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H^∞-Optimal Control and Related Minimax Design Problems

A Dynamic Game Approach

Second Edition

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Preface

One of the major concentrated activities of the past decade in control theory has been the development of the so-called "H^∞-optimal control theory," which addresses the issue of worst-case controller design for linear plants subject to unknown additive disturbances and plant uncertainties, including problems of *disturbance attenuation*, *model matching*, and *tracking*. The mathematical symbol "H^∞" stands for the *Hardy space* of all complex-valued functions of a complex variable, which are analytic and bounded in the open right-half complex plane. For a linear (continuous-time, time-invariant) plant, the H^∞ norm of the transfer matrix is the maximum of its largest singular value over all frequencies.

Controller design problems where the H^∞ norm plays an important role were initially formulated by *George Zames* in the early 1980's, in the context of sensitivity reduction in linear plants, with the design problem posed as a mathematical optimization problem using an (H^∞) operator norm. Thus formulated originally in the frequency domain, the main tools used during the early phases of research on this class of problems have been operator and approximation theory, spectral factorization, and (Youla) parametrization, leading initially to rather complicated (high-dimensional) optimal or near-optimal (under the H^∞ norm) controllers. Follow-up work in the mid 1980's have shown, however, that the maximum *McMillan degree* of these controllers is in fact in the order of the *McMillan degree* of the overall system transfer matrix, and further work has shown that in a time-domain characterization of these controllers (generalized) Riccati equations of the type that arises in linear-quadratic differential games play a key role. These findings have prompted further accelerated research on the topic, for more direct time-domain (state-space) derivation of these results — a direction which has also led to more general formulations, including time-varying plants and finite design horizons.

Among different time-domain approaches to this class of worst-case design problems, the one that uses the framework of *dynamic* (*differential*) *game theory* seems to be the most natural. This is so because the original H^∞-optimal control problem (in its equivalent time-domain formulation) is in fact a *minimax* optimization problem, and hence a *zero-sum game*, where the controller can be viewed as the *minimizing player* and disturbance as the *maximizing player*. Using this framework, we present in this book a complete theory that encompasses continuous-time as well as discrete-time systems, finite as well as infinite horizons, and several different measurement schemes, including closed-loop perfect state, delayed perfect state, sampled state, closed-loop imperfect state, delayed imperfect state and sampled imperfect state information patterns. We also discuss extensions of the linear theory to nonlinear systems, and derivation of lower dimensional controllers for systems with regularly and singularly perturbed dynamics.

This is the second edition of our 1991 book with the same title, which, besides featuring a more streamlined presentation of the results included in the first edition, and at places under more refined conditions, also contains substantial new material, reflecting new developments in the field since 1991. Among these are the nonlinear theory (Section 4.6; Chapters 5 and 6); connections between H^∞-optimal control and risk sensitive stochastic control problems (Section 4.7); H^∞ filtering for linear and nonlinear systems (Section 7.4); and robustness considerations in the presence of regular and singular perturbations (Chapter 8). We have also included a rather detailed description of the relationship between frequency- and time-domain approaches to robust controller design (Section 1.3), and a complete set of results on the existence of value and characterization of optimal policies in infinite-horizon LQ differential games (Section 9.2), in addition to the complete set of results for finite-horizon LQ differential games (involving the study of existence and nonexistence of conjugate points) (Section 9.1).

As stated in the preface of the first edition, the theory is now at a stage where it can easily be incorporated into a second-level graduate course in a control curriculum, that would follow a basic course in linear control theory covering LQ and LQG designs. The framework adopted in this book makes such an ambitious plan possible, and indeed, both authors have taught such courses during the last couple of years, at the *University of Illinois, Urbana-*

Champaign; *Ecole Superieure des Sciences de l'Informatique, University of Nice*; *Institut Superieur d'Informatique et d'Automatique, Ecole des Mines de Paris, Sophia Antipolis*; and *Ecole Polytechnique, Paris.*

For the most part, the only prerequisite for the book is a basic knowledge of linear control theory, at the level of, say, the texts by *Kwakernaak and Sivan (1972)* or *Anderson and Moore (1990)*. No background in differential games, or game theory in general, is required, as the requisite concepts and results have been developed in the book at the appropriate level. The book is written in such a way that makes it possible to follow the theory for the continuous- and discrete-time systems independently, so that a reader who is interested only in discrete-time systems, for example, may skip Chapters 4 and 5 without any disruption in the development of the theory. On the other hand, for the benefit of the reader who is interested in both continuous- and discrete-time results, we have included statements in each relevant chapter, that place the comparisons in proper perspective.

In addition to re-expressing our gratitute to the individuals and institutions mentioned in the preface of the first edition of the book, we thank here also to many of our colleagues who have provided us with many valuable comments and suggestions since the publication of the 1991 volume. The first author would also like to acknowledge the Senior Scientist Fellowship he received from the French Government during his 1994-95 sabbatical stay at INRIA, Sophia Antipolis, which facilitated the collaboration of the two authors in the writing of this second edition; the hospitality he received from INRIA during his stay, and the excellent working environment he found there contributed immensely toward the completion of this project. The collaborative research of the two authors on the topic of this book was conducted under an NSF-INRIA international program, which is also gratefully acknowledged.

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