Optimization based control of networks of discretized PDEs: application to traffic engineering

Paola Goatin (Inria), Nikolaos Bekiaris-Liberis (UC Berkeley)
Maria Laura Delle Monache (Inria), Jack Reilly (UCB), Samitha Samaranayake (UCB), Walid Kirchene (UCB), Alexandre Bayen (UCB)

Workshop BIS 2014

Paris, June 17, 2014

Outline

Outline

1. Associated Team ORESTE
2. A macroscopic model for ramp metering
3. Discretized system
4. Quick review of the adjoint method
5. Numerical results
6. Application to MPC-based ramp metering control
7. Application to optimal re-routing
8. Perspectives

ORESTE (Optimal RErouting Strategies for Traffic mangEment) is an associated team between Inria project-team OPALE and the Connected Corridors project at UC Berkeley.

- Paola Goatin (PI)
- Maria Laura Delle Monache

- Alexandre Bayen (PI)
- Jack Reilly
- Samitha Samaranayake
- Walid Krichene
- Nikolaos Bekiaris-Liberis

Other fundings:

- ERC Starting Grant TRAM3
- France Berkeley Fund

Traffic Modeling and Management:
Trends and Perspectives

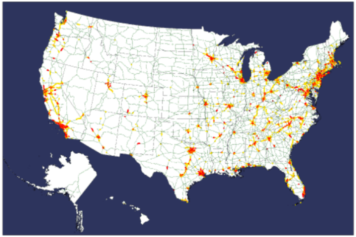
Inria Sophia Antipolis, March 20-22, 2013

- 14 invited speakers
- 10 contributed talks
- ~40 participants

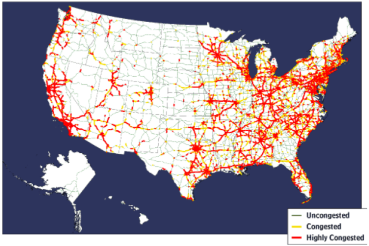
Road traffic congestion

- Congestion in the US in 2011 [Urban Mobility Report, 2012]
  - $121 billions in wasted time and fuel (1% of GDP)
  - 5.5 billions hours of delay
  - 56 billions pounds of additional C02 emissions

- Federal Highway Administration forecast



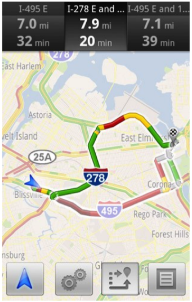
2002



2035

- 37% of total delay occours outside the peak hours

- Spread of smart phones and GPS navigation devices
  - US smartphone penetration is at 65%
- Real-time information
  - accurate live information (<5 minute delay)
  - high granularity (>1 minute update frequency)
- Adaptive control
  - stochastic routing
  - dynamic re-routing



ORESTE project goals

- Optimize traffic flow in corridors
  - ramp metering
  - re-routing strategies
- Modeling approach:
  - macroscopic traffic flow models
  - discrete adjoint method for gradient computation

Other ramp-metering models

- ALINEA
  [Papageorgiou - Hadj-Salem - Blosseville, TRB, 1991]
  - Closed-loop
  - Only uses state estimation (no forward-sim model)
  - Local policy (single ramp), extensions (HERO) for multi-ramp using heuristics
- Continuous adjoint approach
  [Jacquet - Canudas de Wit - Koenig, Proc IFAC, 2005]
  - Derive and discretize
  - Onramps modeled as source terms
  - Control the number of car entering the highway
- Link-node CTM with ramp metering problem as LP
  [Muralidharana - Horowitz, ACC, 2012]
  - For $n \times m$ junctions
  - Priorities are proportional to demand (junction solver not "self-similar")
  - Relaxation of the junction model

Outline

Traffic flow models

Three possible approaches

- Microscopic Models
  - ODE system: each single car is modeled
  - Large number of parameters
  - Numerical simulations
- Kinetic Models
  - Distribution functions of the macroscopic quantities
  - Boltzmann-like equations
- Macroscopic Models
  - Traffic flow modeled as a fluid, i.e., PDEs from fluid dynamics
  - Few parameters
  - Analytical solutions
  - Suitable for optimization and control problems

LWR model

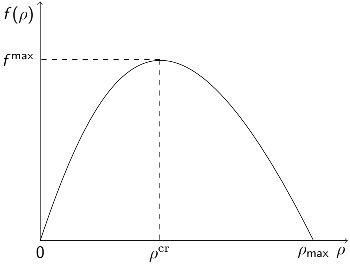
[Lighthill-Whitham '55, Richards '56]

Non-linear transport equation: PDE for mass conservation

$$\partial_t \rho + \partial_x f(\rho) = 0 \qquad x \in \mathbb{R}, t > 0$$

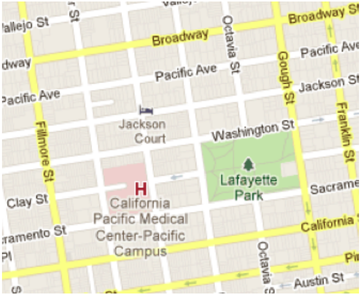
- $\rho \in [0, \rho_{\max}]$ mean traffic density
- $f(\rho) = \rho v(\rho)$ flux function

Empirical flux-density relation: fundamental diagram

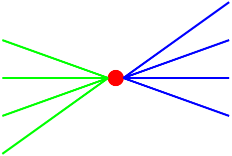


Greenshields '35

Extension to networks



$\Longrightarrow$



m incoming arcs

n outgoing arcs

junction

- LWR on arcs
- Maximization of flux through the junction subject to:
  - Mass conservation
  - Distribution/priority parameters

[Coclite-Garavello-Piccoli, SIAM J. Math. Anal., 2005]

- Two incoming roads:
  - Upstream mainline $I_1 =]-\infty, 0[$
  - Onramp R_1
- Two outgoing roads:
  - Downstream mainline $I_2 =]0, +\infty[$
  - Offramp R_2

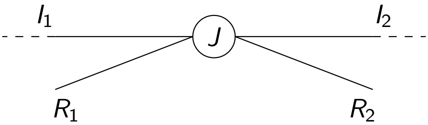


Figure : Junction modeled

A model for ramp metering

Coupled PDE-ODE model:

- Classical LWR on each mainline I_1, I_2

$$\partial_t \rho + \partial_x f(\rho) = 0, \quad (t, x) \in \mathbb{R}^+ \times I_i,$$

- Dynamics of the onramp described by an ODE (buffer)

$$\frac{dl(t)}{dt} = F_{\mathrm{in}}(t) - \gamma_{\mathrm{r1}}(t), \quad t \in \mathbb{R}^+,$$

  - $l(t) \in [0, +\infty[$ length of the onramp queue
  - $F_{\mathrm{in}}(t)$ flux entering the onramp
  - $\gamma_{\mathrm{r1}}(t)$ flux leaving the onramp (through the junction)

- Coupled at junction

A model for ramp metering

Coupled PDE-ODE model:

- Classical LWR on each mainline I_1, I_2

$$\partial_t \rho + \partial_x f(\rho) = 0, \quad (t,x) \in \mathbb{R}^+ \times I_i,$$

- Dynamics of the onramp described by an ODE (buffer)

$$\frac{dl(t)}{dt} = F_{\mathrm{in}}(t) - \gamma_{\mathrm{r1}}(t), \quad t \in \mathbb{R}^+,$$

  - $l(t) \in [0, +\infty[$ length of the onramp queue
  - $F_{\mathrm{in}}(t)$ flux entering the onramp
  - $\gamma_{\mathrm{r1}}(t)$ flux leaving the onramp (through the junction)

- Coupled at junction

A model for ramp metering

Coupled PDE-ODE model:

- Classical LWR on each mainline I_1, I_2

$$\partial_t \rho + \partial_x f(\rho) = 0, \quad (t, x) \in \mathbb{R}^+ \times I_i,$$

- Dynamics of the onramp described by an ODE (buffer)

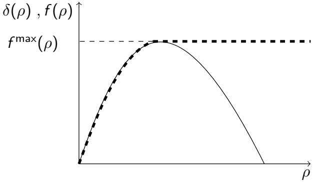
$$\frac{dl(t)}{dt} = F_{\mathrm{in}}(t) - \gamma_{\mathrm{r1}}(t), \quad t \in \mathbb{R}^+,$$

  - $l(t) \in [0, +\infty[$ length of the onramp queue
  - $F_{\mathrm{in}}(t)$ flux entering the onramp
  - $\gamma_{\mathrm{r1}}(t)$ flux leaving the onramp (through the junction)

- Coupled at junction

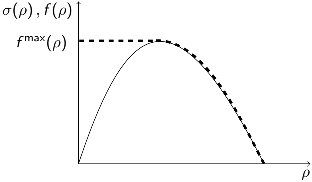
Junction solver: Demand & Supply

$$\delta(\rho_1) = \begin{cases} f(\rho_1) & \text{if } 0 \leq \rho_1 < \rho^{\mathrm{cr}}, \\ f^{\max} & \text{if } \rho^{\mathrm{cr}} \leq \rho_1 \leq 1, \end{cases}$$



$$d(F_{\mathrm{in}}, l) = \begin{cases} \gamma_{\mathrm{r1}}^{\max} & \text{if } l(t) > 0, \\ \min\left(F_{\mathrm{in}}(t), \gamma_{\mathrm{r1}}^{\max}\right) & \text{if } l(t) = 0, \end{cases}$$

$$\sigma(\rho_2) = \begin{cases} f^{\max} & \text{if } 0 \leq \rho_2 \leq \rho^{\mathrm{cr}}, \\ f(\rho_2) & \text{if } \rho^{\mathrm{cr}} < \rho_2 \leq 1, \end{cases}$$



Junction solver: flux maximization

1. Mass conservation: $f(\rho_1(t,0-)) + \gamma_{\mathrm{r1}}(t) = f(\rho_2(t,0+)) + \gamma_{\mathrm{r2}}(t)$
2. $f(\rho_2(t,0+))$ maximum subject to 1 and

$$f(\rho_2(t,0+)) = \min\Big((1-\beta)\delta(\rho_1(t,0-)) + d(F_{\mathrm{in}}(t), l(t)), \sigma(\rho_2(t,0+))\Big)$$

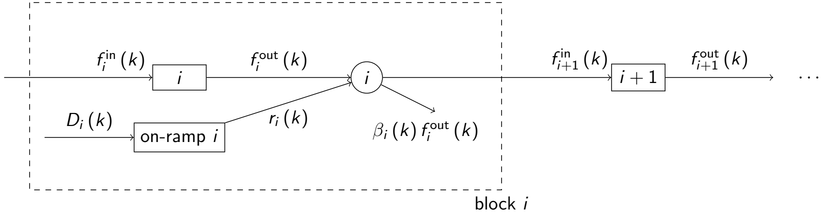
3. **Right of way parameter** $P \in]0,1[$ to ensure uniqueness:

$$f(\rho_2(t,0+)) = P\, f_1(\rho(t,0-)) + (1-P)\,\gamma_{r1}$$

4. Offramp treated as a sink (infinite capacity)
5. No flux from the onramp to the offramp is allowed

Outline

Ramp metering discretized system $H(\vec{\rho}, \vec{u}) = 0$



- Dynamics and junctions solutions based on the model described earlier
- Piecewise affine system
- Control parameter u_i is a constraint on the ramp inflow $r_i(k)$

Ramp metering discretized system $H(\vec{\rho}, \vec{u}) = 0$

Lower triangular forward system $H(\vec{\rho}, \vec{u}) = 0$:

- $h_{i,k} = \rho_i(k) - \rho_i(k-1) - \{\text{flux update equations for cell i and time step } k\}$
- H system of concatenated $h_{i,k}$
- H lower triangular
- Very efficient to solve $H^T x = b$

Ramp metering discretized system $H(\vec{\rho}, \vec{u}) = 0$

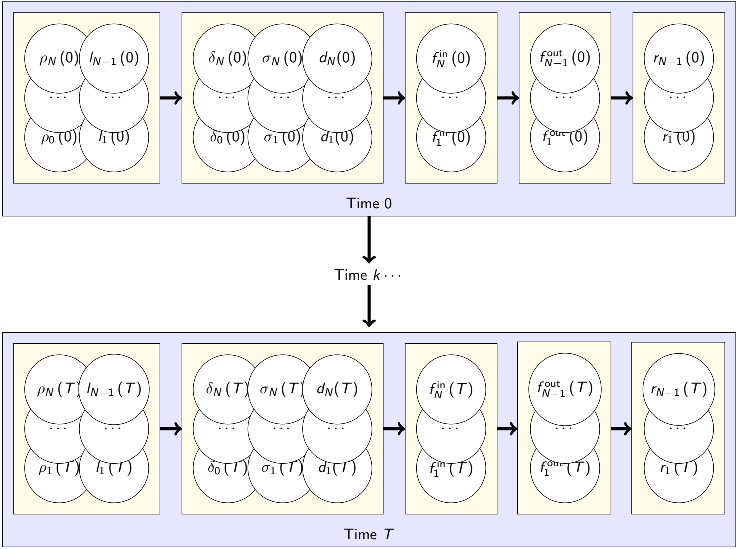


Figure : Dependency diagram of the variables in the system.

Lower triangular forward system $H(\vec{\rho}, \vec{u}) = 0$

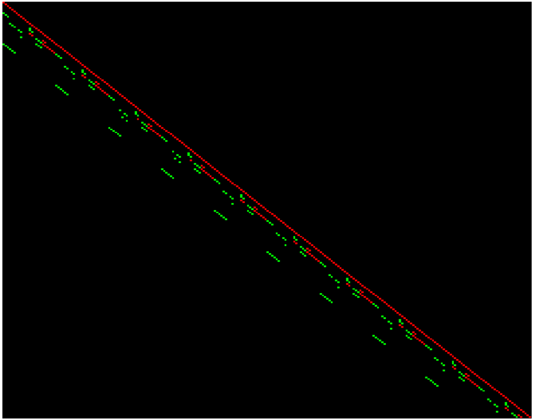


Figure : $\frac{\partial H}{\partial \vec{\rho}}$ matrix

Outline

Optimization of a PDE-constrained system

Optimization problem

$$\begin{array}{ll} \text{minimize}_{\vec{u}\in\mathcal{U}} & J(\vec{\rho},\vec{u}) \\ \text{subject to} & H(\vec{\rho},\vec{u}) = 0 \end{array}$$

- $\vec{\rho} \in \mathcal{X} \subseteq \mathbb{R}^{NT}$: state variables
- $\vec{u} \in \mathcal{U} \subseteq \mathbb{R}^{MT}$: control variables

Want to do gradient descent: How to compute the gradient $\frac{\partial J}{\partial \vec{\rho}} \nabla_{\vec{u}}\vec{\rho} + \frac{\partial J}{\partial \vec{u}}$?

On trajectories, $H(\vec{\rho},\vec{u}) = 0$ constant, thus $\frac{\partial H}{\partial \vec{\rho}} \nabla_{\vec{u}}\vec{\rho} + \frac{\partial H}{\partial \vec{u}} = 0$

Adjoint system:

$$\frac{\partial H}{\partial \vec{\rho}}^T \lambda = -\frac{\partial J}{\partial \vec{\rho}}^T$$

Then

$$\frac{\partial J}{\partial \vec{\rho}} \nabla_{\vec{u}}\vec{\rho} = -\lambda^T \frac{\partial H}{\partial \vec{\rho}} \nabla_{\vec{u}}\vec{\rho} = \lambda^T \frac{\partial H}{\partial \vec{u}}$$

The structure of the forward system influences the efficiency adjoint system solving

Solving for λ

$$\frac{\partial H}{\partial \vec{\rho}}^T \lambda = -\frac{\partial J}{\partial \vec{\rho}}^T$$

Since $\frac{\partial H}{\partial \vec{\rho}}$ is lower triangular, $\frac{\partial H}{\partial \vec{\rho}}^T$ is an upper triangular matrix

$\implies$ The adjoint system can be solved efficiently using backward substitution

Complexity reduction from $O((NT)^2 MT)$ to $O((NT)^2 + (NT)(MT))$

Sparsity $\implies O(NT + MT)$

Exploiting system structure

The structure of the forward system influences the efficiency adjoint system solving

Solving for λ

$$\frac{\partial H}{\partial \vec{\rho}}^T \lambda = -\frac{\partial J}{\partial \vec{\rho}}^T$$

Since $\frac{\partial H}{\partial \vec{\rho}}$ is lower triangular, $\frac{\partial H}{\partial \vec{\rho}}^T$ is an upper triangular matrix

$\implies$ The adjoint system can be solved efficiently using backward substitution

Complexity reduction from $O((NT)^2 MT)$ to $O((NT)^2 + (NT)(MT))$

Sparsity $\implies O(NT + MT)$

Optimization algorithm

Algorithm 1 Gradient descent loop

Pick initial control $\vec{u}_{init} \in U_{\mathrm{ad}}$

while not converged **do**

$\vec{\rho} = forwardSim(\vec{u}, IC, BC)$ solve for state trajectory (forward system)

$\lambda = adjointSln(\vec{\rho}, \vec{u})$ solve for adjoint parameters (adjoint system)

$\nabla_u J = \lambda^T \frac{\partial H}{\partial \vec{u}} + \frac{\partial J}{\partial \vec{u}}$ compute the gradient (search direction)

$\vec{u} \leftarrow \vec{u} - \alpha \nabla_u J$, $\alpha \in (0, 1)$ s.t. $\vec{u} \in U_{\mathrm{ad}}$ update the control $\vec{u}$ (update step)

end while

Remarks on descrete adjoint

Strengths:

- Requires only one solution of adjoint system, independent of $\dim(\vec{u})$ (unlike finite differences or sensitivity analysis).
- Extends existing simulation code
- General technique, can be applied in very different settings

Weaknesses:

- Optimization of the discretized problem, rather than approximation of the optimal solution of the continuous problem

Outline

Numerical Results: case study

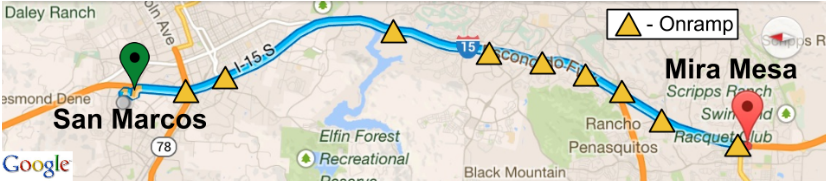


Figure : I15 South, San Diego: 31 km

$N = 125$ links

$M = 9$ onramps

$T = 1800$ time-steps

$\Delta t = 4$ seconds (120 minutes time interval)

Numerical Results

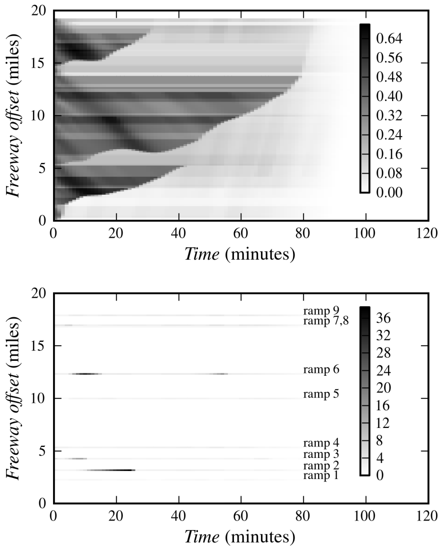


Figure : Density and queue lengths without control

Numerical Results

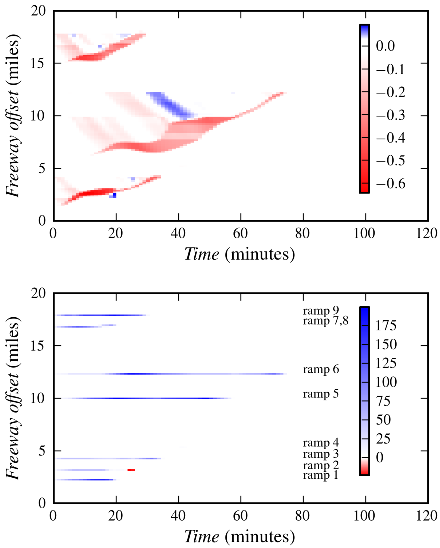
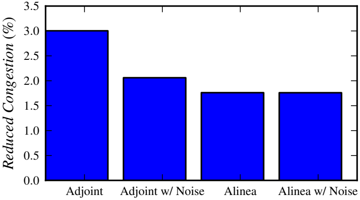


Figure : Density and queue difference with control

Performance under noisy input data: MPC loop

- initial conditions at time t and boundary fluxes on T_h (noisy inputs)
- optimal control policy on T_h
- forward simulation on $T_u \leq T_h$ using optimal controls and exact initial and boundary data
- $t \to t + T_u$

Comparison with ALINEA (local feedback control without boundary conditions)



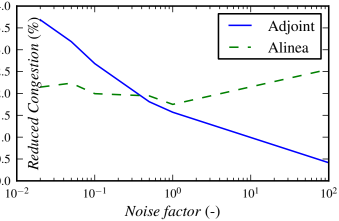
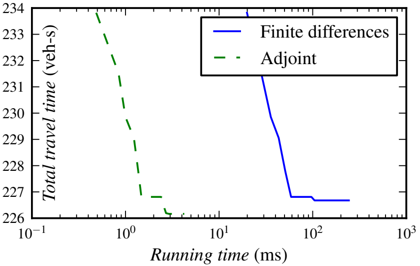


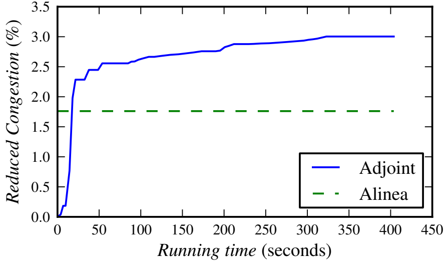
Figure : Congestion reduction and noise robustness

Running time

Convergence



Simulation time



Outline

Predictive/Coordinated Ramp Metering in Connected Corridors

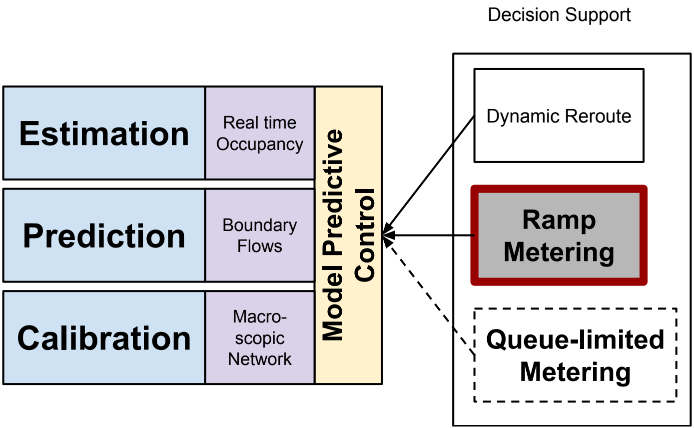
UC Berkeley - PATH

CALIFORNIA PATH

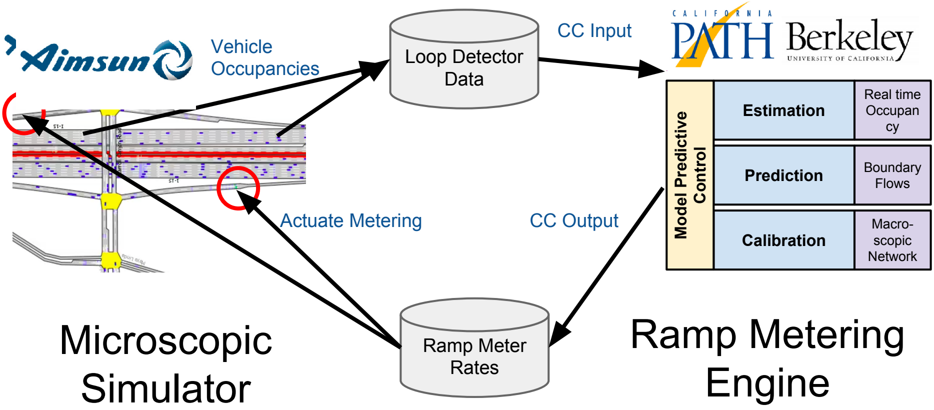
Berkeley

UNIVERSITY OF CALIFORNIA

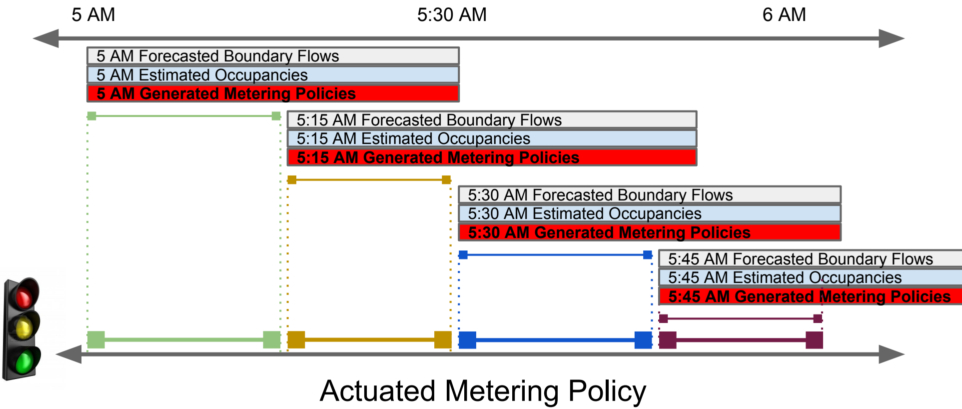
Connected Corridors Framework



Simulation Framework

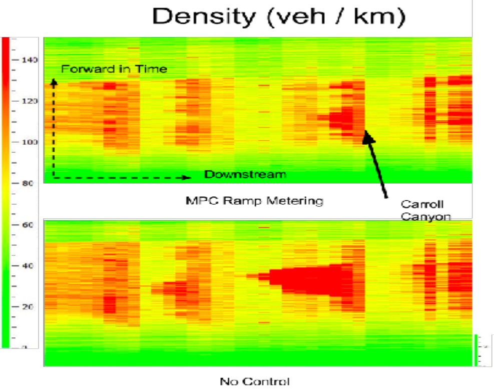


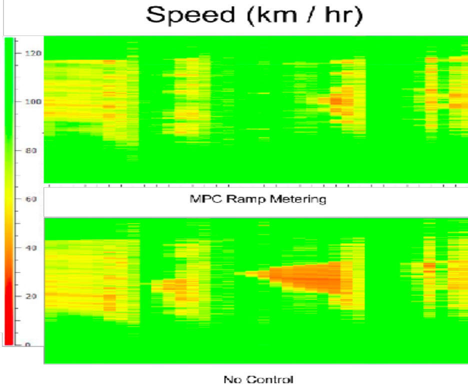
Model Predictive Control: Ramp Metering



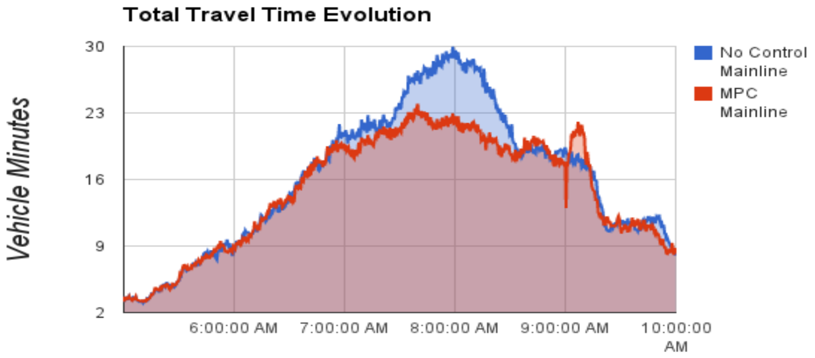
I15 South Simulation

Contour Summaries





I15 South: Total Travel Time-Mainline



Outline

Application to optimal re-routing

Multi-commodity flow:

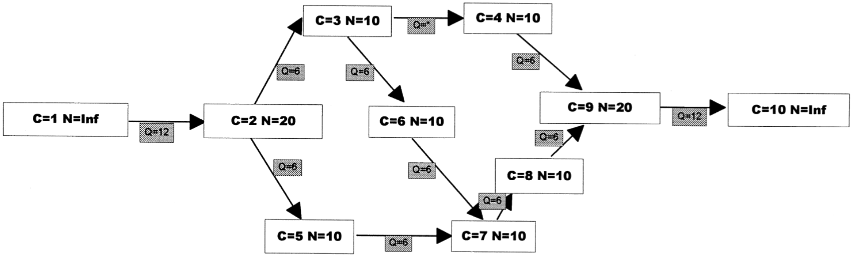
$$\rho_i(k) = \sum_{c \in \mathcal{C}} \rho_{i,c}(k)$$

accounting for *compliant* $c_c \in \mathcal{CC}$ and *non-compliant* c_n users

Goal:

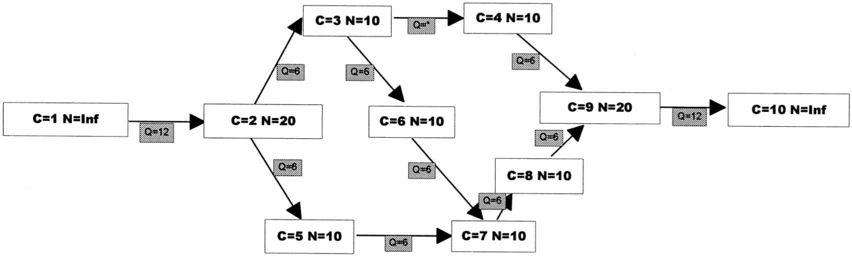
Control compliant users to optimize traffic flow

Example [Ziliaskopoulos, 2001]:



- Normal conditions: link capacity between cell 3 and cell 4 $= 6$
  Optimal total travel time: 190.9875
- Incident between cell 3 and cell 4: capacity goes to 0
  Optimal total travel time: 229.1973 (otherwise 254.758)

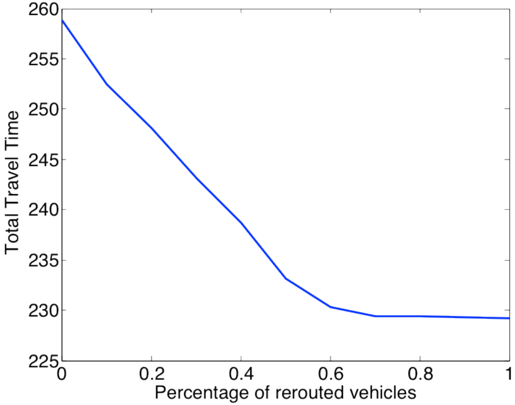
Example [Ziliaskopoulos, 2001]:



- Normal conditions: link capacity between cell 3 and cell 4 = 6
  Optimal total travel time: 190.9875
- Incident between cell 3 and cell 4: capacity goes to 0
  Optimal total travel time: 229.1973 (otherwise 254.758)

Application to optimal re-routing

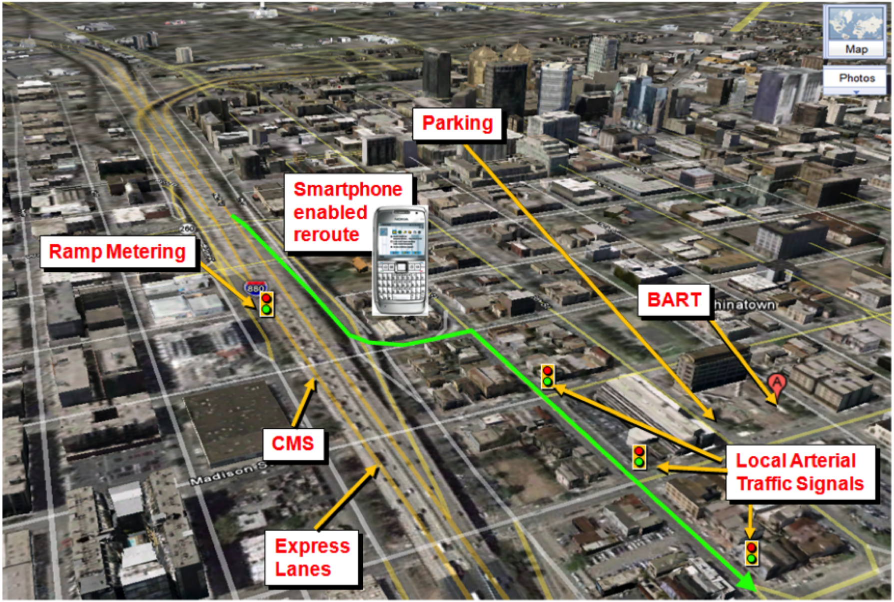
With partial control:



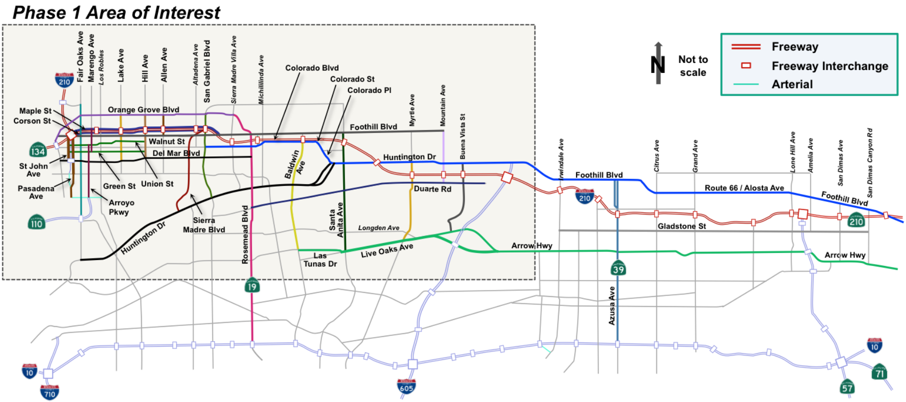
Total travel time reduction as a function of the percentage of vehicles that are rerouted:
almost optimal allocation can be achieved by controlling $\sim$60% of the demand

Outline

California Connected Corridors project

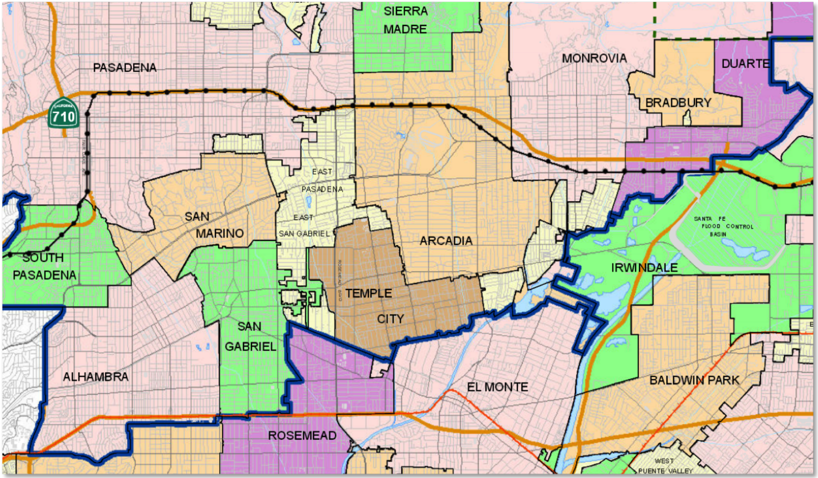


Testbed for deployment in California: SCOPE

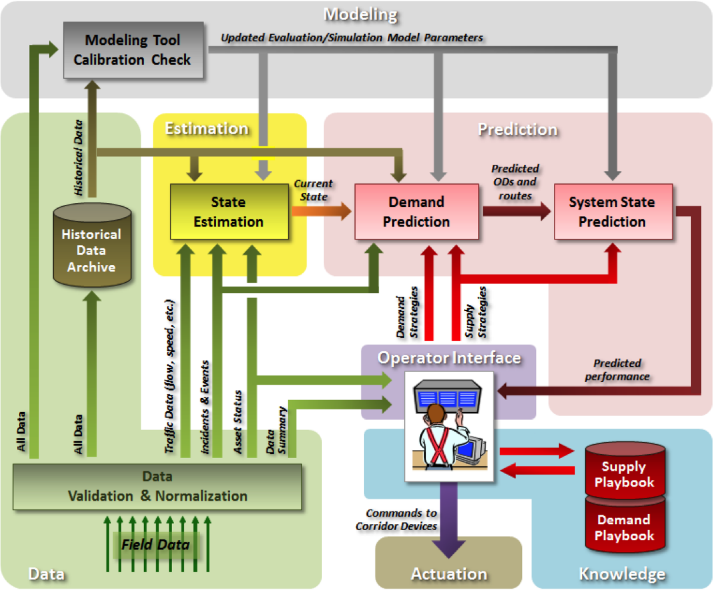


CPS operational decision support system: SENSING

CPS operational decision support system: CONTROL



CPS operational decision support system: DSS



Perspectives for the future of ORESTE

- Models developed as part of the project feed "estimation" and "prediction"
- Accurate modeling is required to perform efficient traffic management
- Perspectives for 2014 - 2015:
  - New member on secondment at Inria Sophia
  - 2015: IPAM workshop at UCLA, co-organized by Berkeley and Inria Sophia as part of "New Directions in Mathematical Approaches for Traffic Flow Management" trimester
- $\sim$10 joint publications (journals, conferences)

Thank you for your attention!

Perspectives for the future of ORESTE

- Models developed as part of the project feed "estimation" and "prediction"
- Accurate modeling is required to perform efficient traffic management
- Perspectives for 2014 - 2015:
  - New member on secondment at Inria Sophia
  - 2015: IPAM workshop at UCLA, co-organized by Berkeley and Inria Sophia as part of "New Directions in Mathematical Approaches for Traffic Flow Management" trimester
- ∼10 joint publications (journals, conferences)

Thank you for your attention!