Workshop

BIS 2012

Paris, May 21-22, 2012

ORESTE

Optimal REroute Strategies for Traffic managEment

Paola GOATIN (INRIA) & **Aude HOFLEITNER** (UC Berkeley)

May 22, 2012

ORESTE is an associated team between OPALE project-team at INRIA and the Mobile Millennium / Integrated Corridor Management (ICM) team at UC Berkeley focused on traffic management. With this project, we aim at processing GPS traffic data with up-to-date mathematical techniques to optimize traffic flows in corridors. More precisely, we seek for optimal reroute strategies to reduce freeway congestion employing the unused capacity of the secondary network. The project uses macroscopic traffic flow models and a discrete approach to solve the corresponding optimal control problems. The overall goal is to provide constructive results that can be implemented in practice.

SUMMARY

1. Macroscopic models for road traffic
2. Finite volume schemes
3. Estimation: the *Mobile Millennium* project
4. Network optimization: *Integrated Corridor Management*
5. Perspectives

1
Macroscopic models for road traffic

First-order models

Lighthill-Whitham-Richards '56:

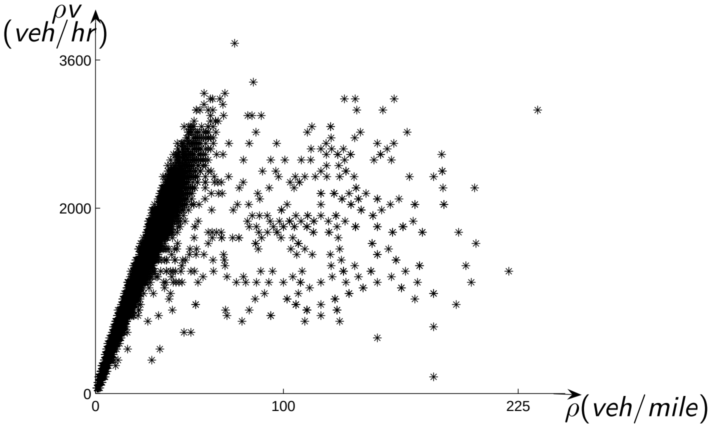
PDE for mass conservation

$$\partial_t \rho + \partial_x(\rho v) = 0, \qquad v = v(\rho)$$

$t > 0$	time	$\rho = \rho(t, x)$	vehicle density
$x \in \mathbb{R}$	space	$v = v(t, x)$	mean velocity

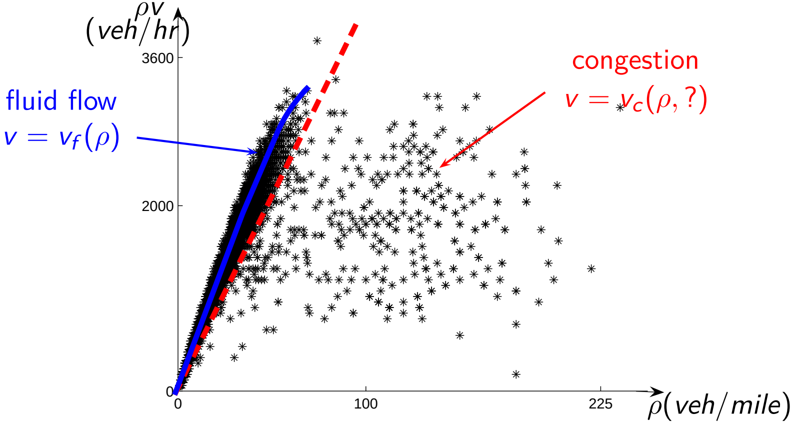
$\Longrightarrow$ number of vehicles is conserved!

Experimental data



Viale Muro Torto, Roma

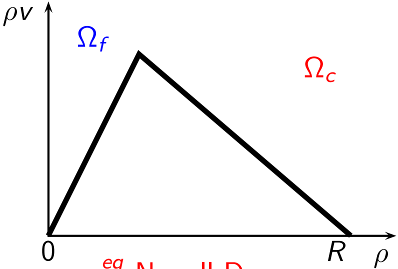
Experimental data



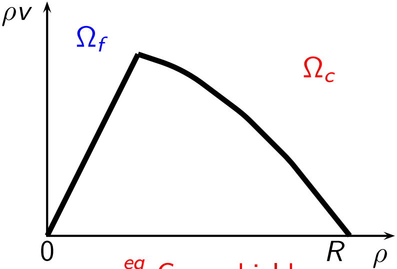
Viale Muro Torto, Roma

Phase transitions

congestion = perturbation from equilibrium



v_c^{eq} Newell-Daganzo



v_c^{eq} Greenshields

Fluid flow: in Ω_f

$$\partial_t \rho + \partial_x(\rho v_f) = 0$$

$$v_f(\rho) = V$$

Congestion: in Ω_c

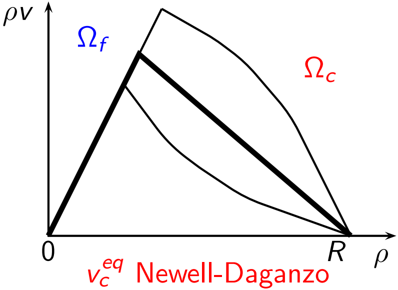
$$\begin{cases} \partial_t \rho + \partial_x(\rho v_c) = 0 \\ \end{cases}$$

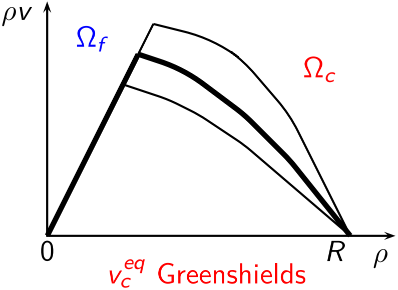
$$v_c(\rho, q) = v_c^{eq}(\rho)$$

(Blandin - Work - Goatin - Piccoli - Bayen '11)

Phase transitions

congestion = perturbation from equilibrium





Fluid flow: in Ω_f

$$\partial_t \rho + \partial_x(\rho v_f) = 0$$

$$v_f(\rho) = V$$

Congestion: in Ω_c

$$\begin{cases} \partial_t \rho + \partial_x(\rho v_c) = 0 \\ \partial_t q + \partial_x(q v_c) = 0 \end{cases}$$

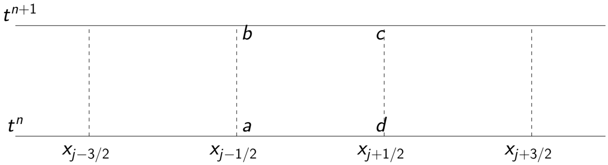
$$v_c(\rho, q) = v_c^{eq}(\rho)(1+q)$$

(Blandin - Work - Goatin - Piccoli - Bayen '11)

2
Finite volume schemes

Classical Godunov scheme

Conservation on rectangular cells:

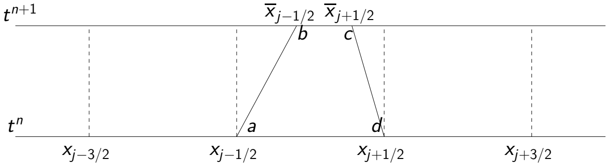


$$u_j^{n+1} = u_j^n - \frac{\Delta t}{\Delta x}\left(f_{j+1/2}^n - f_{j-1/2}^n\right) = \frac{1}{\Delta x}\int_{x_{j-1/2}}^{x_{j+1/2}} u_\nu(t^{n+1}-, x)\,dx$$

$\Omega_f \cup \Omega_c$ non convex $\Longrightarrow u_j^{n+1} \notin \Omega_f \cup \Omega_c$ in general

Modified Godunov scheme

Conservation on modified cells:



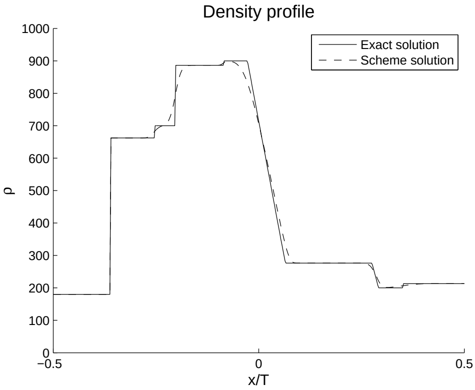
$$\overline{\mathbf{u}}_j^{n+1} = \frac{\Delta x}{\Delta x_j}\mathbf{u}_j^n - \frac{\Delta t}{\Delta x_j}\left(\overline{\mathbf{f}}_{j+1/2}^{n,-} - \overline{\mathbf{f}}_{j-1/2}^{n,+}\right) = \frac{1}{\Delta x}\int_{\overline{x}_{j-1/2}}^{\overline{x}_{j+1/2}} u_\nu(t^{n+1}-, x)\,dx$$

with the numerical fluxes

$$\overline{\mathbf{f}}_{j+1/2}^{n,\pm} = \mathbf{f}(\mathbf{v_r}(\sigma_{j+1/2}^{\pm}; \mathbf{u}_j^n, \mathbf{u}_{j+1}^n)) - \sigma_{j+1/2}\mathbf{v_r}(\sigma_{j+1/2}^{\pm}; \mathbf{v}_j^n, \mathbf{v}_{j+1}^n)$$

$+$ sampling

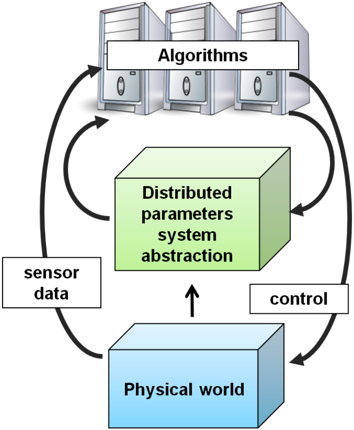
Modified Godunov scheme



3
Estimation: the *Mobile Millennium* project

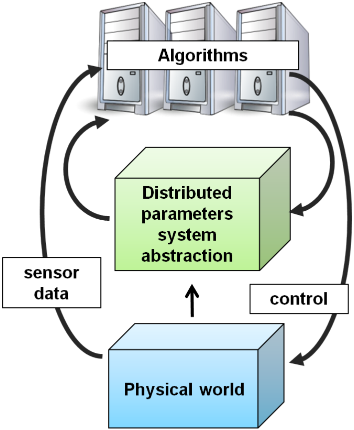
Estimation/control of large scale physical systems

- Model of the physical world
  - Partial differential equation (PDE)
  - Statistical model



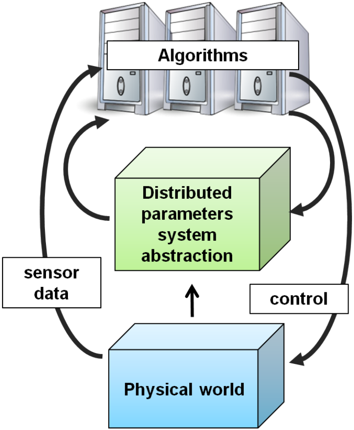
Estimation/control of large scale physical systems

- Model of the physical world
  - Partial differential equation (PDE)
  - Statistical model
- Integration of sensor data in the model
  - Numerical algorithms
  - Inference



Estimation/control of large scale physical systems

- Model of the physical world
  - Partial differential equation (PDE)
  - Statistical model
- Integration of sensor data in the model
  - Numerical algorithms
  - Inference
- Closing the loop on the physical system
  - Network optimization
  - Control



Estimation/control of large scale physical systems

Model
Mathematical abstraction of the dynamics

$+$

Data
(Indirect) measurements of the state of the system

$\Longrightarrow$

Estimation and control
Best knowledge given the data.
Adaptive control strategies

Available data

GPS measurements sent by a fleet of 500 probe vehicles (source: www.cabspotting.org)

Data

(Indirect) measurements of the state of the system

Available data

Map matching and path reconstruction between successive GPS measurements.

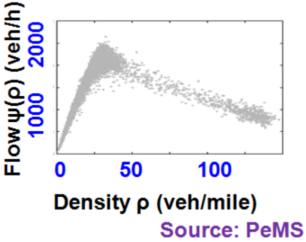
Data

(Indirect) measurements of the state of the system

Physical system

Model

Mathematical abstraction of how the system evolves



The function ψ is called "flux function" or fundamental diagram.

Model characteristics

- Hydrodynamic and horizontal queuing theory

$$\frac{\partial \rho}{\partial t} + \frac{\partial \psi(\rho)}{\partial x} = 0$$

Physical system

Model

Mathematical abstraction of how the system evolves

Model characteristics

- Hydrodynamic and horizontal queuing theory

$$\frac{\partial \rho}{\partial t} + \frac{\partial \psi(\rho)}{\partial x} = 0$$

- On arterials: intersections with unknown signal parameters (signal locations are known)

Physical system

Model

Mathematical abstraction of how the system evolves

Model characteristics

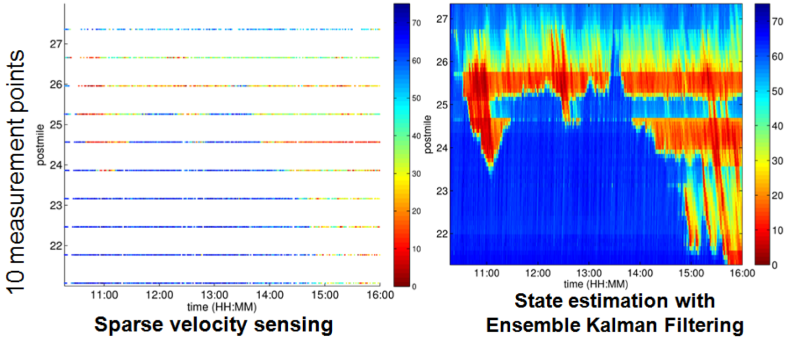
- Hydrodynamic and horizontal queuing theory

$$\frac{\partial \rho}{\partial t} + \frac{\partial \psi(\rho)}{\partial x} = 0$$

- On arterials: intersections with unknown signal parameters (signal locations are known)
- Errors and noise: model inaccuracies, pedestrians, driving behavior,...

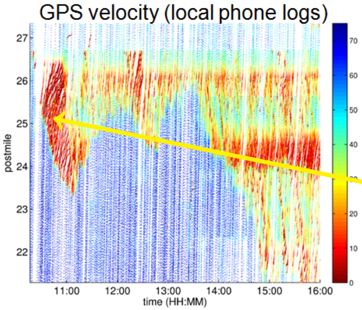
Estimation: Highway traffic

Mobile Century Experiment



Estimation: Highway traffic

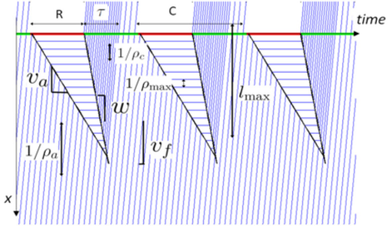
Real-time validation



(2-5% of traffic)

Estimation: arterial traffic

Deterministic solution of the PDE: horizontal queues



Assumptions

- Triangular fundamental diagram
- Uniform arrivals
- Periodic dynamic (cycle time)

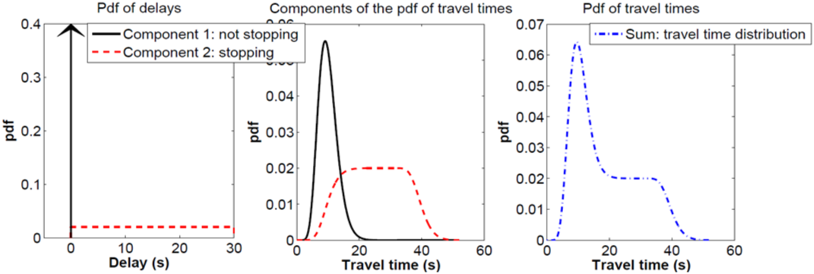
Derivation of the pdf of travel times:

travel time = delay + free flow travel time

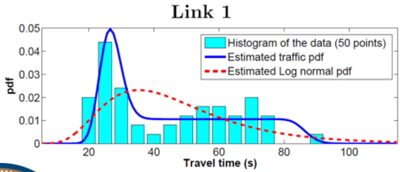
- Pdf of delay: Derived from the solution of the PDE:
  - Non stopping vehicle: zero delay
  - Stopping vehicles: uniform delay
- Pdf of free flow travel times: Model of driving behavior

Estimation: arterial traffic

Probability distribution of travel times



Experimental validation:



Network setting:

- Conservation of vehicles at intersections
- Learning and inference with a Dynamic Bayesian Network

4
Network optimization:
Integrated Corridor Management

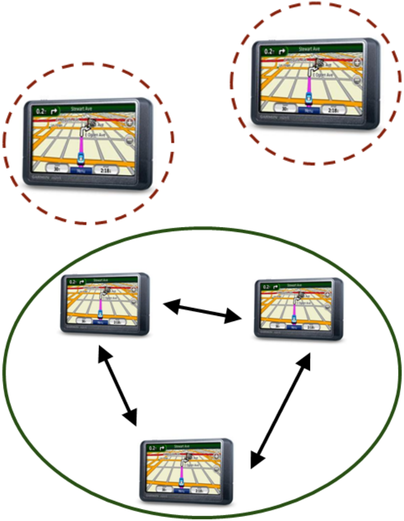
Routing Users "Altruistically"

Today's Routing Algorithms:

- Route individuals optimally
- Leads to greedy, inefficient traffic conditions

Societal Routing:

- Routing algorithms optimize over all users
- At the expense of some users, some of the time



Travel-time reduction via flow reroutes

Save total travel-time by rerouting a fraction of the users

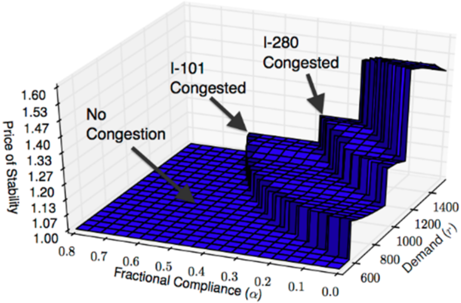
- Re-route some drivers from the shorter congested route to the longer uncongested route
- Incentives and games
- Overall social gain

Fractional Compliance: Stackelberg Games

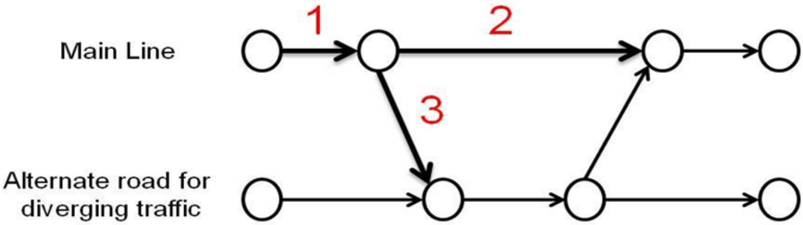
Price of Anarchy: Nash eq. Cost/Social optimum cost

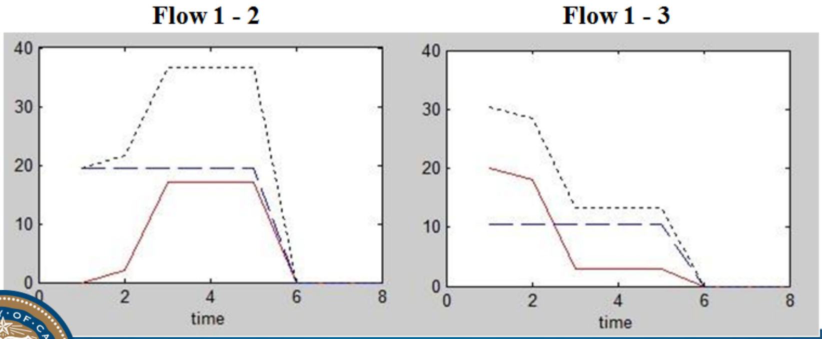
Stackelberg games:

- Optimally route social drivers?
- In the presence of greedy drivers?



Simple example: SO DTA with partial compliance





5
Perspectives

Project plan

Optimal reroute strategies for traffic management:

- Formulation of a PDE constrained optimization problem, with an objective function that encodes network efficiency.
- Solution algorithms for the relaxed (convex) optimization problem.
- Construction of solutions to the original problem from the solution of the relaxed problem.
- Implementation on sub-networks of the corridor of interest, simulations and validation.

Thanks for your attention