TD6

Exo 3

$$\int_0^t \frac{W_s^{(i)}}{R_s} dW_s^i = B^{(i)}$$ Martingale de carré $\int$: $E\left(B^{(i)2}\right) < \infty$

$$\Leftarrow \quad E\int_0^t \left(\frac{1}{R_s} W_s^{(i)}\right)^2 ds \leq t \qquad 0 \leq t < \infty$$

Ito $\int$ a un sens

$$\mu\{0 \leq s \leq t \quad R_s = 0\} = 0 \qquad \text{ps}$$

Rappel $\quad \mathcal{Z}_\omega = \{0 \leq t < \infty \ / \ W_t(\omega) = 0\}$

$$\mu(\mathcal{Z}_\omega) = 0 \qquad \text{ps}$$

Alors $(d-1)/R_s$ définit ps $/s$ ps$/\omega$

$$f_n(R_t^2) = f_n(R_0^2) + J_t(\varepsilon) + K_t(\varepsilon) + \sum_{i=1}^d I_t^{(i)}(\varepsilon)$$

$$I_t^{(i)}(\varepsilon) = \int_0^t \left\{ 1(R_s^2 \geq \varepsilon)\frac{1}{R_s} + 1(R_s^2 < \varepsilon)\frac{1}{2\sqrt{\varepsilon}}\left(3 - \frac{R_s^2}{\varepsilon}\right)\right\} W_s^i \, dW_s^i$$

$$J_t(\varepsilon) = \int_0^t 1(R_s^2 \geq \varepsilon)\frac{d-1}{2R_s} ds$$

$$K_t(\varepsilon) = \int_0^t 1(R_s^2 < \varepsilon)\frac{1}{4\sqrt{\varepsilon}}\left[3d - (d+2)\frac{R_s^2}{\varepsilon}\right] ds$$

TCV monotone : $\displaystyle \lim_{n\to\infty} J_t\left(\frac{1}{n}\right) = \int_0^t \frac{d-1}{2R_s} ds$ ps

$$0 \leq E K_t^2(\varepsilon) \leq \left(\frac{3d}{4\sqrt{\varepsilon}}\right)^2 \int_0^t P(R_s^2 < \varepsilon) ds$$

$$\leq \int_0^{2\pi}\int_0^{\sqrt{\varepsilon}} \frac{1}{2\pi s} e^{-r^2/2s} r\,dr\,d\theta$$

$$\sqrt{\varepsilon} \quad -r^2/2s$$