ON THE HUGHES MODELS OF CROWD DYNAMICS

Optimization of the evacuation

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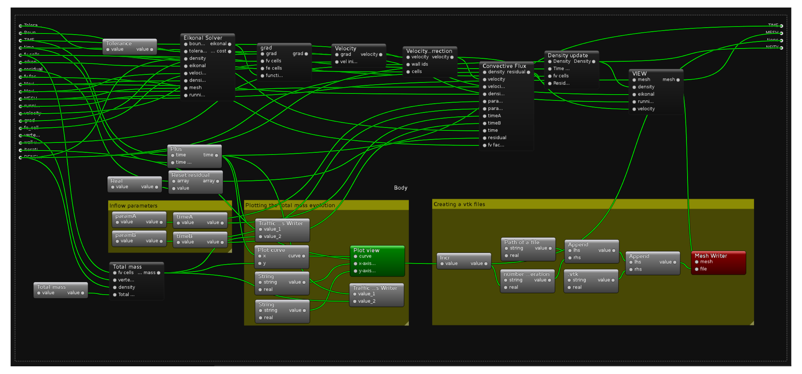
THE NUM3SIS PROJECT

Facing the increase of simulation use, computational facilities and numerical methods in industries and in research and development, a new **modular architecture** based on a **platform** and **plugins** has been developed allowing:

- **visual programming**
- **interactive computation/visualization** on screen or virtual reality facilities



Example: num3sis composition of the "while loop" for the Hughes model

MATHEMATICAL MODEL OF A PEDESTRIAN FLOW

Domain: two-dimensional walking facility Ω with exits within which pedestrians can freely move in any direction

Model: Hughes model [1] consisting of a nonlinear conservation law in which the flow flux is implicitly dependent on the density through the Eikonal equation

$$\rho_t + div\left(\rho\vec{V}(\rho)\right) = 0$$

where

$\rho = \rho(x, y, t)$ - density of pedestrians

$\vec{V}(\rho) = v(\rho)\vec{N}$ - density dependent velocity of pedestrians

Speed of pedestrians: $v(\rho) = v_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right)$

- pedestrians move with the maximal speed $v_{\max}$ in the absence of other pedestrians
- there exists a maximal density of the crowd at which pedestrians don't move

Direction of the motion: $\vec{N} = -\frac{\nabla\phi}{|\nabla\phi|}$ where the potential ϕ models the common sense of the task and is given by the Eikonal equation

$$\begin{cases} |\nabla\phi| = \frac{1}{v(\rho)} \\ \phi(x, y, t) = 0 \text{ on exits} \end{cases}.$$

- pedestrians tend to minimize their estimated travel time to the exit
- pedestrians temper their estimated travel time by avoiding extremely high densities

SPACE DISCRETIZATION

Mixed finite-volume/element approximation over unstructured mesh on a bounded domain $\Omega \subset \mathbb{R}^2$

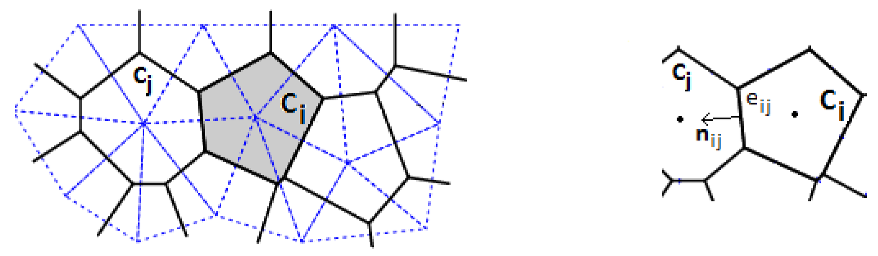


Illustration of Voronoi type vertex centered cells

Numerical scheme:

- **Conservation law:**

$$\frac{d}{dt}\rho_i = -\frac{1}{|C_i|}\sum_{j\in J(i)} |e_{ij}|\mathcal{F}\left(\rho_i, \rho_j, \vec{n}_{ij}\right)$$

where $\rho_i = \frac{1}{|C_i|}\int_{C_i}\rho(x, y, t)dxdy$ and $\mathcal{F}\left(\rho_i, \rho_j, \vec{n}_{ij}\right)$ is a numerical flux function computed in a 1d mode across the facet e_{ij} between the cells C_i and C_j along the normal direction $\vec{n_{ij}}$

$$\textbf{Lax-Friedrichs flux: } \mathcal{F}\left(\rho_i, \rho_j, \vec{n}_{ij}\right) = \frac{1}{2}\left[\vec{F}(\rho_i)\cdot\vec{n}_{ij} + \vec{F}(\rho_j)\cdot\vec{n}_{ij} - \alpha(\rho_j - \rho_i)\right]$$

where $F(\rho)$ is the analytical flux and $\alpha = \max_{\rho_i,\rho_j}\frac{d}{d\rho}(\rho v(\rho))$

- **Eikonal equation:** finite element discretization based on a solution of simplified localized Dirichlet problem solved by variational principle [2] and Gauss-Seidel iterative method to solve the resulting system on of nonlinear equations
- **Gradient:**

$$\nabla\phi_i = \frac{1}{|C_i|}\sum_{T_{ij}\in C_i}\frac{|T_{ij}|}{3}\sum_{k\in T_{ij}}\phi_k\nabla P_k|_{T_{ij}}$$

T_{ij} - is a finite element which has the node i as vertex

P_k - is a P_1 basis function associated with the vertex k

NUMERICAL PROCEDURE

1. Solve the Eikonal equation to obtain ϕ^n
2. Calculate the gradient $\nabla\phi^n$ and the residuals $R_i^n = \frac{1}{|C_i|}\sum_{j\in J(i)}|e_{ij}|\mathcal{F}\left(\rho_i^n, \rho_j^n, \vec{n}_{ij}\right)$
3. Use an explicit first-order time integration to obtain

$$\rho_i^{n+1} = \rho_i^n - \Delta t R_i^n,$$

with $\Delta t = 0.5\frac{\min_i|C_i|}{v_{\max}}$

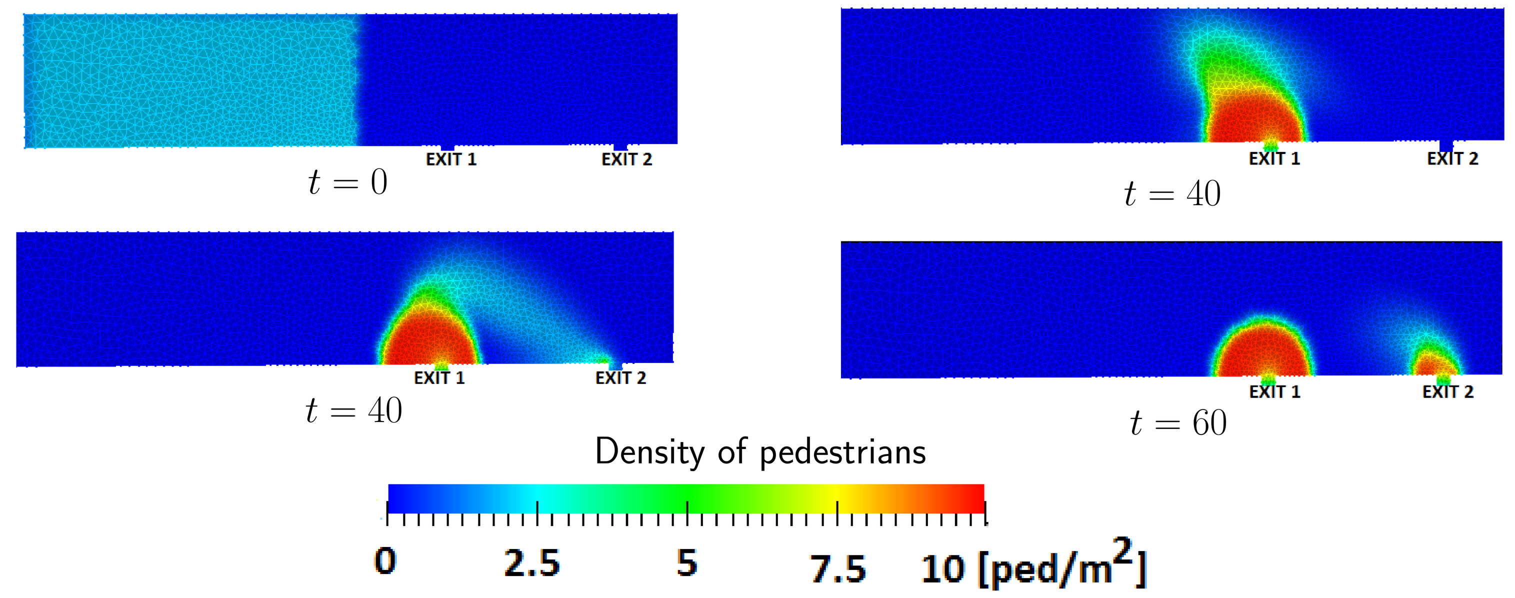
REFERENCES

1. R. L. Hughes. A continuum theory for the flow of pedestrians. *Transportation Research Part B: Methodological*, 36(6):507-535, 2002
2. F. Bornemann, Ch. Rasch. Finite-element discretization of static Hamilton-Jacobi equations based on a local variational principle. *Computing and Visualization in Science*, 9(2):57-69, 2006

SIMULATION: TWO EXITS PROBLEM

We consider a domain in the form of a platform $\Omega = 20m \times 100m$ with two exits and a homogeneously distributed group of pedestrians near one of them

- Maximal speed of pedestrians: $v_{\max} = 2$ m/s
- Maximal density of pedestrians: $\rho_{\max} = 10$ ped/m^2
- Size of the exits: $d_{exit} = 2$ m

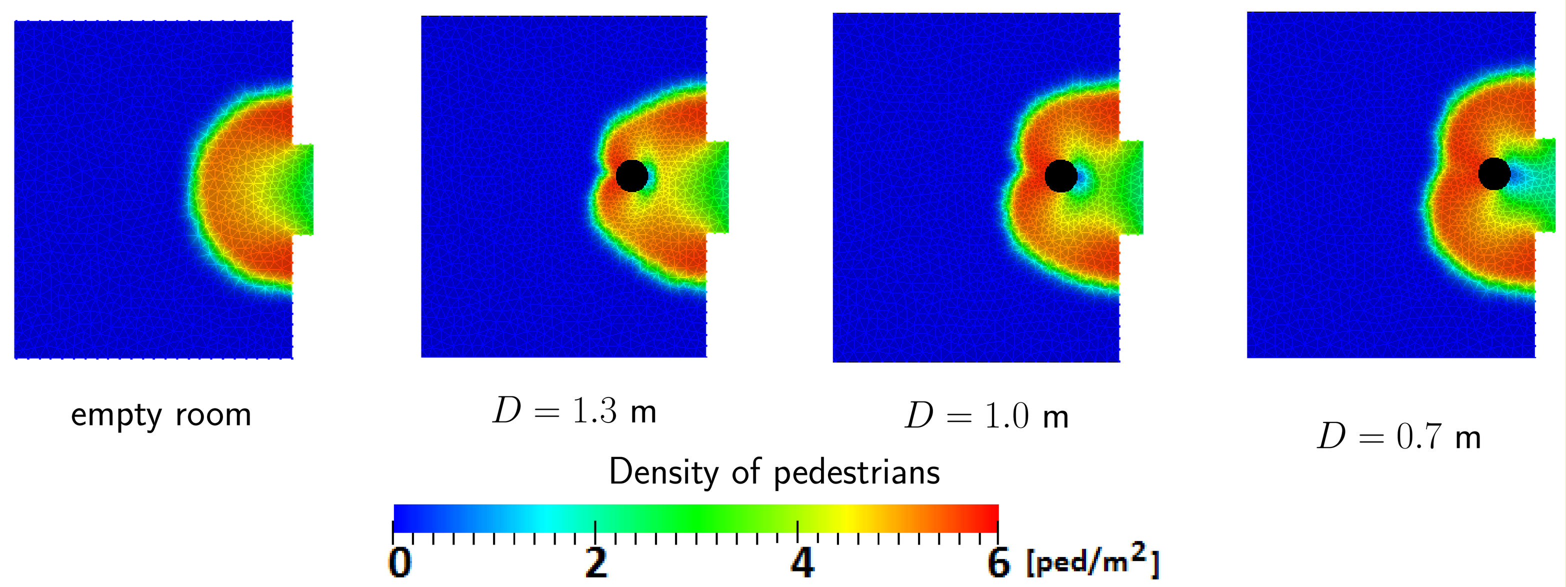


Density of pedestrians

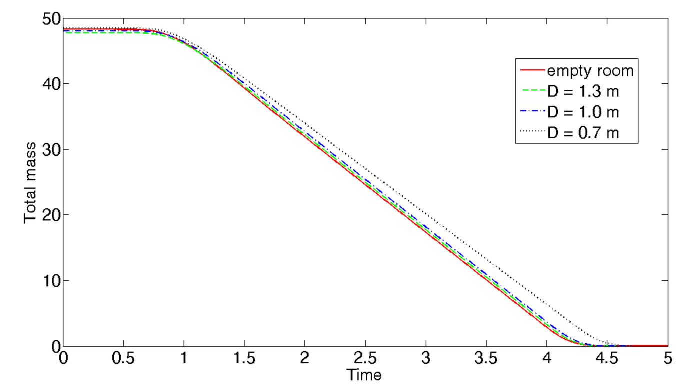
SIMULATION: THE EFFECT OF A COLUMN IN EVACUATION

We consider a room with one exit and a homogeneously distributed group of pedestrians inside who want to leave the facility as soon as possible. We analyze an effect of an obstacle placed in front of the exit (method known in granular materials to increase the flow through a whole)

- Maximal speed of pedestrians: $v_{\max} = 4$ m/s
- Maximal density of pedestrians: $\rho_{\max} = 6$ ped/m^2
- Size of the exit: $d_{exit} = 1.8$ m
- Radius of the column: $r = 0.3$ m
- D - distance between the center of the column and the center of the exit



Density of pedestrians



Time evolution of the total mass of pedestrians still remaining in the facility