To generate identification data, we simulated this system using the feedback law

\[ u(t) = r(t) - (-0.95y(t) - 2)y(t) = r(t) + 0.95y(t - 2) \]  

which places the closed-loop poles in 0.8618 and 0.6382. In the simulation we used independent, zero-mean, Gaussian white noise reference and noise signals \( \{r(t)\} \) and \( \{e(t)\} \) with variances 1 and 0.01, respectively. \( N = 200 \) data samples were used.

In Table I, we have summarized the results of the identification, the numbers shown are the estimated parameter values together with their standard deviations. For comparison, we have, apart from the model structure (24), used a standard output error model model structure and a second-order ARMAX model structure. As can be seen, the standard output error model structure gives completely useless estimates, and the modified output error and the ARMAX model structures give similar and accurate results.

### V. An Alternative Box–Jenkins Model Structure

The trick to include a modified noise model in the output error model structure is of course also applicable to the Box–Jenkins model structure. The alternative form in this case be

\[ y(t) = \frac{B(q)}{F(q)} u(t) + \frac{F_p(q)C(q)}{F_q(q)} e(t). \]  

An explicit expression for the gradient filters for this predictor can be derived similarly as in the output error case, albeit the formulas will be even messier. We leave the details to the reader.

### VI. Conclusions

In this paper, we have proposed new versions of the well-known output error and Box–Jenkins model structures that can also be used for identification of unstable systems. The new model structures are equivalent to the standard ones, as far as number of parameters and asymptotical results are concerned, but guarantee stability of the predictors.

### References


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**Control of Nonlinear Chained Systems: From the Routh–Hurwitz Stability Criterion to Time-Varying Exponential Stabilizers**

P. Morin and C. Samson

**Abstract**—We show how any linear feedback that stabilizes the origin of a linear chain of integrators induces a simple, continuous time-varying feedback that exponentially stabilizes the origin of a nonlinear chained-form system. The design method is related to a method developed by M’Closkey and Murray to transform smooth feedback yielding slow polynomial convergence into continuous homogeneous ones that give exponential convergence.

**Index Terms**—Asymptotic stability, nonholonomic system, time-varying feedback.

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**I. INTRODUCTION**

Control systems in the so-called chained form have been extensively studied in recent years. This research interest partly stems from the fact that the kinematic equations of many nonholonomic mechanical systems, such as those arising in mobile robotics (unicycle-type carts, car-like vehicles with trailers, etc.), can be converted into this form [12], [16], [18]. This paper addresses the problem of asymptotic stabilization of a given equilibrium point (which corresponds to a fixed configuration for a mechanical system).

Because chained systems do not satisfy Brockett’s necessary condition [1], they cannot be asymptotically stabilized, with respect to any equilibrium point, by means of a continuous pure-state feedback \( u(x) \). In [15], one of the authors proposed and derived smooth time-varying feedback laws \( u(x,t) \) for the stabilization of a unicycle-type vehicle. This proposition showed how the topological obstruction raised by Brockett could be dodged and was the starting point of other studies on time-varying feedback. In [3] and [4], Coron established that most controllable systems can be asymptotically stabilized with this type of feedback. The literature on the subject has since then mostly focused on the explicit design of such stabilizing control laws. Smooth feedback laws yielding slow (polynomial) asymptotic convergence have first been designed (see, e.g., [13], [15]–[17], and [19]). More recently, properties associated with homogeneous systems have been used to obtain feedback laws only continuous but yielding an exponential convergence rate [7], [8], [10], [14].

Lately, M’Closkey and Murray have presented in [9] a method for transforming smooth time-varying stabilizers into homogeneous continuous ones. The method is best suited for driftless systems for which it applies systematically. The construction relies upon the initial knowledge of an adequate Lyapunov function coupled with a smooth stabilizing feedback law. More precisely, the exponential stabilizer is obtained by “scaling” the size of the smooth control inputs on a level set of the Lyapunov function. The feedbacks derived in the present paper have been obtained by adapting and combining the core of this method to the control design method earlier proposed by Samson in [16] for the smooth feedback stabilization of chained systems. Although our approach is specific to chained systems, it carries with it two important improvements with respect to [9]. The first one is that the knowledge Manuscript received November 18, 1997; revised December 18, 1998. Recommended by Associate Editor L.-S. Wang.

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The Routh–Hurwitz Criterion

Finally, a sketch of proof of these results is given in Section IV.

factor is still implicitly defined. The second proposition is an adaptation of two propositions. In the first one, the aforementioned scaling is:

explicit function. The implementation of the resulting control law is

will usually have to be done numerically. The first feedback law proposed in the present study is of this type. We also show in a second result, however, that this scaling factor can be replaced by an adequate explicit function. The implementation of the resulting control law is consequently simplified.

This paper is organized as follows. In Section II, some results used further for the design of the control laws are recalled. The two main results and proposed control laws are presented in Section III in the form of two propositions. In the first one, the aforementioned scaling factor is still implicitly defined. The second proposition is an adaptation of the first one to get rid of the implicit definition of the scaling factor. Finally, a sketch of proof of these results is given in Section IV.

II. PRELIMINARY RECALLS

A. Stabilization of a Multi-Order Integrator and the Routh–Hurwitz Criterion

Consider the following linear \((n-1)\)-order integrator 

\[
\left\{ \begin{array}{l}
\dot{x}_i = x_{i+1}, \\
\dot{x}_n = u,
\end{array} \right.
\]

Any linear feedback control

\[
u = -\sum_{i=2}^{n} a_i x_i
\]

asymptotically stabilizes the origin of this system, provided all roots of the characteristic polynomial \(p(s) = s^{n-1} + a_{n-1}s^{n-2} + \cdots + a_3 s + a_2\) associated with the closed-loop system have strictly negative real parts.

The Routh–Hurwitz table associated with this polynomial is

\[
\begin{array}{cccccc}
1 & a_{n-1} & a_{n-3} & \cdots & \cdots & \\
a_n & a_{n-2} & a_{n-4} & \cdots & 0 & \\
b_n & b_{n-2} & \cdots & \cdots & 0 & \\
c_n & c_{n-2} & \cdots & \cdots & 0 & \\
d_n & d_{n-2} & \cdots & \cdots & 0 & \\
\vdots & \vdots & \vdots & \vdots & 0 & \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

with

\[
\begin{align*}
a_k &= 0 & \text{for } k < 2 \\
b_k &= \frac{1}{a_n} (a_{k-2} - a_k a_{k-1}) \\
c_k &= \frac{1}{a_n} (a_k b_{k-2} - b_{k-2} a_{k-2}) \\
d_k &= \frac{1}{b_n c_{k-2} - c_n b_{k-2}} \\
\vdots
\end{align*}
\]

Let \(k \doteq (k_2, \ldots, k_n)\) be defined from the first column of the Routh–Hurwitz table by

\[
(k_n, k_{n-1}, k_{n-2}, \ldots) = (a_n, b_n, c_n, \ldots)
\]

Then, we have the following two lemmas whose proofs can be found in several control textbooks (see [2], for example).

**Lemma 1:** Let \(X_2 = (x_2, x_3, \ldots, x_n)^T\), and consider the linear change of coordinates \(\phi : X_2 \rightarrow Z_2 = (z_2, z_3, \ldots, z_n)^T = \Phi_2 X_2\) defined by

\[
\begin{align*}
z_2 &= x_2 \\
z_3 &= x_3 \\
z_{j+3} &= k_{j+1} z_{j+1} + L_j z_{j+2} & (j = 1, \ldots, n - 3)
\end{align*}
\]

with

\[
L_j \phi_i = (\partial \phi_i / \partial X_2)f
\]

the Lie derivative of \(\phi_i\) along \(f(X_2) = (x_3, x_4, \ldots, x_n, 0)^T\). Then, in the coordinates \(Z_2\), the controlled system (1), (2) becomes

\[
\begin{align*}
\dot{z}_2 &= z_3 \\
\dot{z}_{i+1} &= -k_i z_i + z_{i+2} & (i = 2, \ldots, n - 2) \\
\dot{z}_n &= -k_{n-1} z_{n-1} - k_n z_n.
\end{align*}
\]

Using the fact that the time derivative of the quadratic function \(V_i\) defined by

\[
V_i(Z_2) \doteq Z_2^T D_k Z_2
\]

\[
D_k \doteq \text{Diag} \left( \prod_{i=2}^{n} k_{i}, k_{i+1}, \ldots, k_{n-1}, 1 \right)
\]

along any solution of the system (4) is

\[
\dot{V}_i(Z_2) = -2k_n z_n^2
\]

we easily establish the following Lemma.

**Lemma 2:** The origin \(Z_2 = 0\) of the linear system (4) is asymptotically stable if and only if \(k_i > 0\) for \(i = 2, \ldots, n\).

A corollary of the above two lemmas is the well-known Routh–Hurwitz stability criterion.

**Corollary 1 (Routh–Hurwitz):** All roots of the polynomial \(p(s) = s^{n-1} + a_n s^{n-2} + \cdots + a_3 s + a_2\) have strictly negative real parts if and only if \(k_i > 0\) for \(i = 2, \ldots, n\).

B. Nonexponential Time-Varying Feedback Stabilization of Chained Systems

The prime objective of the previous section was to point out the algebraic operations that transform the chain of integrators involved in the system (1), (2) into the skew-symmetric representation (4) to which the simple Lyapunov function (5) can be associated. In [16], the structural similarity between the linear \(n\)-order integrator system (1) and the following nonlinear chained system:

\[
\begin{align*}
\dot{x}_1 &= u_1 \\
\dot{x}_i &= u_1 x_{i-1}, & (i = 2, \ldots, n-1), \\
\dot{x}_n &= u_2,
\end{align*}
\]

has been used, with the aforementioned transformations, to prove the following result.
C. Homogeneity and Exponential Stabilization

The set of nonlinear systems homogeneous of degree zero with respect to some dilation constitutes a fairly natural extension of the set of linear systems. Some properties of these systems are briefly recalled hereafter. For more details, see, e.g., [5].

For any \( \lambda > 0 \) and any set of real parameters \( r_i > 0 \) (\( i = 1, \ldots, n \)), a “dilation” operator is a map \( \delta(\lambda, :): \mathbb{R}^n \rightarrow \mathbb{R}^n \) defined by

\[
\delta(\lambda, x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda^n x_n).
\]

A function \( f \in C^0(\mathbb{R}^n \times \mathbb{R}) \) is homogeneous of degree \( \tau \geq 0 \) with respect to the dilation \( \delta(\lambda, :), \) if:

\[
\forall \lambda > 0, \quad f(\delta(\lambda, x), t) = \lambda^{-\tau} f(x, t).
\]

A homogeneous norm \( \rho \) associated with this dilation operator is a \( C^0 \) function from \( \mathbb{R}^n \) to \( \mathbb{R}^n \), homogeneous of degree one with respect to the dilation, positive \( \rho(x) > 0, \forall x \), and proper. A consequence of this definition is that \( \rho(x) = 0 \) iff \( x = 0 \). An example of homogeneous norm is

\[
\rho(x) = \left( \sum_{j=1}^{n} |x_j|^{r/j} \right)^{1/p}, \quad \text{with} \quad p > 0.
\]

A differential system \( \dot{x} = f(x, t) \) (or a vector field \( f \)), with \( f \in C^0(\mathbb{R}^n \times \mathbb{R}; \mathbb{R}^n) \), is homogeneous of degree \( \tau \geq 0 \) with respect to the dilation \( \delta(; \lambda) \) if for any \( i = 1, \ldots, n \), the \( i \)th component \( f_i \) of the vector field \( f \) is homogeneous of degree \( \tau + r_i \).

Finaly, let \( f \in C^0(\mathbb{R}^n \times \mathbb{R}^n), \) with \( f(x, :), T \) periodic, define an homogeneous vector field of degree zero with respect to the dilation \( \delta(; \lambda) \). Then, the following two properties are equivalent:

1) the origin \( x = 0 \) of the system \( \dot{x} = f(x, t) \) is asymptotically stable;
2) \( x = 0 \) is globally exponentially stable in the sense that there exist \( \gamma > 0 \) and, for any homogeneous norm \( \rho \), a value \( K \) such that along any trajectory \( x(t) \) (\( t \geq t_0 \)) of the system \( \dot{x} = f(x, t) \),

\[
\rho(x(t)) \leq K \rho(x(t_0)) e^{-\gamma(t-t_0)}.
\]
Then, \( q_0 > 1 \) exists such that, if \( q \geq q_0 \) and \( p > n - 2 + q \), the continuous time-varying feedback control

\[
\begin{cases}
u_1(x, t) = -k_1 x_1 (\sin^2 t + \text{sign}(x_1) \sin t) \\
- k_{n+1} \rho_{n+1}(X_2) \sin t,
\end{cases}
\]

satisfy this inequality, we conjecture this control, combined with the change of coordinates which transforms (1), (2) into (4). Therefore, we deduce from (6) and (9)

\[
\frac{\partial V_x}{\partial x}(W_2) A_{+1} W_2 = \frac{\partial V_x}{\partial x}(\Phi_k W_2) K \Phi_k W_2
\]

with \( K \) the matrix associated with the right-hand side of (4). This proves (20) for \( \mu_1 \geq 1 \). If \( \mu_1 = -1 \), we consider the change of coordinates \( \psi: \mathbb{R}^n \rightarrow \mathbb{R}^{n-1} \) defined by \( \psi(W_2) = (\psi_{1}^{n-1} \cdots \psi_{n-1}^{n-1}) \). It is simple to verify

\[
\psi(A_{-1} W_2) = A_{+1} \psi(W_2).
\]

Moreover, using the definition of the matrix \( \Phi_k \) of change of coordinates introduced in Lemma 1, we also verify

\[
\Phi_k \psi(W_2) = \psi(A_{+1} W_2).
\]

Using (22)–(24), and because, for any \( W_1 \) and \( W_2 \), \( \psi^T(W_1) D_k \psi(W_2) = W_1^{T} D_k W_2 \) because \( D_k \) is diagonal, we have

\[
\frac{\partial V_x}{\partial x}(W_2) A_{+1} W_2 = 2 W_2^{T} \Phi_k^{T} D_k \Phi_k A_{-1} W_2
\]

This proves (20) and completes the proof of i).

Part i) and (13) imply \( x_1 \) is bounded along the trajectories of the system, and because \( \rho_k \) is proper, all trajectories exist on \([0, +\infty)\) and are bounded. To show the asymptotic stability, we apply LaSalle’s invariance principle [6] for time-periodic systems. First, remark, for \( \nu > n - 2 + g, \rho_0^\nu \) is of class \( C^0 \) on \( \mathbb{R}^{n-1} \) because each partial derivative \( \partial^\nu C_0(x_2) \) is homogeneous of degree \( \nu - \gamma > 0 \), and therefore tends to zero as \( |X_2| \) tends to zero. From what precedes, all solutions converge to the largest invariant set \( M \) contained in \( E = \{x: (d/dt) \rho_0^\nu(X_2) = 0\} \). Let us show

\[
M = \{x_0, 0): x_1 \in \mathbb{R}\}
\]

Consider any solution \( x(t) \) of the system contained in \( E \), and assume \( X_2(t) \) is not identically zero. Because \( \rho_k \) is constant and different from zero, this implies \( X_2(t) \neq 0 \) for all \( t \). In view of (21) and Lemma 3, we deduce

\[
|u_1(x(t), t)| \left( \Phi_k W_2^2 \right)^{-1} = 0.
\]

From (13), \( u_1(x(t), t) \) cannot be identically zero because \( \rho_k \) is constant and different from zero. Let \( (t_1, t_2) \) denote a non-empty time-interval on which \( u_1(x(t), t) \neq 0 \). Without loss of generality, we
can assume \( u_1(x(t), t) \) is positive on \((t_1, t_2)\). Then, it comes that
\[
(\Phi_4 W_2)_a(t) = 0 \text{ for } t \in (t_1, t_2).
\]
Because \( W_2 = \delta_1(\lambda, X_2), \) and \( \lambda(X_2(t)) \) is constant, we deduce from (17) \( W_2 = \lambda u_1 A_{-1} W_2 \), so
\[
\Phi_4 W_2 = \lambda u_1 \Phi_4 A_{\lambda} \Phi_4^{-1} W_2 = \lambda u_1 K \Phi_4 W_2.
\]
(28)

Because both \( \lambda \) and \( u_1 \) do not vanish on \((t_1, t_2)\), and in view of the structure of the matrix \( K \) [recall the matrix is associated with the right-hand side of (4)], we easily show from (28) \( (\Phi_4 W_2)_a \), is identically zero on \((t_1, t_2)\) only if all components of \( \Phi_4 W_2 \) are identically zero, that is, if \( X_2(t) \) is identically zero. This proves (26). From the expression of \( u_1(x, t) \) and the system’s equation \( \dot{x}_1 = u_1, x_1(t) \) also converges to zero.

Finally, we easily show the closed-loop system (7)-(13) is homogeneous of degree zero with respect to the dilation \( \delta_\lambda(\lambda, X_2) \). In view of the results of Section II-C, this implies the exponential stability of this system.

B. Proof of Proposition 3

The proof uses the following Lemmas (see [11] for the proofs of these results).

Lemma 4: The map \( \gamma \) defined by
\[
\gamma(x_1, X_2) = (u_1 x_1, u_1 x_1, \ldots, u_1 x_n, \alpha u_1, X_2))
\]

with \( \alpha \) is a Hurwitz-stable matrix, \( \epsilon = \epsilon_1(\mathbb{R}^n) \) exist such that
1) \( \forall X_2 \neq 0, L_{g}(x_1, X_2) = [u_1] \epsilon(x, \epsilon_1(X_2)); \)
2) \( \lim_{t \to +\infty} \sup_{x \in \mathbb{R}^n} \epsilon(x, \epsilon_1(X_2)) = 0, (\alpha = \pm 1, -1). \)

Lemma 6: Consider the system
\[
y = \gamma(t)(A + \epsilon(\gamma) B)y
\]

with \( y \in \mathbb{R}^n, \gamma_1 \in \mathbb{C}^n(\mathbb{R}^n, \mathbb{R}^n) \), \( A \) a Hurwitz-stable matrix, \( \epsilon \in \mathbb{C}^n(\mathbb{R}^n, \mathbb{R}^n) \) exists such that the function \( g(x_1, X_2) \) strictly positive on some nonempty interval \((t_1, t_2)\), and \( \delta \in (0, t_2 - t_1) \), \( \alpha \in \{0, 1\} \), such that
\[
\gamma(t) \geq \gamma_0(t), \forall t \in (t_1, t_2), \text{ and } ||y(t)|| \leq \beta
\]
\[
= 1 + \alpha \gamma(t_1) T P y(t_1)
\]

with \( ||y(t)|| \geq \sup_{\mathbb{R}^n} \epsilon(x, \epsilon_1(X_2)) \). The proof of Proposition 3 involves two steps. First, we show, if \( q \) is large enough, any solution that crosses at some time the set \( X_2 = 0 \) (the same as the set \( Y_2 = 0 \), in view of Lemma 4) remains in this set ever after. For the first state, the first state variable satisfies, after a finite time, the equation \( \dot{x}_1 = -k_1 x_1 \sin^2 t + \sin(x_1) \sin(t) \), and this implies \( x_1(t) \) asymptotically converges to zero (see [10], for example). Therefore, the only solutions that may not converge to zero are those that never cross the set \( X_2 = 0 \). The second step of the proof thus consists in showing any of these solutions asymptotically converges to zero. Exponential stability then results because the controlled system is homogeneous of degree zero with respect to the dilation \( \delta_\lambda(\lambda, X_2) \).

Step 1: If \( X_2(t) \neq 0 \), the derivative of \( Y_2(X_2) \) at time \( t \) is well defined and such that
\[
Y_2 = \frac{u_1}{P_{p, q}}(y_1, \cdots, y_n, -\sum_{i=2}^{n+1} \text{sign}(u_1) y_{i+1} - u_1 y_1)
\]

with \( y_{i+1} = (x_i + 1/p_{p, q}^{-1}) \) denoting the \( i \)th component of the vector \( Y_2 \), and \( B = -\text{diag}(n - 2, n - 3, \ldots, 1, 0) \). In view of (18) and Lemma 5, we can rewrite (31) as
\[
Y_2 = \frac{u_1}{P_{p, q}}(A_{\text{sign}(u_1)} + \epsilon_1, \epsilon_1)(Y_2)B)Y_2.
\]

Let us assume \( X_2(t) \neq 0 \) on some interval \([t_0, t_1] \). The function \( u_1/p_{p, q} \) is well defined on this interval, moreover, in view of the expression of the control \( u_1(x, t) \)
\[
\gamma(t) \leq \frac{u_1}{P_{p, q}}(X_2(t)) \geq \gamma_0(t) \leq \delta_{k+1} |\sin t|.
\]

with the sign of \( u_1 \) being the opposite of the sign of \( \sin t \). The sign of \( u_1 \) thus changes periodically.

Because both \( A_{+1} \) and \( A_{-1} \) are Hurwitz-stable—from (23), the change of coordinates \( Y_2 \rightarrow \mathcal{Y}_2 \) transforms \( A_{+1} \) into \( A_{-1} = P_{p, q} \), s.p.d matrices \( P_{p, q} \) and \( Q_{p, q} \) exist such that, for each value of \( \text{sign}(u_1) \), \( P_{p, q} A_{\text{sign}(u_1)} + A_{\text{sign}(u_1)} P_{p, q} \rightleftharpoons Q_{p, q} \).

This inequality implies the function \( V_{p, q}(Y_2) \rightleftharpoons V_{p, q}(Y_2) \). For example, \( \delta_{k+1} \neq 0 \) for any time \( t \), \( X_2(t) \neq 0 \) for any time \( t \). Suppose on the contrary \( X_2(t_2) \neq 0 \) for any time \( t_2 \). For example, assume \( X_2(t_2) \neq 0 \). By continuity of \( X_2(t) \), \( X_2(t_1) \) with \( t_1 \leq t_2 \) exists such that
\[
0 = X_2(t_1) < X_2(t_2), \text{ for } t 
\]

By possibly decreasing the value of \( t_2 \), we can assume \( u_1 \) is of constant sign on \((t_1, t_2)\). We deduce from (34),
\[
V_{p, q}(Y_2(t_2)) \geq V_{p, q}(Y_2(t_2)).
\]

This is clearly in contradiction with the fact that the function \( V_{p, q}(Y_2) \rightleftharpoons V_{p, q}(Y_2) \). For example, assume \( X_2(t_2) \neq 0 \). By continuity of \( X_2(t) \), \( X_2(t_1) \) with \( t_1 \leq t_2 \) exists such that
\[
0 = X_2(t_1) < X_2(t_2), \text{ for } t 
\]

with \( \alpha \) an even integer, and \( k_{-1} \) any odd integer. Property i) of Proposition 3 follows from (35) with \( \alpha = \max(1 - \alpha_{+1}, 1 - \alpha_{-1}) \) \((0, 1)\), provided \( \gamma \geq \max(q_{+1}, q_{-1}) \). This property, plus the proof of Step 1, clearly imply \( Y_2(t) \) asymptotically converges to zero. The convergence of \( x_1(t) \) to zero then easily follows from the first system's
equation \( \dot{x}_1 = u_1 \) and the expression of the control \( u_1(x, t) \) (see [10], for example).

REFERENCES


Fixed Zeros of Decentralized Control Systems

Konur A. Üneylioğlu, Ümit Özugüner, and A. Bülent Özugüler

Abstract—This paper considers the notion of decentralized fixed zeros for linear, time-invariant, finite-dimensional systems. For an \( N \)-channel plant that is free of unstable decentralized fixed modes, an unstable decentralized fixed zero of Channel \( i (1 \leq i \leq N) \) is defined as an element of the closed right half-plane, which remains as a blocking zero of that channel under the application of every set of \( N = 1 \) controllers around the other channels, which make the resulting single-channel system stabilizable and detectable. This paper gives a complete characterization of unstable decentralized fixed zeros in terms of system-invariant zeros.

Index Terms—Decentralized control, fixed zeros, linear systems, stabilization.

I. INTRODUCTION

The main objective of this paper is to give a definition and a characterization of unstable decentralized fixed zeros of a linear, time-invariant, finite-dimensional plant.

Consider the \( N \)-channel decentralized plant \( Z \) in Fig. 1, which is assumed to be free of unstable decentralized fixed modes [13]. Let \( i \in \{1, \ldots, N\} \) be fixed. Assume, without loss of generality, \( i = 1 \). Let the closed-loop transfer matrix between \( y_1 \) and \( y_i \) be denoted by \( \tilde{Z}_{11} \), where the dependence of \( Z_{11} \) on the controllers \( Z_{22}, \ldots, Z_{NN} \) is suppressed for simplicity.

An unstable decentralized fixed zero of Channel 1 is defined as an element of the closed right half-plane, which remains as a blocking zero \[2], [3]\ of \( \tilde{Z}_{11} \) for the application of every collection of \( N = 1 \) local controllers \( Z_{22}, \ldots, Z_{NN} \), which yield that the partially closed-loop system is stabilizable and detectable around Channel 1.

Decentralized fixed zeros deserve attention because of the performance limitations they impose on various sensitivity minimization problems, which can be explained by referring to Figs. 2 and 3, where \( Z_{11}, \ldots, Z_{NN} \) are local controllers to achieve two objectives: 1) closed-loop stability and 2) minimization of the \( H_\infty \) norm of the transfer matrix between \( w \) and \( z \) in Fig. 2.

In Fig. 2, the signal \( w \) is a noise affecting the first channel observation. In Fig. 3, the signal \( r \) is a reference signal to be tracked by the first channel output \( y_1 \). The transfer matrix between \( r \) and the error signal \( e \) is identical to the one between \( w \) and \( z \) in Fig. 2. It is easy to compute the transfer matrix between \( w \) and \( z \) (or the sensitivity function around Channel 1) equals \( S := (I + \tilde{Z}_{11} Z_{11})^{-1} \). Let \( Z_{11}, Z_{22}, \ldots, Z_{NN} \) be any collection of local controllers satisfying the closed-loop stability. From [8, Remark and Theorem 3.2] (see also Lemma 2 in the next section), the controllers \( Z_{22}, \ldots, Z_{NN} \) yield that the closed-loop system is stabilizable and detectable around Channel 1 in the partially closed-loop configuration of Fig. 1. Then, observe, at each unstable decentralized fixed zero \( \eta_0 \) of Channel 1, \( \|S(\eta_0)\|_\infty = 1 \), regardless of the controllers chosen. In other words, 1) the sensitivity of the closed-loop