Reproduction dynamics in a differential game between foraging predators and hiding preys

Andrei Akhmetzhanov
Institute for Problems in Mechanics
of the Russian Academy of Sciences
Vernadsky Ave. 101-1
119526 Moscow, Russia
e-mail: akhmetzhanov@gmail.com

Pierre Bernhard
Polytech'Nice Sophia Antipolis
Route des Colles 930, BP145
06903 Sophia Antipolis Cedex, France
e-mail: bernhard@polytech.unice.fr

Frédéric Grognard
INRIA, Project COMORE,
Route de Lucioles 2004, BP 93
06902 Sophia Antipolis Cedex, France
e-mail: Frederic.Grognard@sophia.inria.fr

Ludovic Mailleret
INRA, UR880
Route des Chappes 400, BP 167
06903 Sophia Antipolis Cedex, France
e-mail: Ludovic.Mailleret@sophia.inra.fr

Abstract

In this work we investigate seasonal prey-predator models, where the evolution of the system during a season of fixed length is governed by optimal game dynamics with two players. On the one hand, the predator has the choice between foraging the food (eating preys) or reproducing for the next year (laying eggs at a rate proportional to its energy). On the other hand, the prey has a chance to hide from it but in this case it has a negative mortality rate and its population can decrease faster than if it would be foraged by the predator. The preys lay eggs at a constant rate whether they are hiding or eating. The aim of both is to maximize their population (the number of offsprings) for the next season.

Mathematically, this model can be described by the language of non-zero differential games with the following dynamics

$$\begin{aligned}\dot{p} &= -p + nuv,\\ \dot{n} &= -\mu(1-v)n - nzuv\end{aligned}$$

where $\mu > 0$ is natural mortality rate and the predator population is described by two parameters: p, the average weight or the energy of one

population, and z, the size of the population. The size of the prey population is denoted through n. We assume that z remains constant during the season and the prey may only prevent death by eating. The control parameters are defined as follows: $u = 0$ means that predators are laying eggs and $u = 1$ that they are foraging the food; $v = 0$ means that preys are hiding from the predators and $v = 1$ that they are eating.

The aim of the control has been taken in the form

$$J_1 = \int_0^T p(1-u)\, dt \to \max_u, \quad J_2 = \int_0^T n\, dt \to \max_v$$

which represent the total amount of eggs laid during one season by the predators and preys respectively.

For the solution of this game we consider the system of two Hamilton-Jacobi-Bellman equations and use the method of characteristics. After that, we investigate the question of possible existence of the bi-singular region where both players apply mixed strategies, see [1]. In the end we consider the long-term evolution of the system over seasons.

A similar semi-discrete approach has been used in [2], but no optimization of the trajectory has been done. The game-theoretic approach has been recently applied in [3].

This research is supported by Lutins&Co COLOR Project (INRIA, Sophia Antipolis) and RFBR Grant No. 07-01-00418.

References

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