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T. S. Başar, P. Bernhard (Eds.)

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PREFACE

This volume contains fifteen articles on the topic of *differential and dynamic games*, focusing on both theory and applications. All but one of these are based on presentations made at the *Third International Symposium on Differential Games and Applications* held at *INRIA*, Sophia-Antipolis, France, on June 16-17, 1988. The meeting was sponsored by *INRIA*, and involved the participation of *IFAC* and *the IEEE Control Systems Society*.

The first two articles in the volume deal with discrete two-person constant-sum dynamic games, that is games where the action spaces of the players are finite and decisions are made at discrete instants of time. The first paper, which comprises Chapter 1, describes a parlor game named after Emanuel Lasker; it discusses the challenges involved in obtaining a solution for the general case, and computes the value function for a special case of the problem. The second chapter deals with a 'search game' with delayed information for the 'searcher', and obtains expressions for the saddle-point solution in Markovian-type strategies when the discrete positions are either on the line or on the plane, and the 'searcher' is not allowed to use memory. The paper also discusses the implications of allowing the 'searcher' to use memory strategies, and of introducing different types of 'noise' into the perfect (but delayed) measurements of the 'searcher'.

The next six chapters deal with different aspects of zero-sum differential games of the pursuit-evasion type, the class of games introduced and pioneered by Rufus Isaacs in the 1950's. The first of these (chapter 3) outlines a rigorous approach towards formulating and analyzing such games, which involves a discrete-time approximation to the original continuous-time game. The strategy choices by the two players are allowed to depend on the history of the game, and the solution to the original game is captured by letting the length of the small time intervals on which the sequence of discrete-time games is defined go to zero. The next chapter studies the Isaacs partial differential equation that arises in these games, and provides a characterization of its classical and semicontinuous solutions. In this analysis, the author uses contingent epiderivatives, in place of the classical derivatives, which allows him to characterize the solutions which are indicators of closed subsets (corresponding to some desirable behavior), and are only semicontinuous. Chapter 5 also deals with the Isaacs equation, and shows that the capture time of a pursuit-evasion game is the unique viscosity solution of that equation under some singular boundary conditions, and provided that a 'capturability' condition is satisfied. The topic of Chapter 6 is the well-known 'Lion and Man' pursuit-evasion game, the formulation here giving speed advantage to the Lion (the pursuer) by a factor of 1.2. The author shows that the solution to the game features a 'focal surface' of 'radial' positions leading to curved motion by both players until termination, and an 'indifference zone' for Man. Chapter 7 introduces a new approach for the problem of feedback guidance during the pre-launch maneuvering phase of medium range missile combat, which exploits any possible nonoptimal play by the opponent, and is related to the ideas of threat reciprocity, reprisal strategies and a bi-criteria formulation of pursuit-evasion games. Feedback guidance law determination is also the subject of Chapter 8, which obtains a closed-form solution for a coplanar pursuit-evasion game between a coasting missile and a maneuvering target of constant speed, leading to an optimal strategy for the missile as a function of the current and final values of the state variables. The

implementable feedback guidance law constructed from this strategy is shown to perform better than proportional navigation, and only slightly worse than the optimal game strategy.

The next two chapters are devoted to stochastic games. The class of stochastic differential games treated in Chapter 9 are piecewise deterministic, with the stochasticity being due to random jump process which determines the times and the nature of switch from one system (or game) to another. In this context, the author studies the existence and characterization of Nash equilibria under feedback and piecewise open-loop strategies. Chapter 10, on the other hand, deals with discrete discounted-cost zero-sum stochastic games (finite state and action spaces), and presents numerical results on the computation of stationary equilibrium strategies, using a recently proposed modified Newton method.

Chapter 11 deals with an application of the nonzero-sum discrete-time dynamic game theory on a model arising in the European gas market, and shows that under some conditions the open-loop and feedback Nash equilibria coincide. Chapter 12 involves a dynamic game formulation where the players are allowed to switch between cooperative and noncooperative (Nash) modes of play, with cooperation occurring if all players benefit from it (as compared with the Nash solution). In the context of linear-quadratic games, it is shown that the evolution of the game depends very much on the type of Pareto-optimal solution adopted for the cooperative mode of play. The next chapter is on an application of differential game theory in predator-prey systems. Using a Lotka-Volterra formulation, the author studies the scenario where the two players, while seeking conflicting growth objectives, cooperate in designing a self-controlled growth policy in order to reach and maintain a common population size level. Chapter 14 presents an extension of the Nash bargaining scheme to discrete-time and continuous-time dynamic games, which uses a two-level decision structure. At the lower level, a weighted sum of the players' objectives is maximized, while at the upper level the bargaining contract is determined, that satisfies the required rationality axioms.

One topic which is considered to belong to the general area of differential games, and which is not treated in the chapters above, is 'stochastic teams', and especially those with decentralized information patterns. The last chapter of this volume provides a selective annotated bibliography of the literature on this topic, covering the past twenty-five years.

Even though the volume is not exhaustive in the choice of topics within the field of differential games and its applications, it does cover a variety of areas and presents recent developments on topics of current interest. Therefore, it should be useful to researchers in differential and dynamic games, systems and control, operations research and mathematical economics. We thank the authors for their contributions to this volume, and to all participants at the *Third International Symposium on Differential Games and Applications* for their active involvement in the discussions during and after the paper presentations.

Our thanks also go to *INRIA*, for both the financial and moral support without which the Differential Games Meeting where these papers were presented could not have been organized. Last, but not least, our sincere thanks go to Ms. Catherine Juncker and Ms. Jacqueline Tchobanian for their invaluable assistance during both the organizational and the execution phases of the Symposium.

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