2n\alpha$, and we call $\omega^\star$ the solution sought. Equation (8) may be re-written as

$$\frac{1}{\rho\delta^2}\omega^\star = n\frac{(n-2)\omega^\star + 2}{(n+1-\omega^\star)^2} = g_n(\omega^\star).$$

Expectedly, we have $g_n(1) = 1$ and $g_n'(1) = 1$. We also have $g_n(0) = 2n/(n+1)^2$. Furthermore, as a simple calculation shows, for all n and all $\omega \le 1$, $g_n''(\omega) > 0$. As a consequence, $g_n(\cdot)$ is a convex function. Hence,

$$\forall \omega \in [0,1], \quad g_n(\omega) \le g_n(0) + \omega(g_n(1) - g_n(0)).$$

Thus, $g_n(\cdot)$ being strictly convex,

$$\forall \omega \in (0,1), \quad g_n(\omega) < g_n(0) + \omega(1 - g_n(0)) = G_n(\omega).$$

Let

$$\omega_1 = \frac{\rho\delta^2 g_n(0)}{1 - \rho\delta^2 + g_n(0)}.$$

so that $G_n(\omega_1) = \omega_1/\rho\delta^2$. Now, observe that $\omega_1 < 1$ so that

$$g_n(\omega_1) < G_n(\omega_1) = \frac{\omega_1}{\rho\delta^2}.$$

The continuous function $g_n(\omega) - \omega/\rho\delta^2$ is positive for $\omega = 0$ and negative for $\omega = \omega_1$. It follows from the intermediate value theorem that it vanishes at some $\omega = \omega^\star$ between 0 and ω_1 (and only once because it is convex). We have therefore established that $0 < \omega^\star < \omega_1$, which goes to zero with $g_n(0)$ as n goes to infinity.

The intuition for this proof is pictured in the following graphic, which is an enlargment, with a magnification factor $2n$, of the lower part of Fig. 1.

B Linear Plus Quadratic Production Costs

The question arises[4] as to whether the somewhat nonintuitive results remain true if the production costs include a quadratic term. We would then have

$$\Pi_i = \sum_{t=1}^{\infty} \rho^{t-1} \left[\theta P(t-1) + (1-\theta)\left(a_0 - \sum_{k=1}^{n} b_k q_k(t)\right) - c_i - \frac{d_i}{2} q_i(t)\right] q_i(t).$$

A general case with arbitrary coefficients d_i is much more complex to solve than our calculations above. But things simplify in the (unrealistic) case where the d_i/b_i are the same for all producers. This is the case if all producers are identical.

[4] It was asked by an anonymous reviewer.

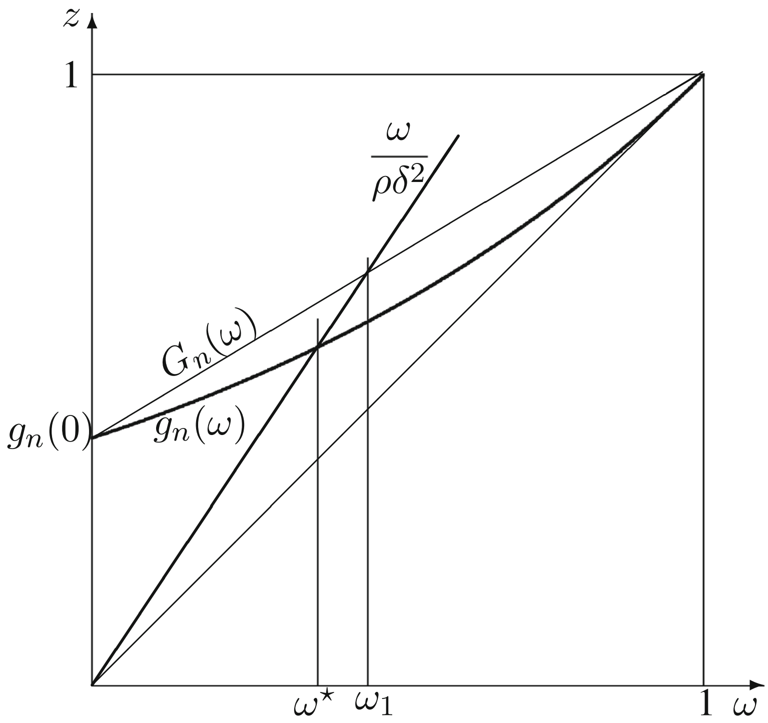

Fig. 3 Investigation of the behavior of $n\alpha$ as $n \to \infty$

Let then $d_i/b_i = f$. Essentially the same calculations as above hold provided that we let

$$\Delta = 1 + f + n - 2n\alpha, \qquad \eta_i = \frac{a_i - \beta_i}{1+f}, \qquad H = \sum_k \eta_k.$$

We still get formula (7) for $r_i^\star$, with

$$R = \frac{n(\delta x - 2\alpha a_0) + (1+f)H}{\Delta}$$

The price dynamics are now

$$x(t+1) = -\frac{n\delta}{\Delta}x(t) + \frac{(n+1+f)a_0 - H}{\Delta}.$$

Finally, we can see that α is solution of equation

$$\frac{1}{\rho\delta^2}\alpha = f_n(\alpha)$$

with now

$$f_n(\alpha) = \frac{n^2\alpha - 2n\alpha + 1 + f}{(n+1+f-2n\alpha)^2}.$$

It still holds that

$$f_n'(\alpha) = n\frac{2n(n-2)\alpha + n^2 - n + 2 + f}{(n+1+f-2n\alpha)^2} > 0,$$

and

$$f_n(0) > 0, \qquad f_n\left(\frac{1}{2n}\right) = \frac{1}{2(n+f)} < \frac{1}{2n}.$$

Therefore, the same analysis as before yields $\alpha < 1/(2n)$, thus $\Delta > n + f$. Hence we see that the price trajectory generated by the equilibrium strategies oscillates around the limit price

$$\bar{P} = \bar{x} = \frac{(n+1+f)a_0 - H}{\Delta + n\delta}.$$

The detailed calculations to get H are somewhat more complicated.

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Declarations

Conflict of interest We declare no conflict of interest.

Ethical Approval Not applicable.

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