Throughput-Optimal Topology Design for Cross-Silo Federated Learning

Othmane Marfoq (Inria&Accenture), Chuan Xu (Inria), Giovanni Neglia (Inria), Richard Vidal (Accenture)
Federated Learning

Federated learning involves “Training statistical models over remote devices or siloed data centers, such as mobile phones or hospitals, while keeping data localized” (Li et al. 2020).
Federated Learning

Federated learning involves “Training statistical models over remote devices or siloed data centers, such as mobile phones or hospitals, while keeping data localized” (Li et al. 2020).
Federated Learning

Federated learning involves “Training statistical models over remote devices or siloed data centers, such as mobile phones or hospitals, while keeping data localized” (Li et al. 2020).
Federated Learning

Federated learning involves “Training statistical models over remote devices or siloed data centers, such as mobile phones or hospitals, while keeping data localized” (Li et al. 2020).
Communication Topology & Training Time

Training Time = \#Iterations \times Iteration Time
Communication Topology & Training Time

\[ \text{Training Time} = \#\text{Iterations} \times \text{Iteration Time} \]

The communication topology has two contrasting effects on the training time:
Communication Topology & Training Time

\[ \text{Training Time} = \# \text{Iterations} \times \text{Iteration Time} \]

The communication topology has two contrasting effects on the training time:

- Fewer iterations
- More connected topology
The communication topology has two contrasting effects on the training time:

- More connected topology leads to fewer iterations.
- Slower iterations increase the training time.

Training Time = \#Iterations \times Iteration Time
The communication topology has two contrasting effects on the training time:

- More connected topology
- Slower iterations

- Fewer iterations

Recent experimental and theoretical works suggest that the second effect may dominate the first one.
Communication Topology & Training Time

The communication topology has two contrasting effects on the training time:

More connected topology

- Fewer iterations
- Slower iterations

Recent experimental and theoretical works suggest that the second effect may dominate the first one.
Problem Formulation

Underlay

Connectivity Graph

Overlay
Problem Formulation

\[ d_o(i, j) = s \times T_c(i) + l(i, j) + \frac{M}{\min\left(\frac{C_{UP}(i)}{|N_i^-|}, \frac{C_{DN}(j)}{|N_j^+|}, A(i', j')\right)} \]
Problem Formulation

\[ d_o(i, j) = s \times T_c(i) + l(i, j) + \frac{M}{\min \left( \frac{C_{UP}(i)}{|N_i^-|}, \frac{C_{DN}(j)}{|N_j^+|}, A(i', j') \right)} \]
Problem Formulation

Underlay

Connectivity Graph

Overlay

\[ d_o(i, j) = s \times T_c(i) + l(i, j) + \min \left( \frac{C_{UP}(i)}{|N^-_i|}, \frac{C_{DN}(j)}{|N^+_j|}, A(i', j') \right) \]

Model size

Latency

Capacities
Problem Formulation

$\text{Underlay}$

$\text{Connectivity Graph}$

$\text{Overlay}$

Computation Time

Latency

Model size

Capacities

$$d_o(i, j) = s \times T_c(i) + l(i, j) + \min \left( \frac{C_{UP}(i)}{|\mathcal{N}_i^-|}, \frac{C_{DN}(j)}{|\mathcal{N}_j^+|}, A(i', j') \right)$$
Problem Formulation

Each silo maintains a local copy of the model. At time $t_i(k)$ silo $i$ starts its $k$-th iteration, it

1) updates the local model through minibatch gradient descent.
2) sends the new model to its out-neighbors in the overlay.
3) aggregates the models received from its in-neighbors into a new local model.
Problem Formulation

Each silo maintains a local copy of the model. At time $t_i(k)$ silo $i$ starts its $k$-th iteration, it

1) updates the local model through minibatch gradient descent.
2) sends the new model to its out-neighbors in the overlay.
3) aggregates the models received from its in-neighbors into a new local model.

This is a **synchronous system**. The following recurrence holds:

$$t_i(k + 1) = \max_{j \in N_i^+ \cup \{i\}} (t_j(k) + d_o(i, j))$$
Problem Formulation

The duration of an iteration at silo $i$ is defined as $\tau_i = \lim_{k \to +\infty} t_i(k)/k$. 
Problem Formulation

The duration of an iteration at silo $i$ is defined as $\tau_i = \lim_{k \to +\infty} t_i(k)/k$.

**Max-plus algebra & synchronization theory** show that:

- $\tau_i$ does not depend on the specific silo.
- $\tau_i$ is the cycle time of the graph $G_o$, defined as $\tau(G_o) = \max_{\gamma} \frac{d_o(\gamma)}{|\gamma|}$, where $\gamma$ is a circuit of $G_o$. 
Analysis

Minimal Cycle Time (MCT)

**Input:** A strong directed graph $G_c = (V, E_c)$,

\[ \{ C_{UP}(i) C_{DN}(j), l(i, j), A(i', j'), T_c(i), \forall (i, j) \in E_c \} \]

**Output:** Strong spanning subdigraph of $G_c$ with minimal cycle time

From Max-Plus algebra & Synchronization theory
Analysis

**Minimal Cycle Time (MCT)**

**Input:** A strong directed graph $G_c = (V, E_c)$,
\[ \{C_{UP}(i)C_{DN}(j), l(i, j), A(i', j'), T_c(i), \forall (i, j) \in E_c\} \]

**Output:** Strong spanning subdigraph of $G_c$ with minimal cycle time

From Max-Plus algebra & Synchronization theory
Analysis

Minimal Cycle Time (MCT)

**Input:** A strong directed graph $G_c = (V, E_c)$, 
\[ \{(C_{UP}(i)C_{DN}(j), l(i, j), A(i', j'), T_c(i), \forall (i, j) \in E_c)\} \]

**Output:** Strong spanning subdigraph of $G_c$ with minimal cycle time

The proposed algorithms output either a **ring** or a **tree** with constrained degree.

<table>
<thead>
<tr>
<th>Network</th>
<th>Conditions</th>
<th>Algorithm</th>
<th>Complexity</th>
<th>Guarantees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge-capacitated</td>
<td>Undirected $G_o$</td>
<td>Prim’s Algorithm [80]</td>
<td>$O(</td>
<td>E_c</td>
</tr>
<tr>
<td>Edge/Node-capacitated</td>
<td>Euclidean $G_c$</td>
<td>Christofides’ Algorithm [69]</td>
<td>$O(</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>Euclidean $G_c$</td>
<td>Algorithm 1 (App. D)</td>
<td>$O(</td>
<td>E_c</td>
</tr>
</tbody>
</table>
Numerical Experiments

We considered three real topologies from Rocketfuel engine (Exodus and Ebone) and from The Internet Topology Zoo [48] (Géant), and two synthetic topologies (AWS North-America and Gaia) built from the geographical locations of AWS data centers.
Numerical Experiments

We considered three real topologies from Rocketfuel engine (Exodus and Ebone) and from The Internet Topology Zoo [48] (Géant), and two synthetic topologies (AWS North-America and Gaia) built from the geographical locations of AWS data centers.

Table 2: Datasets and Models. Mini-batch gradient computation time with NVIDIA Tesla P100.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Task</th>
<th>Samples (x 10^3)</th>
<th>Batch Size</th>
<th>Model</th>
<th>Parameters (x 10^3)</th>
<th>Model Size (Mbits)</th>
<th>Computation Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEMNIST [14]</td>
<td>Image classification</td>
<td>805</td>
<td>128</td>
<td>2-layers CNN</td>
<td>1,207</td>
<td>4.62</td>
<td>4.6</td>
</tr>
<tr>
<td>Sentiment140 [30]</td>
<td>Sentiment analysis</td>
<td>1,600</td>
<td>512</td>
<td>GloVe [82]+ LSTM [37]</td>
<td>4,810</td>
<td>18.38</td>
<td>9.8</td>
</tr>
<tr>
<td>iNaturalist [99]</td>
<td>Image classification</td>
<td>450</td>
<td>16</td>
<td>ResNet-18 [35]</td>
<td>11,217</td>
<td>42.88</td>
<td>25.4</td>
</tr>
</tbody>
</table>
Numerical Experiments

We considered three real topologies from Rocketfuel engine (Exodus and Ebone) and from The Internet Topology Zoo [48] (Géant), and two synthetic topologies (AWS North-America and Gaia) built from the geographical locations of AWS data centers.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Task</th>
<th>Samples (x 10^3)</th>
<th>Batch Size</th>
<th>Model</th>
<th>Parameters (x 10^3)</th>
<th>Model Size (Mbits)</th>
<th>Computation Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEMNIST [14]</td>
<td>Image classification</td>
<td>805</td>
<td>128</td>
<td>2-layers CNN</td>
<td>1, 207</td>
<td>4.62</td>
<td>4.6</td>
</tr>
<tr>
<td>Sentiment140 [30]</td>
<td>Sentiment analysis</td>
<td>1, 600</td>
<td>512</td>
<td>GloVe [82]+ LSTM [37]</td>
<td>4, 810</td>
<td>18.38</td>
<td>9.8</td>
</tr>
<tr>
<td>iNaturalist [99]</td>
<td>Image classification</td>
<td>450</td>
<td>16</td>
<td>ResNet-18 [35]</td>
<td>11, 217</td>
<td>42.88</td>
<td>25.4</td>
</tr>
</tbody>
</table>

Code: https://github.com/omarfoq/communication-in-cross-silo-fl
Numerical Experiments

Table 3: iNaturalist training over different networks. 1 Gbps core links capacities, 10 Gbps access links capacities. One local computation step ($s = 1$).

<table>
<thead>
<tr>
<th>Network name</th>
<th>Silos</th>
<th>Links</th>
<th>Cycle time (ms)</th>
<th>Ring’s training speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>STAR</td>
<td>MATCHA(+)</td>
</tr>
<tr>
<td>Gaia [36]</td>
<td>11</td>
<td>55</td>
<td>391</td>
<td>228 (228)</td>
</tr>
<tr>
<td>AWS North America [91]</td>
<td>22</td>
<td>231</td>
<td>288</td>
<td>124 (124)</td>
</tr>
<tr>
<td>Géant [27]</td>
<td>40</td>
<td>61</td>
<td>634</td>
<td>452 (106)</td>
</tr>
<tr>
<td>Exodus [64]</td>
<td>79</td>
<td>147</td>
<td>912</td>
<td>593 (142)</td>
</tr>
<tr>
<td>Ebone [64]</td>
<td>87</td>
<td>161</td>
<td>902</td>
<td>580 (123)</td>
</tr>
</tbody>
</table>
Numerical Experiments

Table 3: iNaturalist training over different networks. 1 Gbps core links capacities, 10 Gbps access links capacities. One local computation step ($s = 1$).

<table>
<thead>
<tr>
<th>Network name</th>
<th>Silos</th>
<th>Links</th>
<th>Cycle time (ms)</th>
<th>Ring’s training speed-up vs STAR</th>
<th>vs MATCHA(+)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>STAR</td>
<td>MATCHA(+)</td>
<td>MST</td>
</tr>
<tr>
<td>Gaia [36]</td>
<td>11</td>
<td>55</td>
<td>391</td>
<td>228 (228)</td>
<td>138</td>
</tr>
<tr>
<td>AWS North America [91]</td>
<td>22</td>
<td>231</td>
<td>288</td>
<td>124 (124)</td>
<td>90</td>
</tr>
<tr>
<td>Géant [27]</td>
<td>40</td>
<td>61</td>
<td>634</td>
<td>452 (106)</td>
<td>101</td>
</tr>
<tr>
<td>Exodus [64]</td>
<td>79</td>
<td>147</td>
<td>912</td>
<td>593 (142)</td>
<td>145</td>
</tr>
<tr>
<td>Ebone [64]</td>
<td>87</td>
<td>161</td>
<td>902</td>
<td>580 (123)</td>
<td>122</td>
</tr>
</tbody>
</table>
Table 3: iNaturalist training over different networks. 1 Gbps core links capacities, 10 Gbps access links capacities. One local computation step ($s = 1$).

<table>
<thead>
<tr>
<th>Network name</th>
<th>Silos</th>
<th>Links</th>
<th>Cycle time (ms)</th>
<th>Ring’s training speed-up vs STAR</th>
<th>vs MATCHA(+)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>STAR</td>
<td>MATCHA(+)</td>
<td>MST</td>
</tr>
<tr>
<td>Gaia [36]</td>
<td>11</td>
<td>55</td>
<td>391</td>
<td>228 (228)</td>
<td>138</td>
</tr>
<tr>
<td>AWS North America [91]</td>
<td>22</td>
<td>231</td>
<td>288</td>
<td>124 (124)</td>
<td>90</td>
</tr>
<tr>
<td>Géant [27]</td>
<td>40</td>
<td>61</td>
<td>634</td>
<td>452 (106)</td>
<td>101</td>
</tr>
<tr>
<td>Exodus [64]</td>
<td>79</td>
<td>147</td>
<td>912</td>
<td>593 (142)</td>
<td>145</td>
</tr>
<tr>
<td>Ebone [64]</td>
<td>87</td>
<td>161</td>
<td>902</td>
<td>580 (123)</td>
<td>122</td>
</tr>
</tbody>
</table>
Numerical Experiments

Sparser topologies can lead to a faster convergence even in the absence of congestion.

<table>
<thead>
<tr>
<th>Network name</th>
<th>Silos</th>
<th>Links</th>
<th>Cycle time (ms)</th>
<th>Ring’s training speed-up vs STAR</th>
<th>Ring’s training speed-up vs MATCHA(+)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STAR</td>
<td>MATCHA(+)</td>
<td>MST</td>
<td>δ-MBST</td>
<td>RING</td>
</tr>
<tr>
<td>Gaia [36]</td>
<td>11</td>
<td>55</td>
<td>391 (228)</td>
<td>138 (138)</td>
<td>118</td>
</tr>
<tr>
<td>AWS North America [91]</td>
<td>22</td>
<td>231</td>
<td>288 (124)</td>
<td>90 (90)</td>
<td>81</td>
</tr>
<tr>
<td>Géant [27]</td>
<td>40</td>
<td>61</td>
<td>634 (106)</td>
<td>101 (101)</td>
<td>109</td>
</tr>
<tr>
<td>Exodus [64]</td>
<td>79</td>
<td>147</td>
<td>912 (142)</td>
<td>145 (145)</td>
<td>103</td>
</tr>
<tr>
<td>Ebone [64]</td>
<td>87</td>
<td>161</td>
<td>902 (123)</td>
<td>122 (122)</td>
<td>95</td>
</tr>
</tbody>
</table>

Table 3: iNaturalist training over different networks. 1 Gbps core links capacities. 10 Gbps access links capacities. One local computation step ($s = 1$).
Conclusion

• Synchronization theory & max-plus algebra to model and optimize iteration time.

• In cross-silo setting, replacing server by peer-to-peer communication, results in significant speed ups (×9).

• Counter-intuitively, sparser topologies may lead to faster convergence even in the absence of congestion.
Thank you for your attention

Code: https://github.com/omarfoq/communication-in-cross-silo-fl
Email: othmane.marfoq@inria.fr