Dynamic Coverage of Mobile Sensor Networks

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Abstract

In this paper we study the dynamic aspects of the coverage of a mobile sensor network resulting from continuous movement of sensors. As sensors move around, initially uncovered locations are likely to be covered at a later time. A larger area is covered as time continues, and intruders that might never be detected in a stationary sensor network can now be detected by moving sensors. However, this improvement in coverage is achieved at the cost that a location is covered only part of the time, alternating between covered and not covered. We characterize area coverage at specific time instants and during time intervals, as well as the time durations that a location is covered and uncovered. We further consider the time it takes to detect a randomly located intruder and prove that the detection time is exponentially distributed with parameter $2\lambda r\bar{v_s}$ where λ represents the sensor density, r represents the sensor's sensing range, and $\bar{v_s}$ denotes the average sensor speed. Our results show that sensor mobility brings about unique dynamic coverage properties not present in a stationary sensor network, and that mobility can be exploited to compensate for the lack of sensors to improve coverage. For mobile intruders, we take a game theoretic approach and derive optimal mobility strategies for both sensors and intruders. We prove that the optimal sensor strategy is to choose their directions uniformly at random between $[0, 2\pi)$. The optimal intruder strategy is to remain stationary to maximize its detection time. This solution represents a mixed strategy which is a Nash equilibrium of the zero-sum game between mobile sensors and intruders.

Index Terms

Wireless sensor networks, coverage, detection time, mobility.

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This work is a substantial extension of a previous ACM Mobihoc conference paper. A revision statement is attached at the end of the manuscript.

I. Introduction

Coverage is a critical issue for the deployment and performance of a wireless sensor network, representing the quality of surveillance that the network can provide, for example, how well a region of interest is monitored by sensors, and how effectively a sensor network can detect intruders. It is important to understand how the coverage of a sensor network depends on various network parameters in order to better design and use sensor networks in different application scenarios.

In many applications, sensors are not mobile and remain stationary after their initial deployment. The coverage of such a stationary sensor network is determined by the initial network configuration. Once the deployment strategy and sensing characteristics of the sensors are known, network coverage can be computed and remains unchanged over time.

Recently, there has been increasing interest on building mobile sensor networks. Potential applications abound. Sensors can be mounted on mobile platforms such as mobile robots and move to desired areas [1], [2], [3], [4]. Such mobile sensor networks are extremely valuable in situations where traditional deployment mechanisms fail or are not suitable, for example, a hostile environment where sensors cannot be manually deployed or air-dropped. Mobile sensor networks can also play a vital role in homeland security. Sensors can be mounted on vehicles (e.g., subway trains, taxis, police cars, fire trucks, boats, etc) or carried by people (e.g., policemen, fire fighters, etc). These sensors will move with their carriers, dynamically patrolling and monitoring the environment (e.g., chemical, biological, or radiological agents). In other application scenarios such as atmosphere and under-water environment monitoring, airborne or under-water sensors may move with the surrounding air or water currents. The coverage of a mobile sensor network now depends not only on the initial network configurations, but also on the mobility behavior of the sensors.

While the coverage of a sensor network with stationary sensors has been extensively explored and is relatively well understood, researchers have only recently started to study the coverage of mobile sensor networks. Most of this work focuses on algorithms to relocate sensors in desired positions in order to repair or enhance network coverage [5], [6], [7], [8], [9], [10], [11]. More specifically, these proposed algorithms strive to spread sensors to desired locations to improve coverage. The main differences among these works are how exactly the desired positions of sensors are computed. Although the algorithms can adapt to changing environments and recompute the sensor locations accordingly, sensor mobility is exploited essentially to obtain a new stationary configuration that improves coverage after the sensors move to their desired locations.

In this paper, we study the coverage of a mobile sensor network from a different perspective. Instead of trying to achieve an improved stationary network configuration as the end result of sensor movement, we are interested in the dynamic aspects of network coverage resulting from the continuous movement of sensors. In a stationary sensor network, the covered areas are determined by the initial configuration and do not change over time. In a mobile sensor network, previously uncovered areas become covered as sensors move through them and covered areas become uncovered as sensors move away. As a result, the areas covered by sensors change over time, and more areas will be covered at least once as time continues. The coverage status of a location also changes with time, alternating between being covered and not being covered. In this work, we assume that sensors are initially randomly and uniformly deployed and move independently in randomly chosen directions. Based on this model, we characterize the fraction of area covered at a given time instant, the fraction of area ever covered during a time interval, as well as the time durations that a location is covered and not covered.

Intrusion detection is an important task in many sensor network applications. We measure the intrusion detection capability of a mobile sensor network by the detection time of a randomly located intruder, which is defined to be the time elapsed before the intruder is first detected by a sensor. In a stationary sensor network, an initially undetected intruder will never be detected if it remains stationary or moves along an uncovered path. In a mobile sensor network, however, such an intruder may be detected as the mobile sensors patrol the field. This can significantly improve the intrusion detection capability of a sensor network. In this paper, we characterize the detection time of a randomly located intruder. The results suggest that sensor mobility can be exploited to effectively reduce the detection time of a stationary intruder when the number of sensors is limited. We further present a lower bound on the distribution of the detection time of a randomly located intruder, and show that it can be minimized if sensors move in straight lines.

In some applications, for example, radiation, chemical, and biological agents detections, there is a sensing time requirement before an intruder is detected. We find in this case that too much mobility can be harmful if the sensor speed is above a threshold. Intuitively, if a sensor moves faster, it will cover an area more quickly and detect some intruders sooner, however, at the same time, it will miss some intruders due to the sensing time requirement. To this end, we find there is an optimal sensor speed that minimizes the detection time of a randomly located intruder.

For a mobile intruder, the detection time depends on the mobility strategies of both sensors and intruder. We take a game theoretic approach and study the optimal mobility strategies of sensors and intruder. Given the sensor mobility pattern, we assume that an intruder can choose its mobility strategy so as to maximize its detection time (its lifetime before being detected). On the other hand, sensors choose a mobility strategy that minimizes the maximum detection time resulting from the intruder's mobility strategy. This can be viewed as a zero-sum minimax game between the collection of mobile sensors and the intruder. We prove that the optimal sensor mobility strategy is for sensors to choose their directions uniformly at random between $[0, 2\pi)$. The corresponding intruder mobility strategy is to remain stationary to maximize its detection time. This solution represents a mixed strategy which is a Nash equilibrium of the game between mobile sensors and intruders. If sensors choose to move in any fixed direction (a pure strategy), it can be exploited by an intruder by moving in the same direction as sensors to maximize its detection time. The optimal sensor strategy is to choose a mixture of available pure strategies (move in a fixed direction between $[0, 2\pi)$). The proportion of the mix should be such that the intruder cannot exploit the choice by pursuing any particular pure strategy (move in the same direction as sensors), resulting in a uniformly random distribution for sensor's movement. When sensors and intruders follow their respective optimal strategies, neither side can achieve better performance by deviating from this behavior.

The remainder of the paper is structured as follows. In Section II, we review related work on the coverage of sensor networks. The network model and coverage measures are defined in Section III. In Section IV, we derive the fraction of the area being covered at specific time instants and during a time interval. The detection time for both stationary and mobile intruders are studied in Section V and Section VI, respectively. In Section VI, we also derive the the optimal mobility strategies for sensors and intruders from a game theoretic perspective. Finally, we summarize the paper in Section VII.

II. Related Work

Recently, sensor deployment and coverage related topics have become an active research area. In this section, we present a brief overview of the previous work on the coverage of both stationary and mobile sensor networks that is most relevant to our study. A more thorough survey of the sensor network coverage problems can be found in [12].

Many previous studies have focused on characterizing various coverage measures for stationary sensor networks. In [13], the authors considered a grid-based sensor network and derived the conditions for the sensing range and failure rate of sensors to ensure that an area is fully covered. In [14], the authors proposed several algorithms to find paths that are most or least likely to be detected by sensors in a sensor network. Path exposure of moving objects in sensor networks was formally defined and studied in [15], where the authors proposed an algorithm to find minimum exposure paths, along which the probability of a moving object being detected is minimized. The path exposure problem is further explored in [16], [17], [18]. In [19], [20], [21], the k-coverage problem where each point is covered by at least k sensors was investigated. In [22], the authors defined and derived several important coverage measures for a large-scale stationary sensor network, namely, area coverage, detection coverage, and node coverage, under a Boolean sensing model and a general sensing model. Other coverage measures have also been studied. In [23], [24], the authors studied a metric of quality of surveillance which is defined to be the average distance that an intruder can move before being detected, and proposed a virtual patrol model for surveillance operations in sensor networks. In [25], the authors studied a novel sensor self-deployment problem and introduced an F-coverage evaluation metric, coverage radius, which reflects the need to maximize the distance from F to uncovered areas. The relationship between area coverage and network connectivity is investigated in [26], [27], [28].

While the coverage of stationary sensor networks has been extensively studied and relatively well understood, researchers have started to explore the coverage of mobile sensor networks only recently. In [5], [8], [29], virtual-force based algorithms are used to repel nodes from each other and obstacles to maximize coverage area. In [9], algorithms are proposed to identify existing coverage holes in the network and compute the desired target positions where sensors should move in order to increase the coverage. In [30], a distributed control and coordination algorithm is proposed to compute the optimal sensor deployment for a class of utility functions which encode optimal coverage and sensing policies. In [31], mobility is used for sensor density control such that the resultant sensor density follows the spatial variation of a scalar field in the environment. In [32], the authors considered the carrier-based sensor placement problem and proposed a novel localized algorithm in which mobile robots carry static sensors and drop them at visited empty vertices of a virtual grid for full coverage. In [33], an autonomous planning process is developed to compute the deployment positions of sensors and leader waypoints for navigationally-challenged sensor nodes. In [34], the authors investigated the problem of self-deploying a network of mobile sensors with simultaneous consideration to fault-tolerance (bi-connectivity), coverage, diameter, and quantity of movement required to complete the deployment. In [35], the authors formulated the distance-sensitive service discovery problem for wireless sensor and actor networks, and proposed a novel localized algorithm (iMesh) that guarantees nearby (closest) service selection with a very high probability. The deployment of wireless sensor networks under mobility constraints and the tradeoff between mobility and sensor density for coverage are studied in [36], [37].

Many of these proposed algorithms strive to spread sensors to desired positions in order to obtain a stationary configuration such that the coverage is optimized. The main difference is how the desired sensor positions are computed. In this work we study the coverage of a mobile sensor network from a very different perspective. Instead of trying to achieve an improved stationary network configuration as an end result of sensor movement, we focus on the dynamic coverage properties resulting from the continuous movement of the sensors.

Intrusion detection problem in mobile sensor networks has been considered in a few recent studies, e.g., [38], [39], [40], [41], [42], [43]. In our work we take a stochastic geometry based approach to derive closed-form expressions for the detection time under different network, mobility, and sensing models. In [44], Chin et. al. proposed and studied a similar game theoretic problem formulation for a different network and mobility model.

III. Network and Mobility Models

In this section, we describe the network and mobility model, and introduce three coverage measures for a mobile sensor network used in this study.

A. Sensing Model

We assume that each sensor has a sensing radius, r. A sensor can only sense the environment and detect intruders within its sensing area, which is the disk of radius r centered at the sensor. A point is said to be *covered* by a sensor if it is located in the sensing area of the sensor. The sensor network is thus partitioned into two regions, the covered region, which is the region covered by at least one sensor, and the uncovered region, which is the complement of the covered region. An intruder is said to be *detected* if it lies within the covered region.

In reality, the sensing area of a sensor is usually not of disk shape due to hardware and environment factors. Nevertheless, the disk model can be used to approximate the real sensing area and provide bounds for the real case. For example, the irregular sensing area of a sensor can be lower and upper bounded by its maximum inscribed and minimum circumscribed circles, respectively.

B. Location and Mobility Model

We consider a sensor network consisting of a large number of sensors placed in a 2-dimensional infinite plane. This is used to model a large two-dimensional geographical region. For the initial configuration, we assume that, at time $t = 0$, the locations of these sensors are uniformly and independently distributed in the region. Such a random initial deployment is desirable in scenarios where prior knowledge of the region of interest is not available; it can also result from certain deployment strategies. Under this assumption, the sensor locations can be modeled by a stationary two-dimensional Poisson point process. Denote the density of the underlying Poisson point process as λ. The number of sensors located in a region R, $N(R)$, follows a Poisson distribution with parameter $\lambda\|R\|$, where $\|R\|$ represents the area of the region.

Since each sensor covers a disk of radius r, the initial configuration of the sensor network can be described by a Poisson Boolean model $B(\lambda, r)$. In a stationary sensor network, sensors do not move after being deployed and network coverage remains the same as that of the initial configuration. In a mobile sensor network, depending on the mobile platform and application scenario, sensors can choose from a wide variety of mobility strategies, from passive movement to highly coordinated and complicated motion. For example, sensors deployed in the air or water may move passively according to external forces such as air or water currents; simple robots may have a limited set of mobility patterns, and advanced robots can navigate in more complicated fashions; sensors mounted on vehicles and people move with their carriers, which may move randomly and independently or perform highly coordinated search.

In this work, we consider the following sensor mobility model. Sensors follow arbitrary random curves independently of each other without coordination among themselves. In some cases, when it helps to yield closed-form results and provide insights, we will make the model more specific by limiting sensor movement to straight lines. In this model, the movement of a sensor is characterized by its speed and direction. A sensor randomly chooses a direction $\Theta \in [0, 2\pi)$ according to some distribution with a probability density function of $f_{\Theta}(\theta)$. The speed of the sensor, V_s, is randomly chosen from a finite range $[0, v_s^{\max}]$, according to a distribution density function of $f_{V_s}(v)$. The sensor speed and direction are independently chosen from their respective distributions.

The above models make simplified assumptions for real network scenarios. Our purpose is to obtain analytical results based on the simplified assumptions and provide insight and guideline to the deployment and performance of mobile sensor networks. The Poisson distribution and unit disk model have been widely used in the studies of wireless networks (e.g., coverage and capacity problems) to obtain analytical results. The Poisson spatial distribution is a good approximation for large networks where nodes are randomly and uniformly distributed. For the mobility model, we consider the scenarios where nodes move independently of each other. For example, sensors can be carried by people or mounted on people's vehicles, boats, or animals, etc. These carriers are likely to move independently according to their own activity patterns without much coordination. This is similar to the *uncoordinated mobility model* used in [39]. Note that in some scenarios (e.g., sensors mounted on robots) mobile sensors can communicate with each other and coordinate their moves. In that case the sensors can optimize their movement patterns and provide more efficient coverage than the independent mobility case. In this paper we will focus on the independent mobility model.

Throughout the rest of this paper, we will refer to the initial sensor network configuration as *random sensor network* $B(\lambda, r)$, the first mobility model where sensors move in arbitrary curves as *random mobility model*, and the more specific mobility model where all sensors move in straight lines as *straight-line mobility model*. The shorthand $X \sim \exp(\mu)$ stands for $\mathbb{P}(X < x) =$

$1-\exp(-\mu x)$, i.e., random variable X is exponentially distributed with parameter μ.

C. Coverage measures

To study the dynamic coverage properties of a mobile sensor network, we define the following three coverage measures.

Definition 1: **Area coverage**: The area coverage of a sensor network at time t, $f_a(t)$, is the probability that a given point $x \in \mathbb{R}^2$ is covered by one or more sensors at time t.

Definition 2: **Time interval area coverage**: The area coverage of a sensor network during time interval $[s,t)$ with $s < t$, $f_i(s,t)$, is the probability that given a point $x \in \mathbb{R}^2$, there exists $u \in [s,t)$ such that x is covered by at least one sensor at time u.

Definition 3: **Detection time**: Suppose that an intruder has a trajectory $x(t)$ and that $x(0)$ is uncovered at time $t = 0$. The detection time of the intruder is the smallest $t > 0$ such that $x(t)$ is covered by at least one sensor at time t.

All three coverage measures depend not only on static properties of the sensor network (initial sensor distribution, sensor density and sensing range), but also on sensor movements. The characterization of area coverage at specific time instants is important for applications that require parts of the whole network be covered at any given time instant. The time interval area coverage is relevant for applications that do not require or cannot afford simultaneous coverage of all locations at specific time instants, but prefer to cover the network within some time interval. The detection time is important for intrusion detection applications, measuring how quickly a sensor network can detect a randomly located intruder.

IV. Area Coverage

In this section, we study and compare the area coverages of both stationary and mobile sensor networks. We first analytically characterize the area coverage. We then discuss the implications of our results on network planning and show that sensor mobility can be exploited to compensate for the lack of sensors to increase the area being covered during a time interval. However, we point out, due to the sensor mobility, a point is only covered part of the time; we further characterize this effect by determining the fraction of time that a point is covered. Finally, we discuss the optimal moving strategies that maximize the area coverage during a time interval.

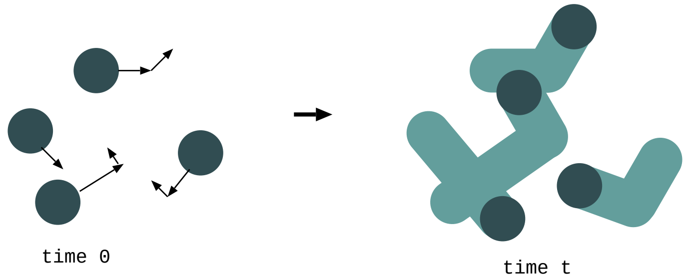

Fig. 1. Coverage of mobile sensor network: the left figure depicts the initial network configuration at time 0 and the right figure illustrates the effect of sensor mobility during time interval $[0,t)$. The solid disks constitutes the area being covered at the given time instant, and the union of the shaded region and the solid disks represents the area being covered during the time interval.

In a stationary sensor network, a location always remains either covered or not covered. The area coverage does not change over time. The effect of sensor mobility on area coverage is illustrated in Figure 1. The union of the solid disks constitutes the area coverage at given time instants. As sensors move around, exact locations that are covered at different time instants change over time. The area that has been covered during time interval $[0,t)$ is depicted as the union of the shaded region and the solid disks. As can be observed, more area is covered during the time interval than the initial covered area. The following theorem characterizes the effect of sensor mobility on area coverage.

Theorem 1: **Consider a sensor network $B(\lambda, r)$ at time $t = 0$, with sensors moving according to the random mobility model.**

1) **At any time instant t, the fraction of area being covered is**

$$f_a(t) = 1 - e^{-\lambda\pi r^2}, \ \ \forall t \geq 0. \tag{1}$$

2) **The fraction of area that has been covered at least once during time interval $[s,t)$ is**

$$f_i(s,t) = 1 - e^{-\lambda\mathbb{E}(\alpha(s,t))}. \tag{2}$$

where $\mathbb{E}(\alpha(s,t))$ is the expected area covered by a sensor during time interval $[s,t)$. When all sensors move in straight lines, we have

$$f_i(s,t) = 1 - e^{-\lambda(\pi r^2 + 2r\bar{v}_s(t-s))}. \tag{3}$$

where $\bar{v}_s$ is the average sensor speed.

3) **The fraction of the time a point is covered is**

$$f_t = 1 - e^{-\lambda\pi r^2}. \tag{4}$$

Proof. Given the initial node placement and the random mobility model, at any time instant t, the locations of the sensors still form a two dimensional Poisson point process of the same density [45, Theorem 9.14]. Therefore, according to [46, Section 3.1.1], the fraction of the area covered at time t remains the same as in the initial configuration, $f_a(t) = 1 - e^{-\lambda\pi r^2}$. More generally, denote the expected area covered by a sensor during time a time interval $[s,t)$ as $\mathbb{E}(\alpha(s,t))$. According to [46, Section 3.1.1], the fraction of area that has been covered at least once is $f_i(s,t) = 1 - e^{-\lambda\mathbb{E}(\alpha(s,t))}$. In particular, when all sensors move in straight lines, each sensor covers a shape of a racetrack whose expected area is $\mathbb{E}(\alpha(s,t)) = \pi r^2 + 2r\bar{v}_s(t-s)$, where $\bar{v}_s$ is the average sensor speed. Thus, we have $f_i(s,t) = 1 - e^{-\lambda(\pi r^2 + 2r\bar{v}_s(t-s))}$.

While an uncovered location will be covered when a sensor moves within distance r of the location, a covered location becomes uncovered as sensors covering it move away. As a result, a location is only covered part of the time. More specifically, a location alternates between being covered and not being covered, which can be modeled as an alternating renewal process. We use the fraction of time that a location is covered to measure this effect. The fraction of time that a location is covered equals the probability that it is covered at any given time instant, $f_t = 1 - e^{-\lambda\pi r^2}$. □

At any specific time instant, the fraction of the area being covered by the mobile sensor network described above is the same as in a stationary sensor network. This is because at any time instant, the positions of the sensors still form a Poisson point process with the same parameters as in the initial configuration. However, unlike in a stationary sensor network, covered locations change over time; areas initially not covered will be covered as sensors move around. Consequently, intruders in the initially uncovered areas can be detected by the moving sensors.

When sensors all move in straight lines, the fraction of the area that has ever been covered increases and approaches one as time proceeds. Later in this section we will prove that, among all possible curves, straight line movement is an optimal strategy that maximizes the area being covered during a time interval. The rate at which the covered area increases over time depends on the expected sensor speed. The faster sensors move, the more quickly the deployed region is covered. Therefore, sensor mobility can be exploited to compensate for the lack of sensors to improve the area coverage over an interval of time. This is useful for applications that do not require or cannot afford simultaneous coverage of all locations at any given time, but need to cover a region within a given time interval. Note that the area coverage during a time interval does not depend on the distribution of sensors movement direction. Based on (3), we can compute the expected sensor speed required to have a certain fraction of the area (f_0) covered within a time interval of length t_0.

$$\overline{v_s} = -\frac{\lambda \pi r^2 + \log(1 - f_0)}{2\lambda r t_0}, \text{ for } f_0 \geq 1 - e^{\lambda \pi r^2}.$$

However, the benefit of a greater area being covered at least once during a time interval comes with a price. In a stationary sensor network, a location is either always covered or not covered, as determined by its initial configuration. In a mobile sensor network, as a result of sensor mobility, a location is only covered part of the time, alternating between covered and not covered. The fraction of time that a location is covered corresponds to the probability that it is covered, as shown in (4). Note that this probability is determined by the static properties of the network configuration (density and sensing range of the sensors), and does not depend on sensor mobility. In the next section, we will further characterize the duration of the time intervals that a location is covered and uncovered.

From the proof of Theorem 1, it is easy to see that area coverage during a time interval is maximized when sensors move in straight lines. This is because, among all possible curves, the area covered by a sensor during time interval $[s,t)$, $\alpha(s,t)$, is maximized when the sensor moves in a straight line. Based on (2), we have the following theorem.

Theorem 2: **In a sensor network $B(\lambda, r)$ with sensors moving according to the random mobility model, the fraction of area covered during any time interval $[s,t)$ is maximized when sensors all move in straight lines.**

It is important to point out that straight line movement is not the only optimal strategy that maximizes the area coverage during a time interval. There is a family of optimal movement patterns that maximize the coverage. We conjecture that the optimal movement patterns have the following properties: 1) the local radius of curvature is greater than the sensing range r

everywhere along the oriented trajectory; 2) if the euclidean distance between two points of the curve is less than $2r$, then the distance between them along the curve is less than πr. When these two properties are satisfied, the sensing disk of a sensor does not overlap with its previously covered areas, and a point will not be covered redundantly by the same sensor. The covering efficiency is thus maximized.

V. Detection Time of Stationary Intruder

The time it takes to detect an intruder is of great importance in many military and security-related applications. In this section, we study the detection time of a randomly located stationary intruder. Detection time for a mobile intruder is investigated in the next section. To facilitate the analysis and illustrate the effect of sensor mobility on detection time, we consider the scenario where all sensors move at a constant speed v_s. More general sensor speed distribution scenarios can be approximated using the results of this analysis.

We assume that intruders do not initially fall into the coverage area of any sensor, and an intruder will be immediately detected when it falls into the sensing range of mobile sensors. Obviously, these intruders will never be detected in a stationary sensor network. In a mobile sensor network, however, an intruder can be detected by sensors passing within a distance r of it, where r is the common sensing range of the sensors. The detection time characterizes how quickly the mobile sensors can detect a randomly located intruder previously not detected. We will first derive the detection time when sensors all move in straight lines. We will then consider the case when sensors move according to arbitrary curves.

Theorem 3: **Consider a sensor network $B(\lambda, r)$ with sensors moving according to the straight-line random mobility model and a static intruder. The sequence of times at which new sensors detect the intruder forms a Poisson process of intensity $2\lambda r\bar{v}_s$, where $\bar{v}_s$ denotes the average sensor speed. As a consequence, the time before the first detection of the intruder is exponentially distributed with the same parameter.**

Proof: We denote by $A(s,t)$ the random region covered by a sensor in the interval $[s,t]$, that was not covered before time s. The shape of this region is illustrated in Figure 2.

We first prove that the number of sensors hitting the intruder in the time interval $[s,t]$ is Poisson distributed with parameter $2\lambda r\bar{v}_s(t-s)$. Suppose without loss of generality that the intruder is

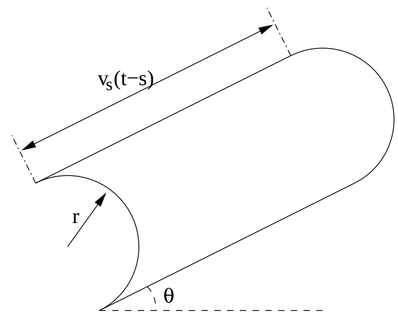

Fig. 2. The region $A(s,t)$ under the straight-line mobility model.

located at the origin. The probability that a sensor initially located at point $x \in \mathbb{R}^2$ hits the intruder within $[s,t]$ is equal to $\mathbb{P}(-x \in A(s,t))$. This probability only depends on the direction and speed of the sensors; in particular, it does not depend on the initial Poisson process giving the positions of the sensors. We can thus define a thinned Poisson process $\Phi(s,t)$ by selecting at time 0 the sensors that will hit the intruder during the interval $[s,t]$. This process is non-uniform and has density

$$\lambda'(x) = \lambda \mathbb{P}(-x \in A(s,t)) .$$

The number of sensors hitting the intruder during $[s,t]$ is equal to the total number of points in the thinned process, which is Poisson distributed with mean

$$\begin{aligned} \mathbb{E}(\mathrm{card}(\Phi(s,t))) &= \int_{\mathbb{R}^2} \lambda'(x) dx \\ &= \lambda \int_{\mathbb{R}^2} \mathbb{P}(-x \in A(s,t))\, dx \\ &= \lambda \int_{\mathbb{R}^2} \mathbb{E}\left(1_{\{-x \in A(s,t)\}}\right) dx \\ &= \lambda \mathbb{E}\left(\int_{\mathbb{R}^2} 1_{\{-x \in A(s,t)\}} dx \right) \\ &= \lambda \mathbb{E}(||A(s,t)||), \end{aligned} \tag{5}$$

where $1_{\{\cdot\}}$ denotes the indicator function of the event $\{\cdot\}$. Furthermore, it is easy to see that $\mathbb{E}(||A(s,t)||) = 2r\bar{v}_s(t-s)$.

Second, we show that the number of sensors hitting the intruder during disjoint time intervals are independent. This is simply done by observing that if $[s_1,t_1] \cap [s_2,t_2] = \emptyset$, each sensor is either selected in $\Phi(s_1,t_1)$ or in $\Phi(s_2,t_2)$ or not selected at all. Therefore, $\Phi(s_1,t_1)$ and $\Phi(s_2,t_2)$ are two independent processes.

Combining the two properties, we conclude that the sequence of times at which the intruder gets hit is a Poisson process.

$\square$

Compared to the case of stationary sensors where an undetected intruder always remains undetected, the probability that the intruder is not detected in a mobile sensor network decreases exponentially over time,

$$P(X \geq t) = e^{-2\lambda r v_s t}.$$

where X represents the detection time of the intruder.

The expected detection time of a randomly located intruder is $E[X] = \frac{1}{2\lambda r v_s}$, which is inversely proportional to the density of the sensors (λ), the sensing range of each sensor (r), and the speed of sensors (v_s). Note that the expected intruder detection time is independent of the sensor movement direction distribution density function, $f^s_\Theta(\theta)$. Therefore, in order to quickly detect a stationary intruder, one can add more sensors, use sensors with larger sensing ranges, or increase the speed of the mobile sensors.

To guarantee that the expected time to detect a randomly located stationary intruder be smaller than a specific value T_0, we have

$$\frac{1}{2\lambda r v_s} \leq T_0$$

or equivalently,

$$\lambda v_s \geq \frac{1}{2rT_0}.$$

If the sensing range of each sensor is fixed, the above formula presents the tradeoff between sensor density and sensor mobility to ensure given expected intruder detection time requirement. The product of the sensor density and sensor speed should be larger than a constant. Therefore, sensor mobility can be exploited to compensate for the lack of sensors, and vice versa.

In the proof of Theorem 1, we pointed out that a location alternates between being covered and not being covered, and then derived the fraction of time that a point is covered. While the time average characterization shows, to a certain extent, how well a point is covered, it does not reveal the duration of the time that a point is covered and uncovered. The time scales of such time durations are also very important for network planning; they present the time granularity of the intrusion detection capability that a mobile sensor network can provide. Theorem 3 now allows us to characterize the time durations of a point being covered and not being covered.

Corollary 1: **Consider a random sensor network $B(\lambda, r)$ at time $t = 0$, with sensors moving according to the straight-line random mobility model. A point alternates between being covered and not being covered. Denote the time duration that a point is covered as T_c, and the time duration that a point is not covered as T_n, we have**

$$T_n \sim \exp(2\lambda r v_s) \tag{6}$$

$$E[T_c] = \frac{e^{\lambda\pi r^2} - 1}{2\lambda r v_s}. \tag{7}$$

Proof. In the proof of Theorem 3, we know that the sequence of times at which a new sensor hits a given point forms a Poisson process of intensity $2\lambda r v_s$. After each sensor hits the point, it immediately covers the point until it moves out of range. There is no constraint on the number of sensors that cover the point. Therefore, the covered/uncovered sequence experienced by the point can be seen as a $M/G/\infty$ queuing process, where the service time of an sensor is the time duration that the sensor covers the point before moving out of range. The idle periods of $M/G/\infty$ queue corresponds to the time duration that the point is not covered. It is known that idle periods in such queues have exponentially distributed durations. Therefore, we have $T_n \sim \exp(2\lambda r v_s)$.

Since a point alternates between being covered and not being covered, the fraction of time a point is covered is

$$f_t = \frac{E[T_c]}{E[T_c] + E[T_n]} = 1 - e^{-\lambda\pi r^2}.$$

The last equality in the above equation is given in (4). Solving for $E[T_c]$, we obtain (7).

Let T denote the period of a point being covered and not being covered, i.e., $T = T_c + T_n$. The expected value of the period is

$$E[T] = E[T_c] + E[T_n] = e^{\lambda\pi r^2}/2\lambda r v_s.$$

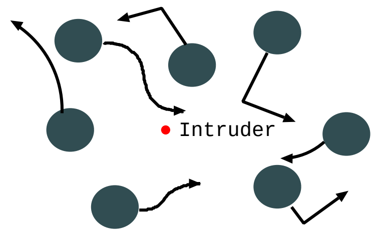

Fig. 3. Mobile sensor network with sensors moving along arbitrary curves.

□

Above we obtain the detection time of a stationary intruder when sensors all move in straight lines. In practice, mobile sensors do not always move in straight lines; they may make turns and move in different curves, as depicted in Figure 3. Next, we establish the optimal sensor moving strategy to minimize the detection time of a stationary intruder.

Theorem 4: **Consider a sensor network $B(\lambda, r)$ at time $t = 0$, with sensors moving according to the random mobility model at a fixed speed v_s. The detection time of a randomly located stationary intruder, X, is minimized in probability if sensors all move in straight lines.**

Proof: From Equation (5), we know that the number of sensors detecting the intruder during the interval $[0,t]$ is Poisson distributed with mean $\mathbb{E}(||A(0,t)||)$. Thus we have

$$\mathbb{P}(X \leq t) = \mathbb{P}(\text{card}(\Phi(0,t)) \geq 1) = 1 - \exp(-\mathbb{E}(||A(0,t)||)),$$

which is a increasing function of $\mathbb{E}(||A(0,t)||)$. As $\mathbb{E}(||A(0,t)||)$ is maximized when sensors move along straight lines, the probability of detecting the intruder is also maximized.

□

Similar to the arguments on the optimal strategies for area coverage in Section IV, straight line movement is not the only optimal strategy that minimizes the detection time. There is a family of moving patterns that can minimize the detection time, where straight line movement is one of them.

In the above analysis, we have assumed that an intruder is immediately detected when it is hit by the perimeter of a sensor, regardless of the time duration (t_s) it stays in the sensing range of the sensor. In many intrusion detection applications, for example, radiation, chemical, and biological threats, due to the probabilistic nature of the phenomenon and the sensing mechanisms, an intruder will not be immediately detected once it enters the sensing range of a sensor. Instead, it will take a certain amount of time to detect the intruder. If the sensing time is too short, an intruder may escape undetected. To account for this sensing time requirement, we define t_d to be the minimum sensing time in order for a sensor to detect an intruder. Obviously, it is only interesting when $0 \leq t_d \leq 2r/v_s$. Otherwise, the sensing time of an intruder by a sensor will be smaller than the minimum requirement t_d, and the intruder will never be detected. In order to yield closed-form results and provide insights, we will consider the straight-line random mobility model.

Theorem 5: **Consider a sensor network $B(\lambda, r)$ at time $t = 0$, with sensors moving according to the straight-line random mobility model at a fixed speed v_s. An intruder is detected iff the sensing time t_s is at least t_d, i.e., $t_s \geq t_d$. Let Y be the detection time of a randomly located stationary intruder initially not located in the sensing area of any sensor, we have**

$$Y = t_d + T \tag{8}$$

where

$$\begin{aligned} T &\sim \exp(2\lambda r_{\text{eff}} v_s) && (9) \\ r_{\text{eff}} &= \sqrt{r^2 - \frac{v_s^2 t_d^2}{4}}. && (10) \end{aligned}$$

Proof: We assume without loss of generality that the intruder is located at the origin. We observe first that a sensor covers the intruder for a time longer than t_d if and only if the distance between its trajectory and the origin is less than r_{eff} (see Figure 4). We call such sensors *valid* sensors.

Similarly as in Theorem 1, we define a thinned Poisson process $\Phi_{\text{eff}}(0,t)$ by selecting the sensors that will detect the intruder during the interval $[0,t]$. To do so, we define the *effective* covered area $A_{\text{eff}}(0,t)$ of a sensor as the area covered by the disk of radius r_{eff} centered on it. Then, the probability that a sensor initially located at x detects the intruder during the interval $[0,t]$ is $\mathbb{P}(-x \in A_{\text{eff}}(0,t))$. By (5) we find that the expected number of points in $\Phi_{\text{eff}}(0,t)$ is

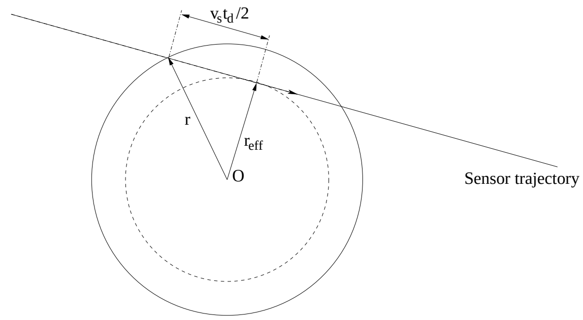

Fig. 4. Effective radius of a mobile sensor.

$\lambda\mathbb{E}(||A_{\mathrm{eff}}(0,t)||) = 2\lambda r_{\mathrm{eff}} v_s$. Denoting by T the time before a valid sensor covers the intruder, we get

$$\mathbb{P}(T \leq t) = \mathbb{P}(\mathrm{card}(\Phi_{\mathrm{eff}}(0,t)) \geq 1) = 1 - \exp(-2\lambda r_{\mathrm{eff}} v_s).$$

Then, the intruder is finally detected by the system after a time $T + t_d$.

$\square$

In (8), the detection time has two terms, namely, a constant term t_d and an exponentially distributed random variable with mean $E[T] = 1/(2\lambda r_{\mathrm{eff}} v_s)$. The first term t_d is a direct consequence of the minimum sensing time requirement. After the perimeter of a sensor hits an intruder, it takes a minimum sensing time of t_d to detect the intruder, and hence the constant delay. By Theorem 3, the second term corresponds to the detection time in the case where there is no minimum sensing time requirement but sensors have a reduced sensing radius of r_{eff}. This is again a consequence of the minimum sensing time requirement and the effect is illustrated in Figure 4. An intruder will only be detected by a mobile sensor if the trajectory of the sensor falls within r_{eff} from the intruder. The above two effects of minimum sensing time requirement result in an increased expected detection time compared to the case without minimum sensing time requirement. Since $t_d > 0$ and $r_{\mathrm{eff}} < r$, we have

$$E[Y] = t_d + 1/(2\lambda r_{\mathrm{eff}} v_s) > 1/(2\lambda r v_s) = E[X].$$

Sensor speed has two opposite effects on an intruder's detection time.

- On one hand, as sensors move faster, uncovered areas will be covered more quickly and this tends to speed up the detection of intruders.

- On the other hand, the effective sensing radius r_{eff} decreases as sensors increase their speed due to the sensing time requirement, making intruders less likely to be detected.

In the following, we present the optimal sensor speed that minimizes the expected detection time. Excess mobility will be harmful when the sensor speed is larger than the optimal value.

Theorem 6: **Under the scenario in Theorem 5, the optimal sensor speed minimizing the expected detection time of a randomly located intruder is**

$$v_s^* = \sqrt{2}r/t_d. \tag{11}$$

Proof. Let $dY/dv_s = 0$, we have $v_s^* = \sqrt{2}r/t_d$, and the second order derivative $\frac{d^2Y}{dv_s^2}|_{v_s^*} < 0$. The corresponding minimum expected detection time is

$$E[Y^*] = (1+2\lambda r^2)t_d/2\lambda r^2.$$

$\square$

In real world applications, the minimum required sensing time depends on a number of components: sensing mechanism (underlying physical, chemical, biological processes), hardware (CPU, ADC, memory, clock rate, etc) and software (operating systems) configurations. While the response time of some sensors is small (e.g., accelerometer MMA73x0L by Freescale Semiconductor Inc. has a response time of less than 1 ms) and the effect on the detection time is negligible, other sensors (e.g., certain optical biosensors, chemical sensors) have a response time of several seconds or longer []. In this case, the effect of the minimum required sensing time on detection time of intruders cannot be ignored. In a real sensor network system, one will need to measure the minimum required sensing time for the application and determine if the effect is negligible.

VI. Detection Time of Mobile Intruder

In this section, we consider the detection time of a mobile intruder, which depends not only on the mobility behavior of the sensors but also on the movement of the intruder itself. Intruders can adopt a wide variety of movement patterns. In this work, we will not consider specific intruder movement patterns. Rather, we approach the problem from a game theoretic standpoint and study the optimal mobility strategies of the intruders and sensors.

For simplicity we assume that an intruder will be immediately detected when it falls into the sensing range of mobile sensors. Note that the analysis and results of the detection time requirement presented in the previous section can be readily adapted in this part of the study. From Theorem 4, the detection time of a stationary sensor intruder is minimized when sensors all move in straight lines. This result can be easily extended to a mobile intruder using similar arguments in the reference framework where the intruder is stationary. From the perspective of an intruder, since it only knows the mobility strategy of the sensors (sensor direction distribution density function) and does not know the locations and directions of the sensors, changing direction and speed will not help prolong its detection time. In the following, we will only consider the case where sensors and intruders move in straight lines.

Given the mobility model of the sensors, $f_\Theta(\theta)$, an intruder chooses the mobility strategy that maximizes its expected detection time. More specifically, an intruder chooses its speed $v_t \in [0, v_t^{\max})$ and direction $\theta_t \in [0, 2\pi)$ so as to maximize the expected detection time. The expected detection time is a function of the sensor direction distribution density, intruder speed, and intruder moving direction. Denote the resulting expected detection time as $\max_{v_t,\theta_t} E[X(f_\Theta(\theta), \theta_t, v_t)]$; the sensors then choose the mobility strategy (over all possible direction distributions) that minimizes the maximum expected detection time. This can be viewed as a zero-sum minimax game between the collection of mobile sensors and the intruder, where the payoffs for the mobile sensors and intruder are $-E[X(f_\Theta, \theta_t, v_t)]$ and $E[X(f_\Theta, \theta_t, v_t)]$, respectively.

To find the optimal mobility strategies for mobile sensors and the intruder, we consider the following minimax optimization problem:

$$\min_{f_\Theta} \max_{\theta_t, v_t} E[X(f_\Theta, \theta_t, v_t)]. \tag{12}$$

To solve the minimax optimization problem, we first characterize the detection time of an intruder moving at a constant speed in a particular direction.

Theorem 7: **Consider a sensor network $B(\lambda, r)$ at time $t = 0$, with sensors moving according to the straight-line random mobility model at a fixed speed v_s. Let X be the**

detection time of an intruder moving at speed v_t along direction θ_t. Denote

$$c = v_t/v_s, \;\; \hat{c} = 1 + c$$
$$w(u) = \sqrt{1 - \tfrac{4c}{\hat{c}^2}\cos^2\tfrac{u}{2}}$$
$$\overline{v}_s = v_s \hat{c} \int_0^{2\pi} w(\theta - \theta_t) f_\Theta(\theta) d\theta.$$

We have

$$X \sim \exp(2\lambda r \overline{v}_s). \qquad (13)$$

Proof: To prove this theorem, we put ourselves in the frame of reference of the intruder and look at the speeds of the sensors. Thus, if a sensor has an absolute speed vector v_s, its speed vector in the new frame of reference is simply $v_s - v_t$, where v_t denotes the intruder's absolute speed vector. Let θ_s denote the direction of v_s and θ_t the direction of v_t.

In the new frame of reference, the intruder is static. Denote $c = v_t/v_s$, $\hat{c} = 1 + c$, and $w(u) = \sqrt{1 - \frac{4c}{\hat{c}^2}\cos^2\frac{u}{2}}$. Using the Law of Cosines, the relative speed of the sensor can be computed as

$$\begin{aligned} ||v_s - v_t|| &= \sqrt{v_s^2 + v_t^2 - 2v_s v_t cos(\theta_s - \theta_t)} \\ &= v_s \hat{c} w(\theta_s - \theta_t) \end{aligned}$$

We know from Equation (5) that

$$\mathbb{P}(X \leq t) = \mathbb{P}(\text{card}(\Phi(0,t)) \geq 1) = 1 - \exp(-\lambda \mathbb{E}(||A(0,t)||)).$$

Therefore, if $\mathbb{E}(||A(0,t)||)$ is a linear function of t, then X is exponentially distributed. We get

$$||A(0,t)|| = 2r||v_s - v_t||t = 2rtw(\theta_s - \theta_t),$$

so that

$$\begin{aligned} \mathbb{E}(||A(0,t)||) &= 2rt \int_0^{2\pi} w(\theta - \theta_t) f_\Theta(\theta) d\theta \\ &= 2rt\overline{v}_s \end{aligned}$$

where $\overline{v}_s = v_s \hat{c} \int_0^{2\pi} w(\theta - \theta_t) f_\Theta(\theta) d\theta$, which can be viewed as the average effective sensor speed in the reference framework where the intruder is stationary. Therefore, the detection time is exponentially distributed with rate $2\lambda r\overline{v}_s$.

$\square$

From Theorems 3 and 7, it can be noted that the detection times of both stationary and mobile intruders follow exponential distributions, and that the parameters are of the same form, except that the sensor speed is now replaced by the effective sensor speed for the mobile intruder case.

Assuming that the sensor density and sensing range are fixed, since the intruder detection time follows an exponential distribution with mean $1/(2\lambda r\overline{v}_s)$, maximizing the expected detection time corresponds to minimizing the effective sensor speed $\overline{v}_s$. In the following, we derive the optimal intruder mobility strategies for two special sensor mobility models.

Sensors move in the same direction θ_s: $f_\Theta(\theta) = \delta(\theta - \theta_s)$.

Using the fundamental property of the delta function $\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$, we have

$$\begin{aligned}\overline{v}_s &= v_s\hat{c}\int_0^{2\pi} w(\theta-\theta_t)\delta(\theta-\theta_s)d\theta \\ &= v_s\hat{c}w(\theta_s-\theta_t).\end{aligned}$$

We need to choose a proper θ_t and v_t that minimizes the above effective sensor speed $\overline{v}_s$. First, it is easy to see that we require $\theta_t = \theta_s$. Now, we have

$$\overline{v}_s = v_s\hat{c}\sqrt{1-\frac{4c}{\hat{c}^2}} = |v_t - v_s|$$

and $\overline{v}_s$ is minimized when

$$v_t = \begin{cases} v_s & \text{if } v_t^{\max} \geq v_s \\ v_t^{\max} & \text{otherwise.}\end{cases}$$

The above results show, quite intuitively, that the intruder should move in the same direction as the sensors at a speed closest matching the sensor speed. If the maximum intruder speed is larger than the sensor speed, the intruder will not be detected since it chooses to move at the same speed and in the same direction as the sensors. In this case, the detection time is infinity. Otherwise, if the maximum intruder speed is smaller than the sensor speed, the intruder should move at the maximum speed in the same direction of the sensors. The expected detection time

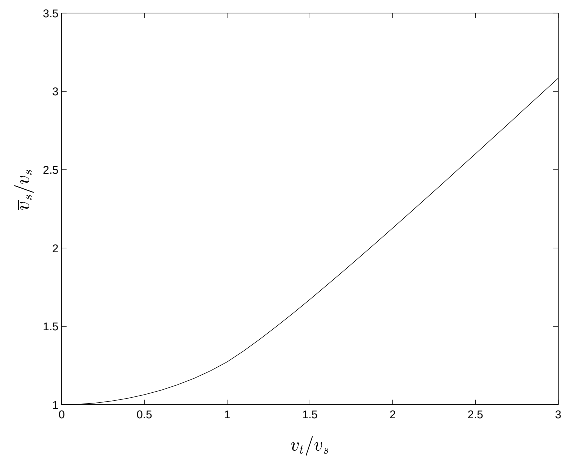

Fig. 5. Normalized effective relative sensor speed $\overline{v}_s/v_s$ as a function of $c = v_t/v_s$

is $\frac{1}{2\lambda r(v_s - v_t^{\max})}$.

Sensors move in uniformly random directions: $f_\Theta(\theta) = \frac{1}{2\pi}$.

Figure 5 plots the normalized effective sensor speed $\overline{v}_s/v_s$ as a function of $c = v_t/v_s$, the ratio of the intruder speed to the sensor speed. The effective sensor speed is an increasing function of c, and is minimized when $c = 0$, or $v_t = 0$. Therefore, if each sensor uniformly chooses its moving direction from 0 to 2π, the maximum expected detection time is achieved when the intruder does not move. The corresponding expected detection time is $\frac{1}{2\lambda r v_s}$. The optimal intruder mobility strategy in this case can be intuitively explained as follows. Since sensors move in all directions with equal probability, the movement of the intruder in any direction will result in a larger relative speed and thus a smaller first hit time in that particular direction. Consequently, the minimum of the first hit times in all directions (detection time) will become smaller.

We now present the solution to the minimax game between the collection of mobile sensors and the intruder in the following theorem.

Theorem 8: **Consider a sensor network $B(\lambda, r)$ at time $t = 0$, with sensors moving according to the random mobility model at a fixed speed v_s. For the game between the collection of mobile sensors and the mobile intruder, the optimal sensor strategy is for each sensor to choose a direction according to a uniform distribution, i.e., $f_\Theta(\theta) = \frac{1}{2\pi}$. The optimal mobility strategy of the intruder is to stay stationary. This solution constitutes a Nash equilibrium of the game.**

Proof.

In the game between the collection of mobile sensors and the intruder, mobile sensors need to choose an optimal moving direction distribution $f_\Theta(\theta)$ so as to detect the intruder as soon as possible, while the intruder chooses its speed $v_t \in [0, v_t^{\max})$ and direction $\theta_t \in [0, 2\pi)$ so as to stay undetected as long as possible. Denote the resulting expected detection time as $E[X(f_\Theta(\theta), \theta_t, v_t)]$. The game payoffs for the mobile sensors and intruder are $-E[X(f_\Theta, \theta_t, v_t)]$ and $E[X(f_\Theta, \theta_t, v_t)]$, respectively.

For sensors, minimizing intruder detection time is equivalent to maximizing the effective sensor speed after an intruder selects the optimal speed and direction. We first prove for any given intruder speed v_t, that among all possible sensor direction distributions, the minimum effective sensor speed resulted from the optimal intruder direction choice, $\min_{\theta_t} \overline{v}_s$, is maximized when sensors choose directions according to a uniform distribution. The formal statement is described as follows.

Denote the uniform distribution density as $f_\Theta^{\text{uniform}} = 1/2\pi$. From Theorem 7, the effective sensor speed is a function of sensor direction distribution density, intruder speed and direction, $\overline{v}_s(f_\Theta(\theta), \theta_t, v_t) = \int_0^{2\pi} w(\theta - \theta_t) f_\Theta(\theta) d\theta$.

We will prove that

$$\min_{\theta_t, v_t} \overline{v}_s(f_\Theta(\theta), \theta_t, v_t) \le \min_{\theta_t, v_t} \overline{v}_s(f_\Theta^{\text{uniform}}, \theta_t, v_t) \tag{14}$$

for all $f_\Theta(\theta)$.

First, let us consider the right-hand side of (14). We have

$$\begin{aligned} \overline{v}_s(f_\Theta^{\text{uniform}}, \theta_t, v_t) &= \frac{1}{2\pi}\int_0^{2\pi} w(\theta - \theta_t)\, d\theta \\ &= \frac{1}{2\pi}\int_{-\theta_t}^{2\pi-\theta_t} w(u)\, du \\ &= \frac{1}{2\pi}\int_0^{2\pi} w(u)\, du \end{aligned}$$

for all θ_t, since the mapping $u \to w(u)$ is periodic with period 2π. This shows that

$$\min_{\theta_t} \overline{v}_s(f_\Theta^{\text{uniform}}, \theta_t, v_t) = \frac{1}{2\pi}\int_0^{2\pi} w(u)\, du. \tag{15}$$

We now come back to the proof of (14). We have

$$
\begin{aligned}
& \min_{\theta_t} \overline{v}_s(f_\Theta(\theta), \theta_t, v_t) \\
\leq \; & \frac{1}{2\pi}\int_0^{2\pi} \overline{v}_s(f_\Theta(\theta), \theta_t, v_t)\, d\theta_t \\
= \; & \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} w(\theta-\theta_t)\, f_\Theta(\theta)\, d\theta d\theta_t \\
= \; & \frac{1}{2\pi}\int_0^{2\pi} f_\Theta(\theta) \left(\int_{\theta-2\pi}^{\theta} w(u)\, du\right) d\theta \\
= \; & \frac{1}{2\pi}\left(\int_0^{2\pi} f_\Theta(\theta)\, d\theta\right)\left(\int_0^{2\pi} w(u)\, du\right) \\
= \; & \frac{1}{2\pi}\int_0^{2\pi} w(u)\, du \\
= \; & \min_{\theta_t} \overline{v}_s(f_\Theta^{\text{uniform}}, \theta_t, v_t)
\end{aligned} \tag{16}
$$

where the last three equalities follow from the fact that $w(u)$ is periodic with period 2π, from the fact that $f_\Theta(\theta)$ is a probability density function on $[0, 2\pi]$, and from (15), respectively.

The proof of (14) is concluded by taking first the minimum over v_t in the left-hand side of (16), then by taking the minimum over v_t in the right-hand side of (16).

It follows that when sensors choose directions according to a uniform distribution, the optimal intruder mobility strategy is to stay stationary, i.e., $v_t = 0$ (since $\overline{v}_s(f_\Theta^{\text{uniform}}, \theta_t, v_t)$ is maximized when $c = 0$ (and equals to 1), i.e. when $v_t = 0$), and θ_t is irrelevant in this case.

Based on the previous discussions on different mobility strategies of sensors and intruders, under the optimal mobility strategies, neither side can improve the payoff by changing the strategy unilaterally. Specifically, when sensors choose their direction uniformly at random, the movement of the intruder in any direction will result in a larger relative speed and thus a smaller first hit time in that particular direction. Consequently, the minimum of the first hit times in all directions (detection time) will become smaller. When the intruder stays stationary, the detection time will not improve if sensors choose a different distribution for the moving direction. Therefore, the solution constitutes a Nash equilibrium of the game. □

This result suggests that in order to minimize the expected detection time of an intruder, sensors should choose their directions uniformly at random between $[0, 2\pi)$. The corresponding optimal mobility strategy of the intruder is to stay stationary. The uniformly random sensor movement represents a mixed strategy which is a Nash equilibrium of the game between mobile sensors and intruders. If sensors choose to move in any fixed direction (pure strategy), it can be exploited by an intruder by moving in the same direction as sensors to maximize its detection time. The optimal sensor strategy is to choose a mixture of available pure strategies (move in a fixed direction between $[0,2\pi)$). The proportion of the mix should be such that the intruder cannot exploit the choice by pursuing any particular pure strategy (move in the same direction as sensors), resulting in a uniformly random distribution for sensor's movement. When sensors and intruders follow their respective optimal strategies, neither side can achieve better performance by deviating from this behavior.

In this study we assume that the goal of the mobile intruder is to maximize the expected detection so that it can stay undetected as long as possible. This is a desirable goal in some applications such as intruder intelligence gathering. In other applications, the intruder may want to pass through a region monitored by sensors (e.g., [22], [47]) or to visit a set of particular locations without being detected. In these cases, the objective of the intruder is different and the corresponding optimal strategy would be different. In general, for a specific application, we will need to first identify the objective of the intruder and then study the corresponding game between the intruder and mobile sensors.

VII. Summary

In this paper, we study the dynamic aspects of the coverage of a mobile sensor network resulting from the continuous movement of sensors. Specifically, we studied the coverage measures related to the area coverage and intrusion detection capability of a mobile sensor network.

For the random initial deployment and the random sensor mobility model under consideration, we showed that while the area coverage at any given time instants remains unchanged, more area will be covered at least once during a time interval. This is important for applications that do not require or cannot afford simultaneous coverage of all locations but want to cover the deployed region within a certain time interval. The cost is that a location is only covered part of the time, alternating between covered and not covered. To this end, we characterized the durations and fraction of time that a location is covered and not covered.

As sensors move around, intruders that will never be detected in a stationary sensor network can be detected by moving sensors. We characterized the detection time of a randomly located stationary intruder. The results suggest that sensor mobility can be exploited to effectively reduce the detection time of an intruder when the number of sensors is limited. We further considered a more realistic sensing model where a minimum sensing time is required to detect an intruder. We find that there is an optimal sensor speed that minimizes the expected detection time. Beyond the optimal speed, excess mobility will be harmful to the intrusion detection performance. Moreover, we discussed the optimal mobility strategies that maximize the area coverage during a time interval and minimize the detection time of intruders.

For mobile intruders, the intruder detection time depends on the mobility strategies of the sensors as well as the intruders. We took a game theoretic approach and obtained the optimal mobility strategy for sensors and intruders. We showed that the optimal sensor mobility strategy is that each sensor chooses its direction uniformly at random in all directions. By maximizing the entropy of the sensor direction distribution, the amount of prior information on sensor mobility strategy revealed to an intruder is minimized. The corresponding intruder mobility strategy is to stay stationary in order to maximize its detection time. This solution represents a Nash equilibrium of the game between mobile sensors and intruders. Neither side can achieve better performance by deviating from their respective optimal strategies.

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