



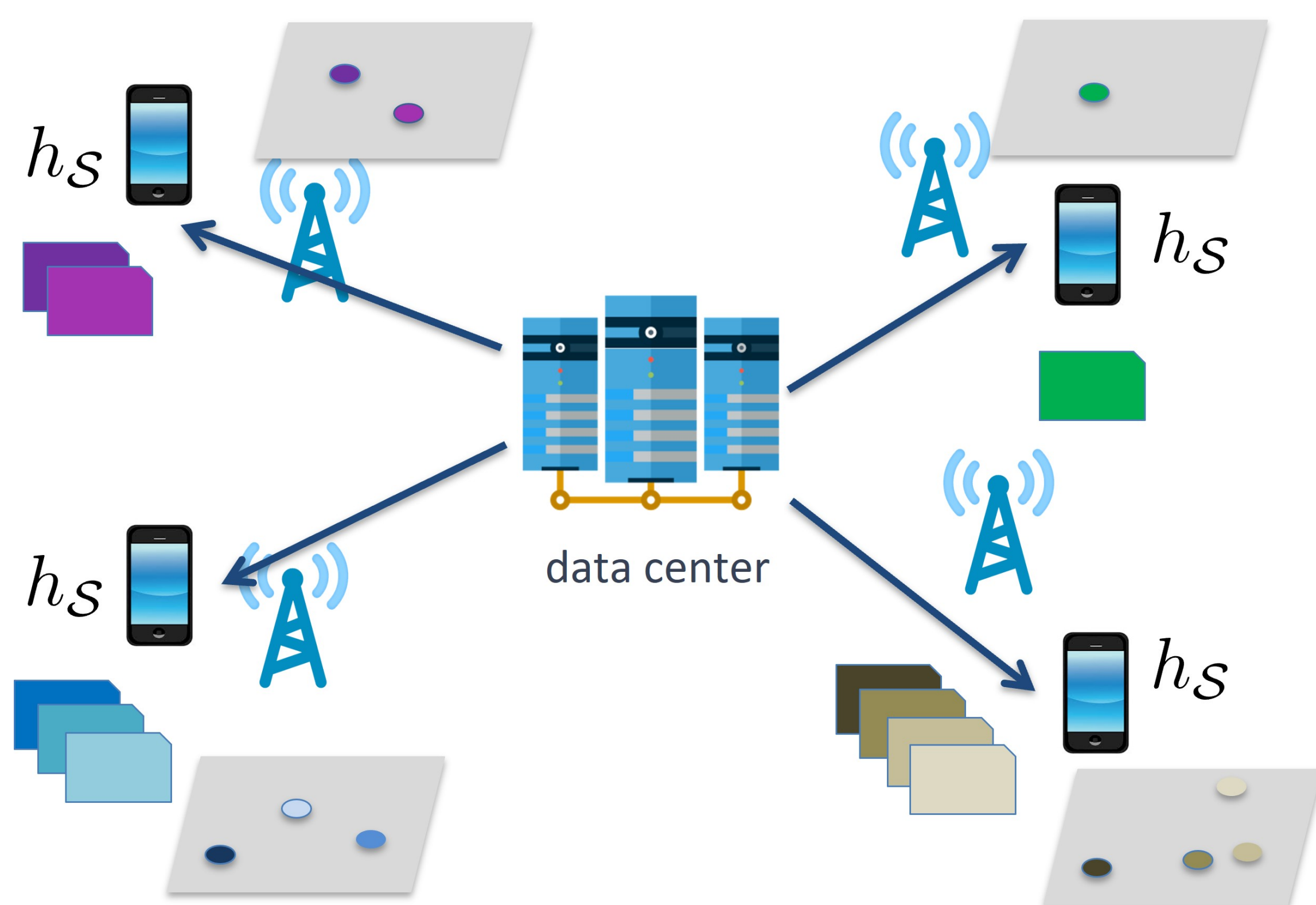
The problem

- We consider M classification (or regression) tasks, one for each client
- Data $\mathcal{S}_m = \{(\mathbf{x}_m^{(i)}, y_m^{(i)})\}_{i=1}^{n_m}$ at client m is drawn from a local distribution \mathcal{D}_m over $\mathcal{X} \times \mathcal{Y}$
- Client $m \in [M]$ wants to learn hypothesis $h_m \in \mathcal{H}_m$ mapping input $\mathbf{x} \in \mathcal{X}$ to a probability distribution over the set \mathcal{Y} :

$$\text{minimize}_{h_m \in \mathcal{H}} \mathcal{L}_{\mathcal{D}_m}(h_m) \triangleq \mathbb{E}_{(\mathbf{x}, y) \sim \mathcal{D}_m} [l(h_m(\mathbf{x}), y)]$$

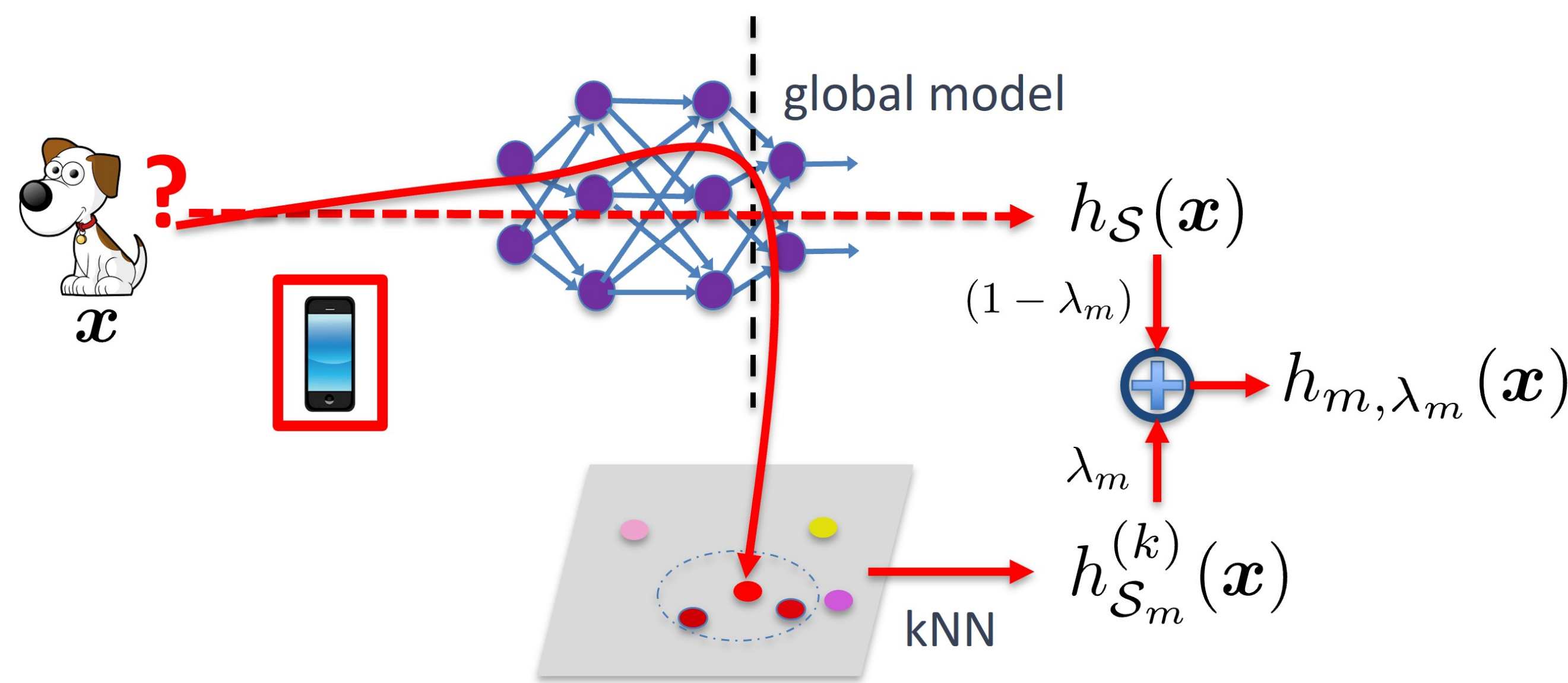
- FedAvg minimizes $\mathbb{E}_{(\mathbf{x}, y) \sim \bar{\mathcal{D}}} [l(h(\mathbf{x}), y)]$, where $\bar{\mathcal{D}} = \sum_{m=1}^M \frac{n_m}{n} \cdot \mathcal{D}_m$ (asymptotically in the total number of samples)
- In many applications, e.g., language modeling, clients' local datasets differ both in size and distribution (*statistical heterogeneity*)
- Clients may differ in their storage and computational capabilities (*system heterogeneity*)

Our algorithm: kNN-Per



- Clients train a global model h_S using a federated learning algorithm, e.g., FedAvg
- Each client creates its local datastore for kNN inference (samples embedded through h_S)
- The global model and the local kNN are interpolated:

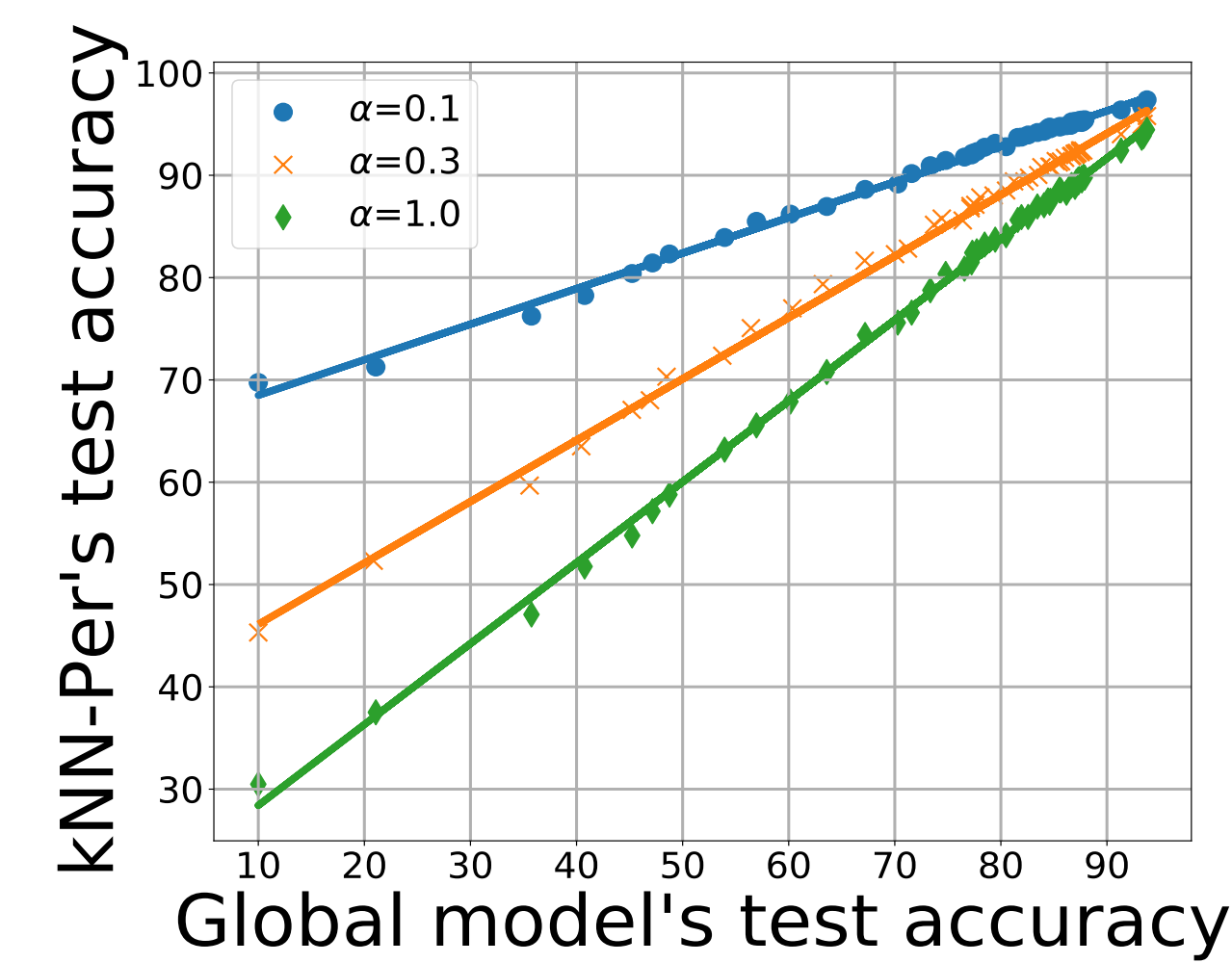
$$h_{m, \lambda_m}(\mathbf{x}) = \lambda_m \cdot h_{S_m}^{(k)}(\mathbf{x}) + (1 - \lambda_m) \cdot h_S(\mathbf{x})$$



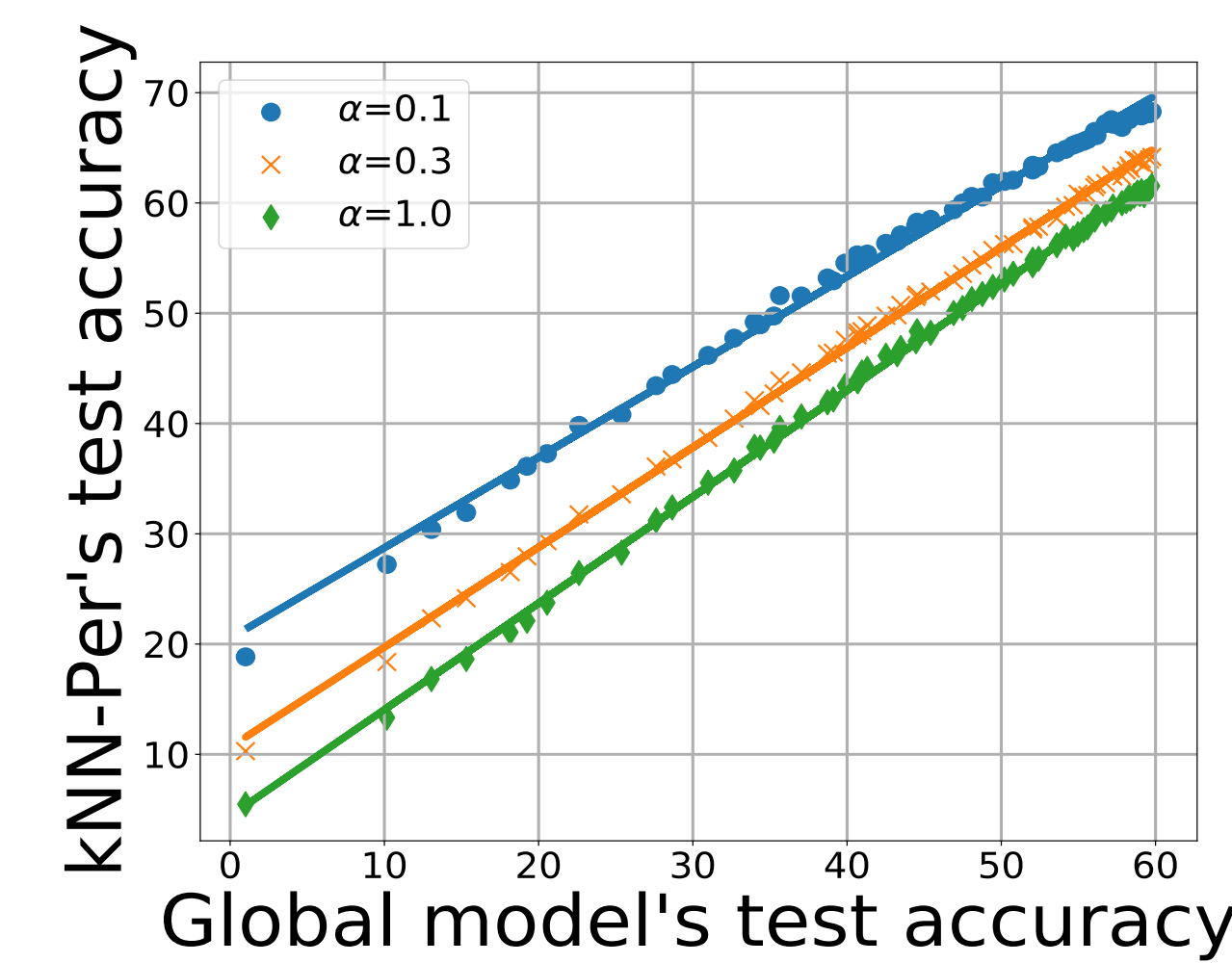
Main assumption

Let $h_m^* \in \arg \min_{h \in \mathcal{H}} \mathcal{L}_{\mathcal{D}_m}(h)$. There exist constants $\gamma_1, \gamma_2 > 0$, such that for any dataset \mathcal{S} drawn from $\mathcal{X} \times \mathcal{Y}$ and any data points $\mathbf{x}, \mathbf{x}' \in \mathcal{X}$, we have

$$\underbrace{\mathbb{P}[y = 1 | \mathbf{x}] - \mathbb{P}[y = 1 | \mathbf{x}']}_{\text{labels' distance}} \leq \underbrace{d(\phi_{h_S}(\mathbf{x}), \phi_{h_S}(\mathbf{x}'))}_{\text{representations' distance}} \times \left(\gamma_1 + \gamma_2 \underbrace{(\mathcal{L}_{\mathcal{D}_m}(h_S) - \mathcal{L}_{\mathcal{D}_m}(h_m^*))}_{\text{global model's quality}} \right)$$



(a) CIFAR-10.



(b) CIFAR-100.

Figure: Effect of the global model quality on the test accuracy of kNN-Per with λ tuned per client.

Generalization bound

Under proper assumptions, there exists $c \in \mathbb{R}$, such that

$$\mathbb{E}_{\mathcal{S} \sim \prod_{m=1}^M \mathcal{D}_m^{n_m}} [\mathcal{L}_{\mathcal{D}_m}(h_{m, \lambda_m})] \leq (1 + \lambda_m) \mathcal{L}_{\mathcal{D}_m}(h_m^*) + c(1 - \lambda_m) \text{disc}_{\mathcal{H}}(\bar{\mathcal{D}}, \mathcal{D}_m) + (1 - \lambda_m) \tilde{\mathcal{O}}\left(\sqrt{\frac{d_{\mathcal{H}}}{n}}\right) + \lambda_m \left(1 + \text{disc}_{\mathcal{H}}(\bar{\mathcal{D}}, \mathcal{D}_m)\right) \cdot \mathcal{O}\left(\frac{\sqrt{p}}{r \sqrt{n_m}}\right) + \lambda_m \cdot \tilde{\mathcal{O}}\left(\sqrt{\frac{d_{\mathcal{H}}}{n}} \cdot \frac{\sqrt{p}}{r \sqrt{n_m}}\right),$$

where $d_{\mathcal{H}}$ is the VC dimension of the hypothesis class \mathcal{H} , $\bar{\mathcal{D}} = \sum_{m=1}^M \frac{n_m}{n} \cdot \mathcal{D}_m$ and $\text{disc}_{\mathcal{H}}$ is the label discrepancy associated to the hypothesis class \mathcal{H} .

Average performance and fairness of personalized model

Dataset	Local	FEDAVG	FEDAVG+	CLUSTEREDFL	DITTO	FEDREP	APFL	kNN-PER (Ours)
FEMNIST	71.0 / 57.5	83.4 / 68.9	84.3 / 69.4	83.7 / 69.4	84.3 / 71.3	85.3 / 72.7	84.1 / 69.4	88.2 / 78.8
CIFAR-10	57.6 / 41.1	72.8 / 59.6	75.2 / 62.3	73.3 / 61.5	80.0 / 66.5	77.7 / 65.2	78.9 / 68.1	83.0 / 71.4
CIFAR-100	31.5 / 19.8	47.4 / 36.0	51.4 / 41.1	47.2 / 36.2	52.0 / 41.4	53.2 / 41.7	51.7 / 41.1	55.0 / 43.6
Shakespeare	32.0 / 16.0	48.1 / 43.1	47.0 / 42.2	46.7 / 41.4	47.9 / 42.6	47.2 / 42.3	45.9 / 42.4	51.4 / 45.4

Table: Test accuracy: average across clients / bottom decile.

Adding compression techniques

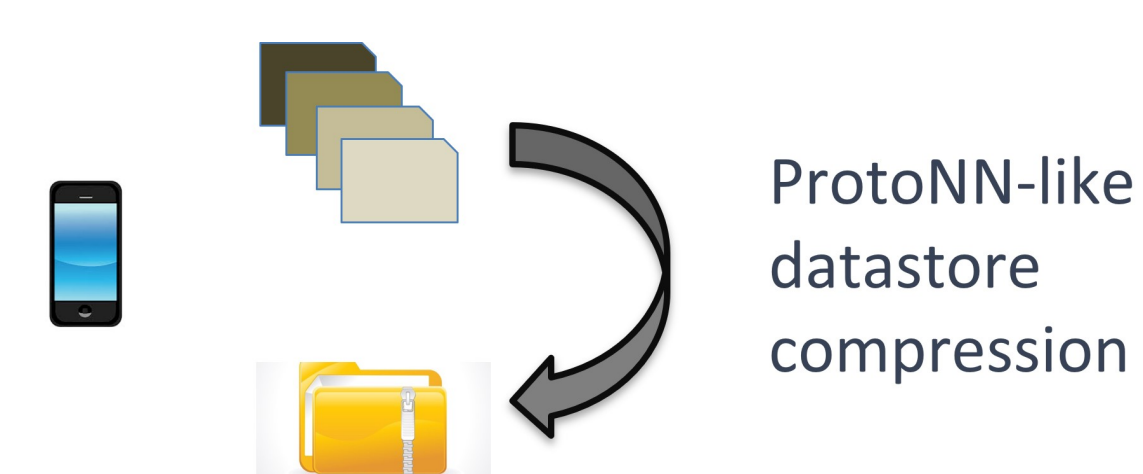
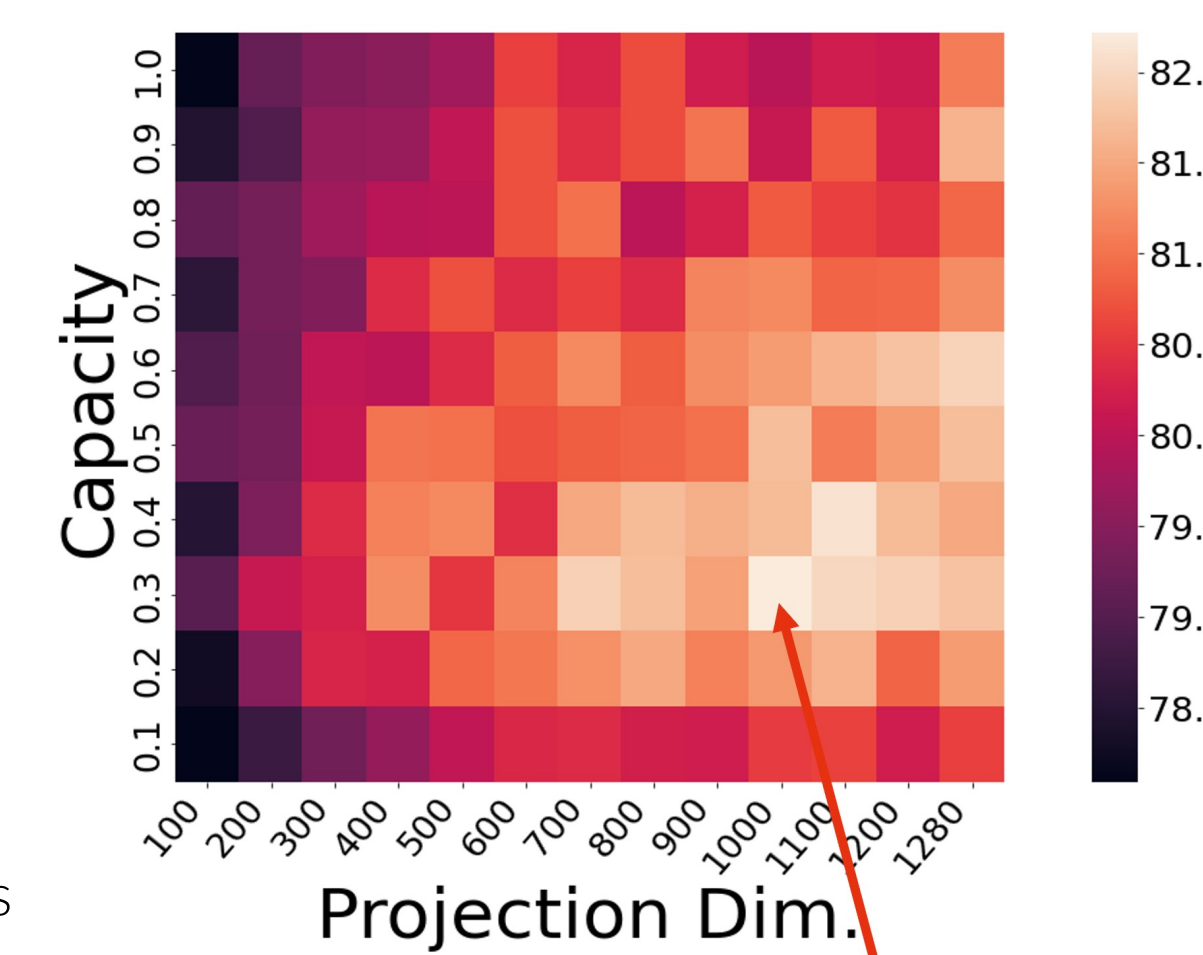


Figure: Test accuracy on CIFAR-10 dataset when the kNN mechanism is implemented through ProtoNN for different values of projection dimension and number of prototypes (expressed as a fraction of the local dataset).



4x memory savings

Effect of local datastore size and data heterogeneity

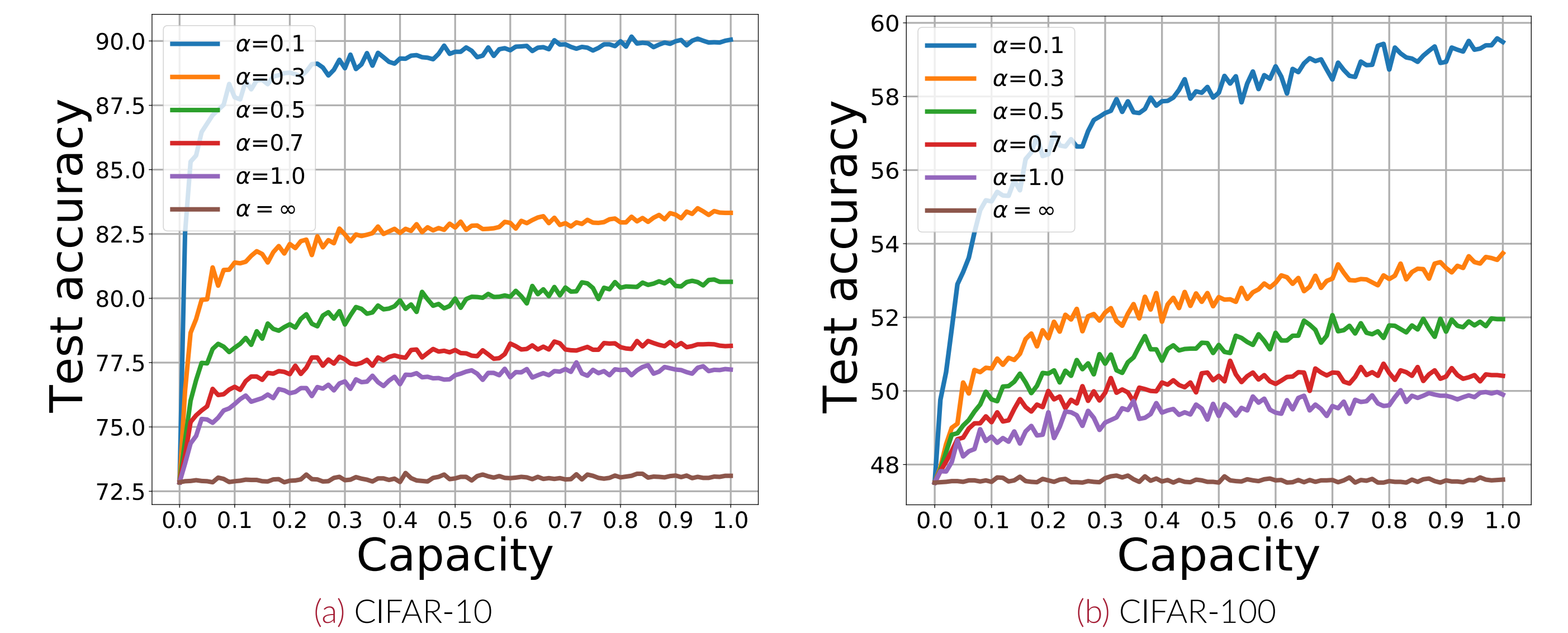


Figure: Accuracy vs capacity (local datastore size). The capacity is normalized with respect to the initial size of the client's dataset partition. Smaller values of α correspond to more heterogeneous data distributions across clients.

Robustness to distribution shift

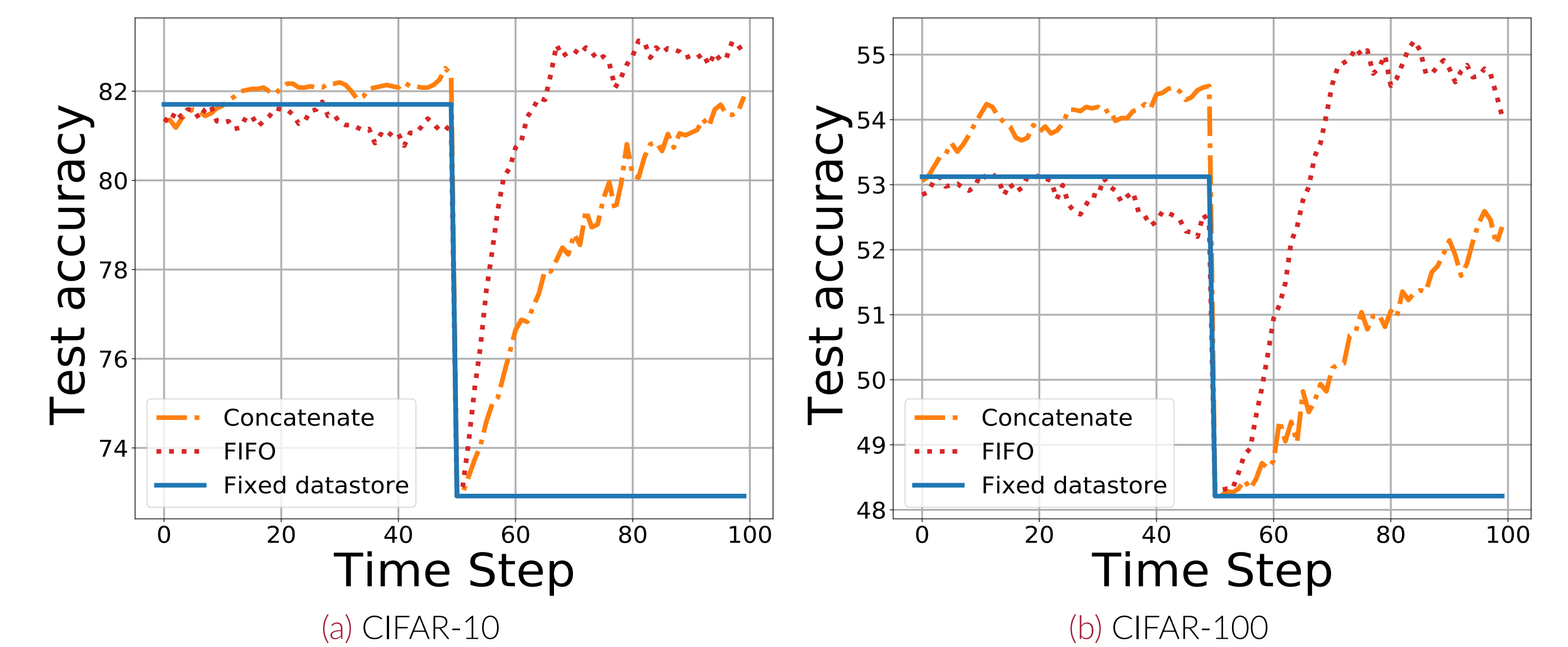
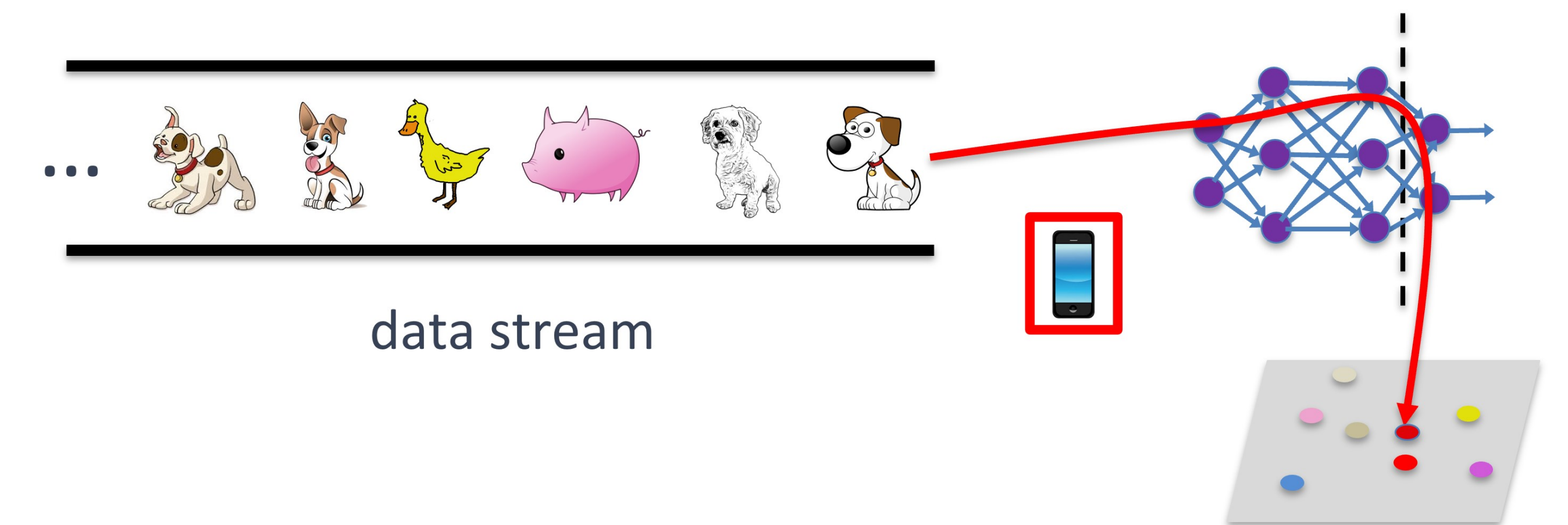


Figure: Test accuracy when a distribution shift happens at time step $t_0 = 50$ for different datastore management strategies.

Conclusions

- kNN-Per offers a simple and effective way to address statistical heterogeneity in FL
- kNN-Per has a limited leakage of private information and can be easily combined with differential privacy techniques
- kNN-Per partially addresses system heterogeneity as data-store's size and approximate kNN's choice can be adapted to client's capabilities
- kNN-Per adapts to data distribution shifts over time by updating the local datastore