Pathwidth and Graph Searching Games

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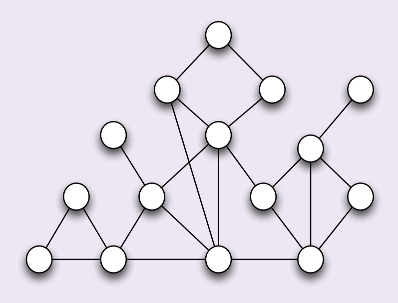
COATI seminar

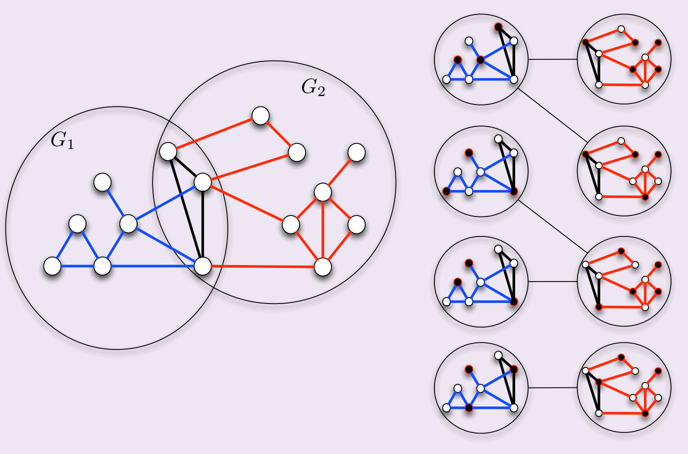
October 8th 2014

Dynamic Programming for Max. Independent Set



Let's compute a maximum independent set of this graph
Brute-force: check all subsets 2^{15}

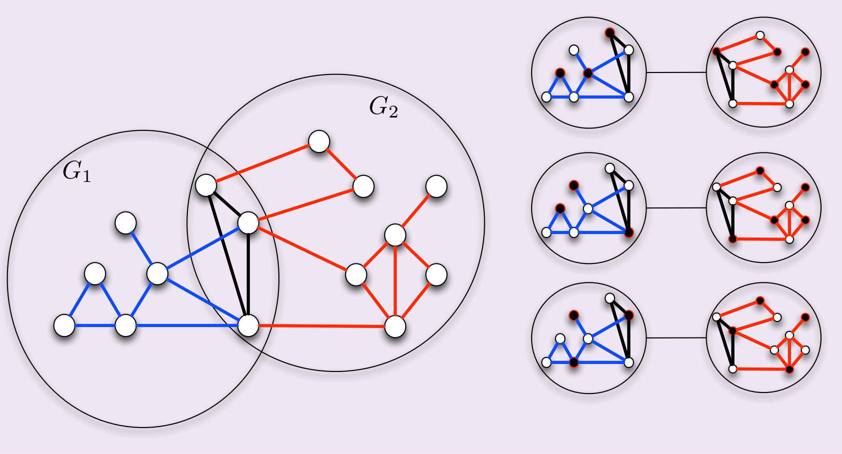
Dynamic Programming for Max. Independent Set



Brute-force: check all subsets 2^{15}

better idea (?): combine IS of G_1 and G_2 $2^8 + 2^{10} + 2^8 * 2^{10}$

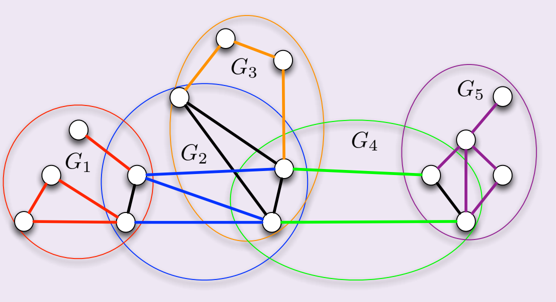
Dynamic Programming for Max. Independent Set



For any indep. set I of the Separator ($G_1 \cap G_2$), find:

- one MIS compatible with I in G_1 2^5
- one MIS compatible with I in G_2 2^7
- combine them 2^3

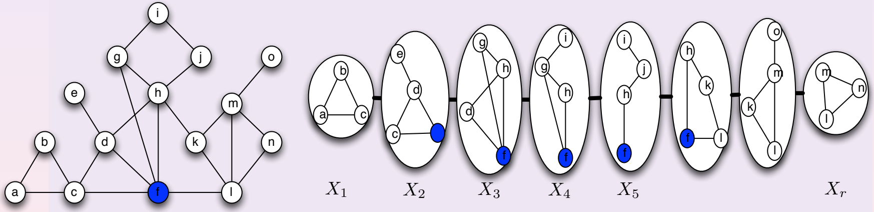
Dynamic Programming for Max. Independent Set



Going further: decompose G into more parts $\Rightarrow \#\ of\ part\ * 2^{O(size\ of\ largest\ part)}$

Path-Decomposition and Pathwidth

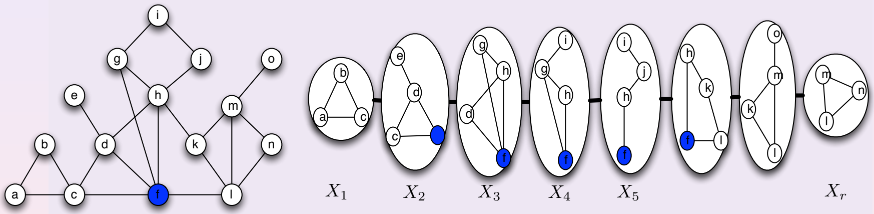
Representation of a graph $G = (V, E)$ as a Path preserving connectivity properties



Sequence $\mathcal{X} = (X_1, \cdots, X_r)$ of "bags" (set of vertices of G)
Important: intersection of two adjacent bags = separator of G

Path-Decomposition and Pathwidth

Representation of a graph $G = (V, E)$ as a Path preserving connectivity properties

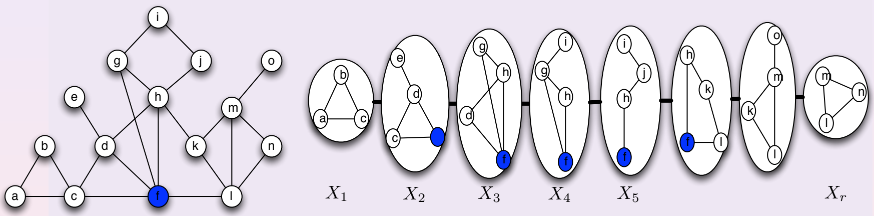


Sequence $\mathcal{X} = (X_1, \cdots, X_r)$ of "bags" (set of vertices of G)
Important: intersection of two adjacent bags = separator of G

- $\bigcup_{i \leq r} X_i = V$
- for any $e = uv \in E$, there is $i \leq r$ such that $u, v \in X_i$
- for any $i \leq j \leq k \leq r$, $X_i \cap X_k \subseteq X_j$.

Path-Decomposition and Pathwidth

Representation of a graph $G = (V, E)$ as a Path preserving connectivity properties

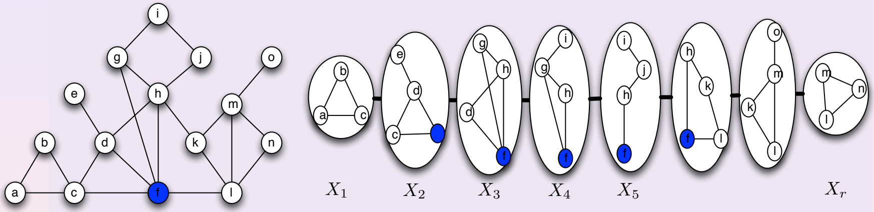


Sequence $\mathcal{X} = (X_1, \cdots, X_r)$ of "bags" (set of vertices of G)
Important: intersection of two adjacent bags = separator of G

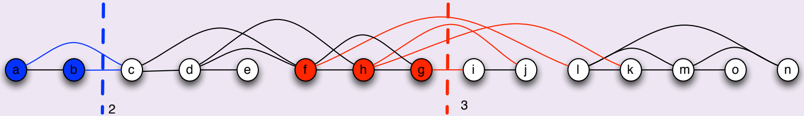
Width of $(T, \mathcal{X})$: $\max_{i \leq r} |X_i| - 1$ $\approx$ size of largest bag
Pathwidth of a graph G, $pw(G)$: min width over all path-decompositions.

Path-Decomposition and Pathwidth

Representation of a graph $G = (V, E)$ as a Path preserving connectivity properties



Equivalent definition:
Ordering of nodes $(v_1, v_2, \cdots, v_n)$ minimizing $\max_{1<i\leq n} |\{j < i \mid v_i v_j \in E\}|$.



Algorithmic Applications and Complexity

Dynamic programming on path decomposition

MSOL Problems: "local" problems are FPT in *pw* [Courcelle'90]
e.g., coloring, independent set: $O(2^{pw} n^{O(1)})$; dominating set $O(4^{pw} n^{O(1)})$...

huge constants may be hidden (at least exponential in *pw*)
"good" decompositions must be computed

Algorithmic Applications and Complexity

Complexity to compute path-decompositions

- NP-complete to compute *pw*
  - in *planar cubic* graphs [Monien, Sudborough'88]
  - in *chordal* graphs [Gustedt'93]
- Not approximable up to additive constant (unless P=NP) [Deo, Krishnamoorthy, Langston'87]
- FPT-algorithm [Bodlaender, Kloks'96]
- Polyomial or Linear in
  - trees [Skodinis'00],
  - cographs [Bodlaender, Möhring'93],
  - split graphs [Gustedt'93], etc.
- Exponential exact algorithm [Coudert,Mazauric,N.'14]

Studying Pathwidth via Graph Searching

Team of Searchers

to Capture an invisible fugitive / Clear a contaminated graph

Rules of Graph Searching [Parsons'76]

Allowed moves

- Place a searcher at a node
- Remove a searcher from a node
- Slide a searcher along an edge

Clearing edges

- when a searcher slides along it

Recontamination

- if no searcher on a path from a clear edge to a contaminated one

Goal: Minimize the number of searchers needed

Studying Pathwidth via Graph Searching

Allowed moves: Place $P(v)$, Remove $R(v)$, Slide $S(e)$
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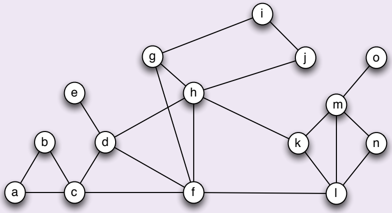

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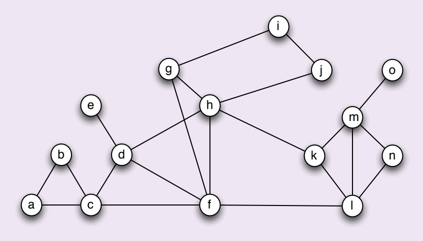

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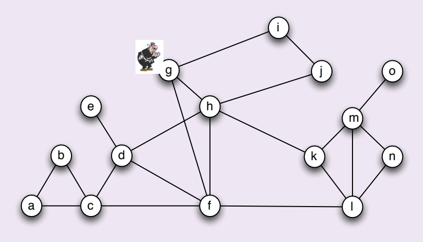

P(g),

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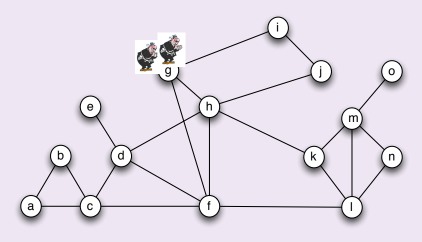

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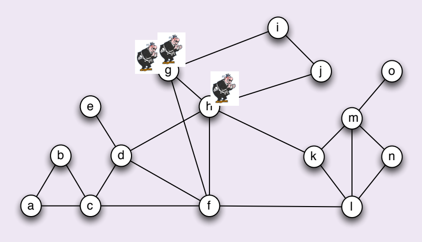

P(g), P(g), P(h),

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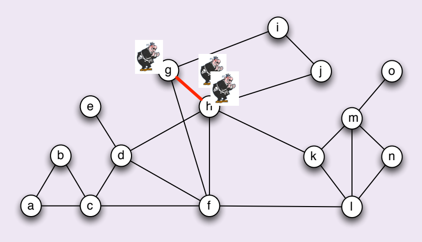

P(g), P(g), P(h), S(gh),

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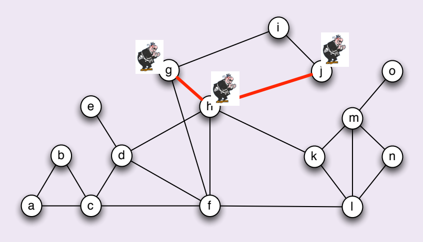

P(g), P(g), P(h), S(gh), S(hj),

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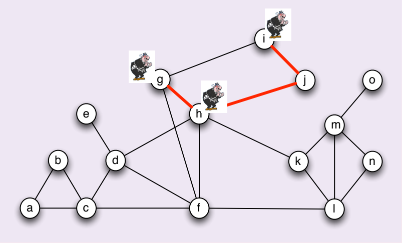

P(g), P(g), P(h), S(gh), S(hj), S(ji),

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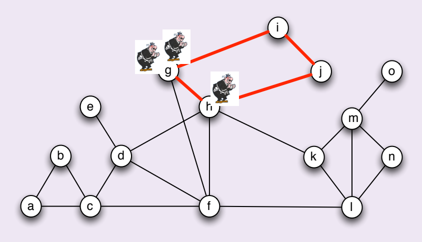

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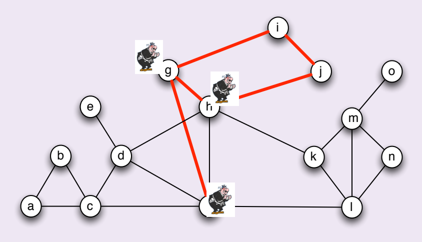

P(g), P(g), P(h), S(gh), S(hj), S(ji), S(ih), S(gf),

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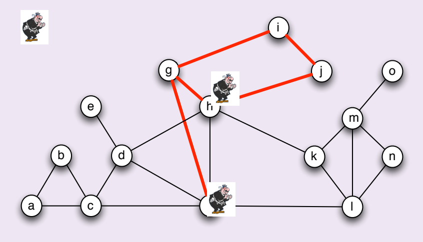

P(g), P(g), P(h), S(gh), S(hj), S(ji), S(ih), S(gf), R(g),

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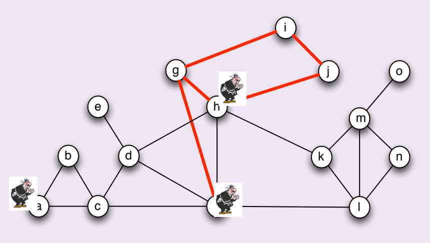

P(g), P(g), P(h), S(gh), S(hj), S(ji), S(ih), S(gf), R(g), P(a),

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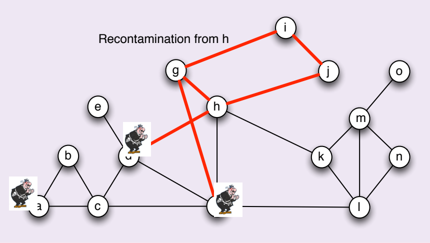

P(g), P(g), P(h), S(gh), S(hj), S(ji), S(ih), S(gf), R(g), P(a), S(hd),

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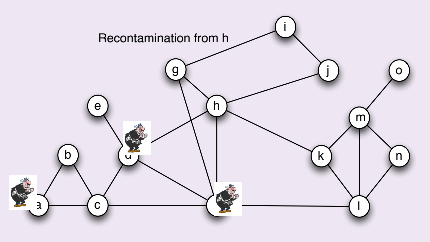

P(g), P(g), P(h), S(gh), S(hj), S(ji), S(ih), S(gf), R(g), P(a), S(hd), Recontamination, let's start again

Studying Pathwidth via Graph Searching

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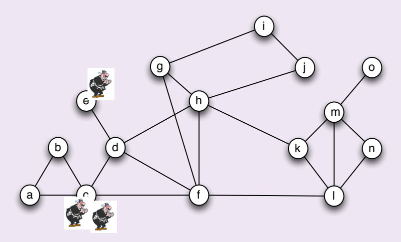

P(c), P(c), P(e),

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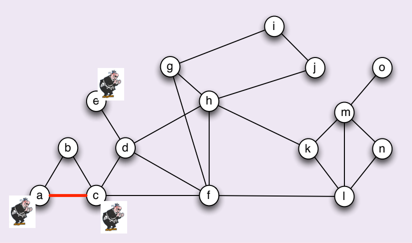

P(c), P(c), P(e), S(ca),

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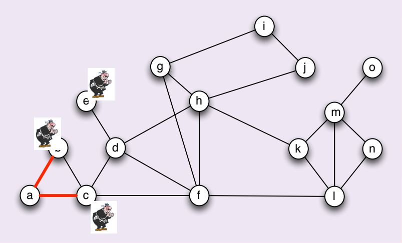

P(c), P(c), P(e), S(ca), S(ab),

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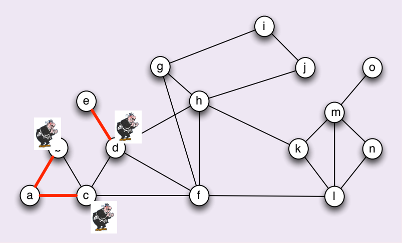

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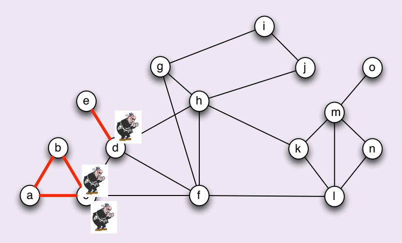

P(c), P(c), P(e), S(ca), S(ab), S(ed), S(bc),

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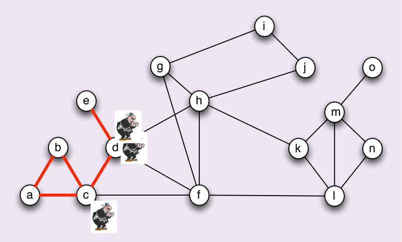

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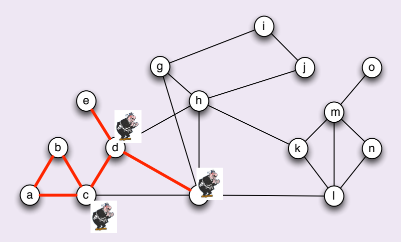

P(c), P(c), P(e), S(ca), S(ab), S(ed), S(bc), S(cd), S(df),

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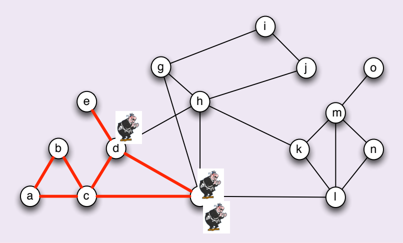

P(c), P(c), P(e), S(ca), S(ab), S(ed), S(bc), S(cd), S(df), S(cf),

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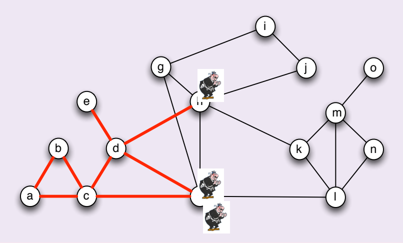

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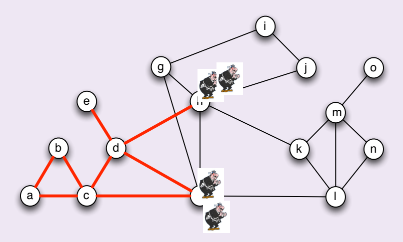

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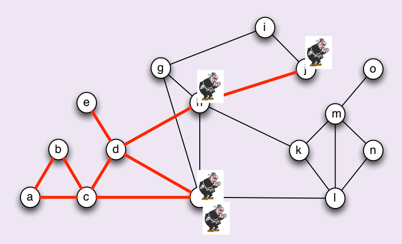

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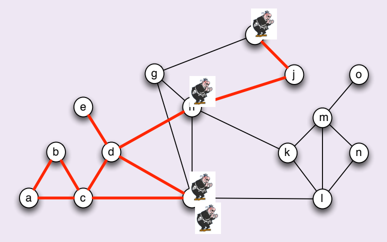

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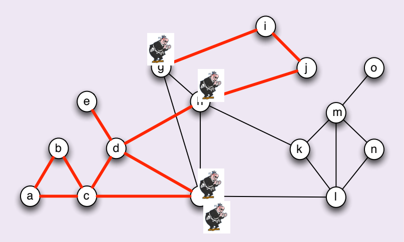

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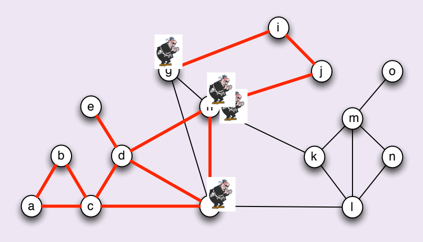

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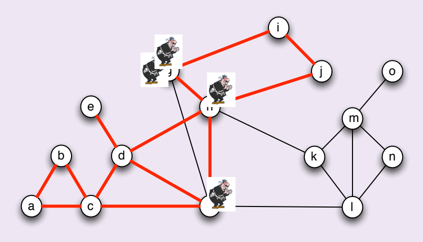

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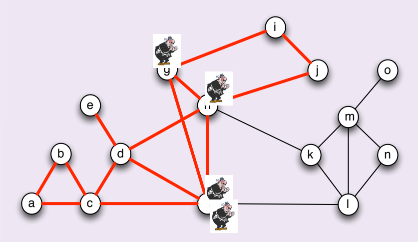

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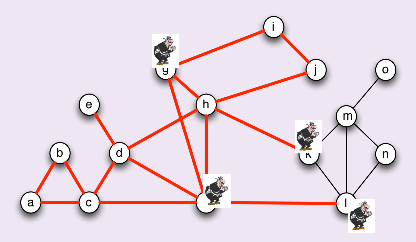

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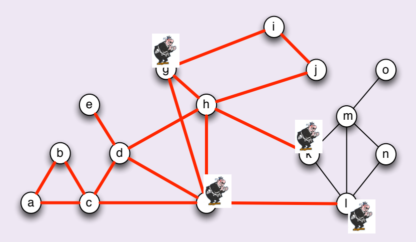

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Relationship with path-decomposition

Induces an sequence on vertices: each time a **contaminated** node becomes occupied

$(c, e, a, b, d, f, h, j, i, g, k, l \cdots)$

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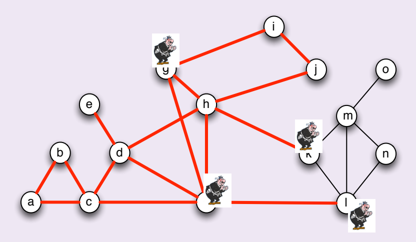

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Induces an sequence on vertices: each time a **contaminated** node becomes occupied

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If there is no recontamination: It is an ordering, i.e., a path-decomposition

Many variants of Graph Searching

		Edge-Search [Parsons'76]	Node-Search [Kirousis-Papdimitriou'86]	Mixed Search [Bienstock, Seymour'91]
Allowed moves	Place	yes	yes	yes
	Remove	yes	yes	yes
	Slide	yes	no	yes
Clearing moves	Slide	yes	no	yes
	2 ends occupied	no	yes	yes
Min. # of searchers		$es(G)$	$ns(G)$	$s(G)$

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Theorem [Bienstock, Seymour'91]

Three previous variants are monotone

i.e., there always exists an optimal strategy without recontamination

Consequence: for any graph G, $ns(G) = pw(G) + 1$.

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Link between variants (easy exercise)

For any graph G

- $s(G) \leq ns(G) \leq s(G) + 1$
- $s(G) \leq es(G) \leq s(G) + 1$
- $es(G) - 1 \leq ns(G) \leq es(G) + 1$

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Star: $s = ns = es = 2$, **Path:** $es = s = 1$, $ns = 2$; **In** $K_{r,r}$**:** $ns = s = r + 1$; $es = r + 2$

Complexity issues

	pathwidth pw (node-search ns)	edge-search es	mixed-search s
planar graphs with bounded maximum degree	NP-complete [Monien, Sudborough'88]		
split graphs	P [Gustedt'93]	P [Peng et al'00]	linear [FominHM10]
star-like graphs with ≥ 2 peripheral nodes per peripheral clique	NP-complete [Gustedt'93]	?	?
cographs	P [Bodlaender, M'93]	linear [GolovachHM12]	P [Heggernes, Mihai'08]

Open Problems

- Graph class where complexity differs ?
- Complexity of deciding if $pw(G)/es(G)/s(G)$ differ ?

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Study new variants of Graph Searching

to understand/approximate Pathwidth ?

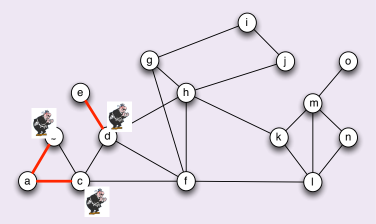
- Connected Graph Searching
- Exclusive Graph Searching

Connected Graph Searching

Connected Graph Searching [Barriere et al.'02]

"cleared" area must be always connected

Connected search number $cs(G)$: # min of Cops



example of non-connected step

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$\forall$ graph G, $cs(G) \leq 2pw(G) + O(1)$ [Dereniowski'12]

not monotone [Yang,Dyer,Alspach DM'09]

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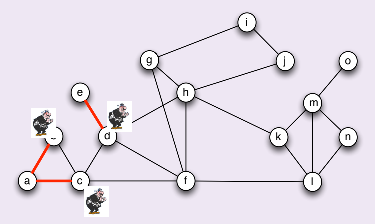

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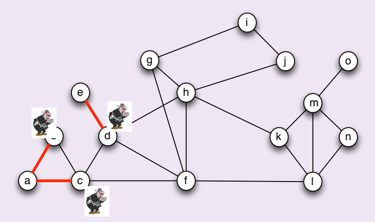

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Approximate Pathwidth via connected Search?

pw is NP-hard in weighted trees [Mihai,Todinca FAW'09]

3-approximation for *cs* in weighted trees [Dereniowski TCS'12]

Other graph classes (chordal,...) ?

Exclusive Graph Searching

Exclusive Graph Searching [Burman,Blin,N.'12]

New Constraint

- Exclusivity: at most one searcher per node

Allowed moves

- Only initially: place some searchers on distinct nodes
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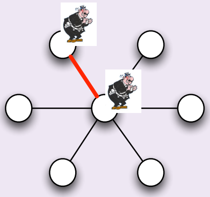

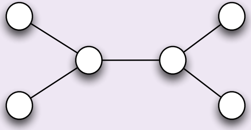

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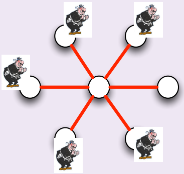
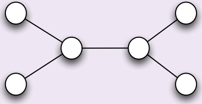


xs(star)$=\Delta - 1$



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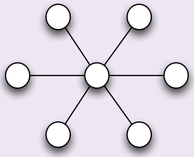

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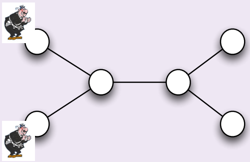

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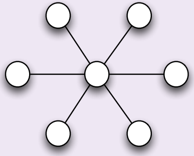

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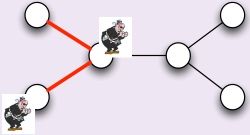

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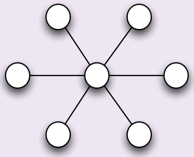

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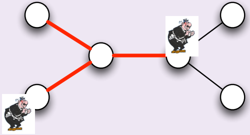

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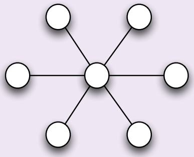

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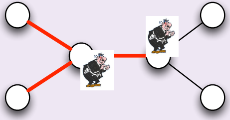

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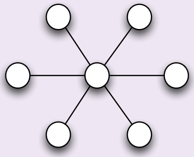

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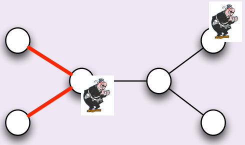

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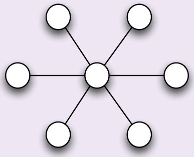

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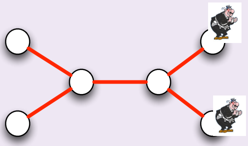

No optimal monotone strategy :(

Results on Exclusive Graph Searching

Exclusive Graph Searching

new constraint: at most one Cop per node at every step [Blin,Burman,N.'13]
(Cops can slide along edges)

$xs(G)$: min # of Cops $mxs(G)$: min # of Cops for monotone strategies

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About complexity: Computing xs is

- NP-hard in planar graphs with max degree 3 [Markou,N.,Pérennes]
- polynomial in trees [Blin,Burman,N.'13]
- linear in cographs [Markou,N.,Pérennes]

	pathwidth [Gustedt'93]	monotone exclusive-search [Markou,N.,Pérennes]
split graphs	**P**	**NP-complete**
star-like graphs with ≥ 2 peripheral nodes per clique	**NP-complete**	**P**

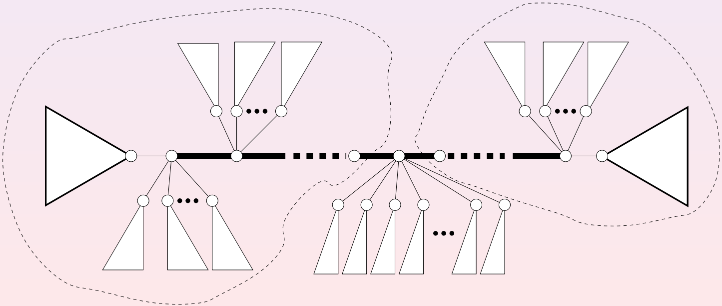
Exclusive Graph Searching in trees

Theorem: characterization of trees with $xs(T) \leq k$ [Blin,Burman,N.'13]

Let $k \geq 1$. For any tree T, $xs(T) \leq k$ iff for any node v:

1. v has degree at most $k+1$;
2. for any branch B at v, $xs(B) \leq k$;
3. for any even $i > 1$, at most i branches B at v have $xs(B) \geq k - i/2 + 1$.

Gives a polynomial-time algorithm using dynamic programming

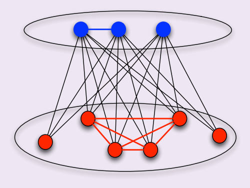


Exclusive Graph Searching in Cograph

Reminder: a graph is a cograph if

- single vertex, or
- disjoint union $G_1 \bigcup G_2$ of 2 cographs, or
- join $G_1 \otimes G_2$ of 2 cographs (add complete bipartite between G_1 and G_2)

The decomposition can be obtained in linear time [Corneil, Perl, Steward'85]



Exclusive search is monotone in cographs and linear-time algo. [Markou,N.,Pérennes]

Let G_1 and G_2 two cographs.

- $xs(G_1 \bigcup G_2) = xs(G_1) + xs(G_2)$
- $xs(G_1 \otimes G_2) \approx \min\{xs(G_1) + |V(G_2)|, xs(G_2) + |V(G_1)|\}$

(small difference if G_1 or G_2 are not connected, but can be well characterized)

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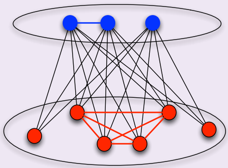
- join $G_1 \otimes G_2$ of 2 cographs (add complete bipartite between G_1 and G_2)

The decomposition can be obtained in linear time [Corneil, Perl, Steward'85]



Exclusive search is monotone in cographs and linear-time algo. [Markou,N.,Pérennes]

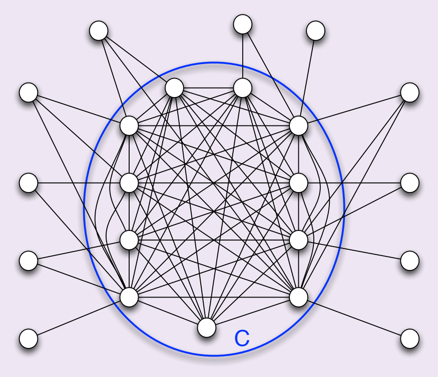
Let G_1 and G_2 two cographs.

- $xs(G_1 \bigcup G_2) = xs(G_1) + xs(G_2)$
- $xs(G_1 \otimes G_2) \approx \min\{xs(G_1) + |V(G_2)|, xs(G_2) + |V(G_1)|\}$

(small difference if G_1 or G_2 are not connected, but can be well characterized)

Exclusive Graph Searching in Split Graphs

Split Graph: $G = (I \cup C, E)$ if C induces a clique and I induces an independent set



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Charaterization of monotone strategies

$mxs(G) \leq k \Leftrightarrow$ it exists a particular strategy

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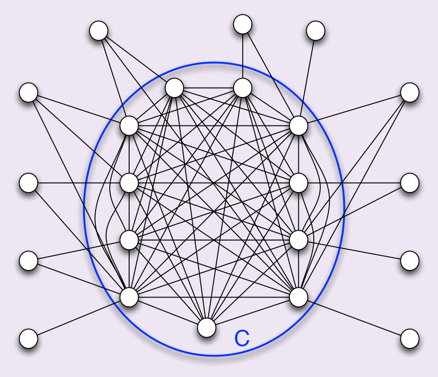

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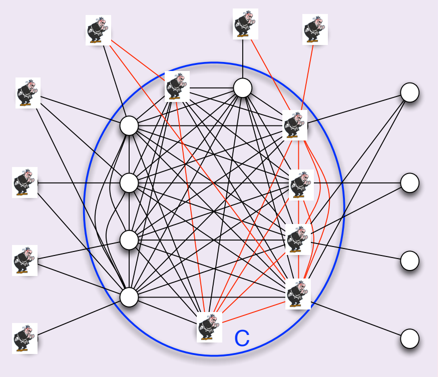

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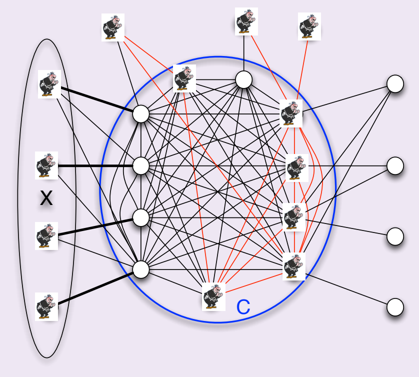

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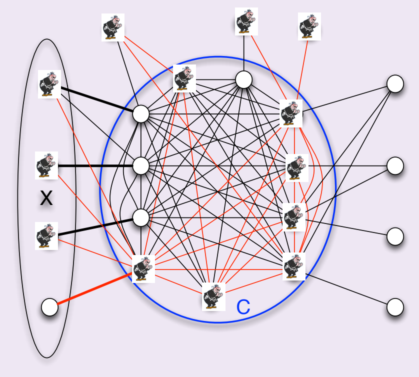

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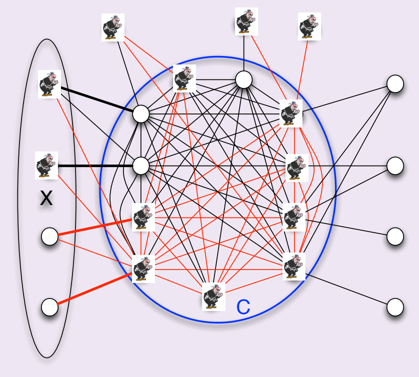

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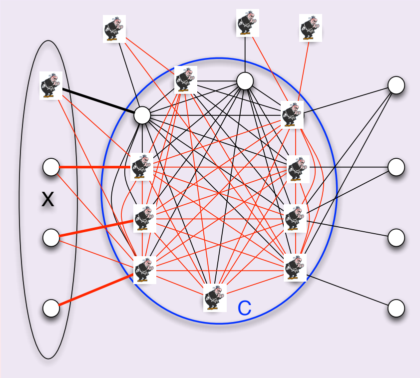

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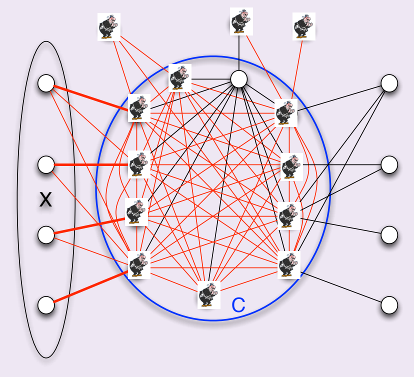

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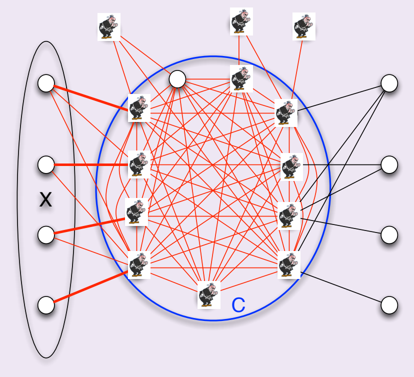

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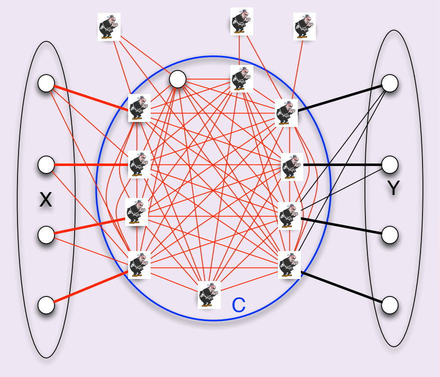

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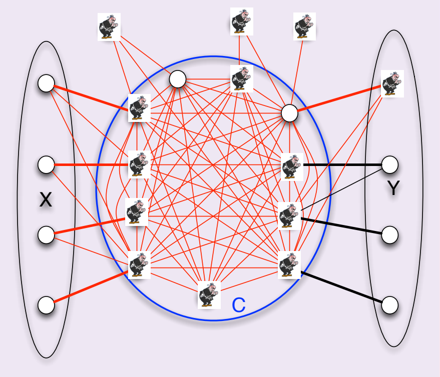

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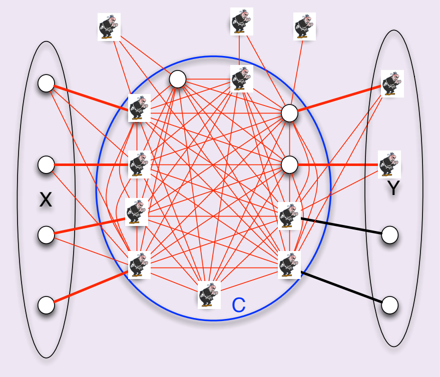

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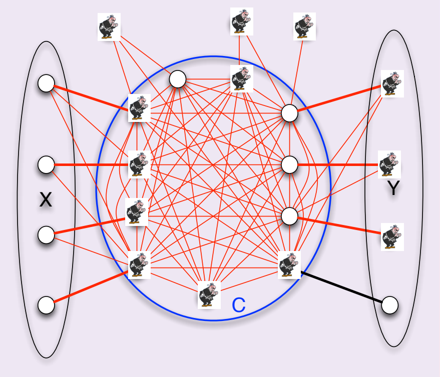

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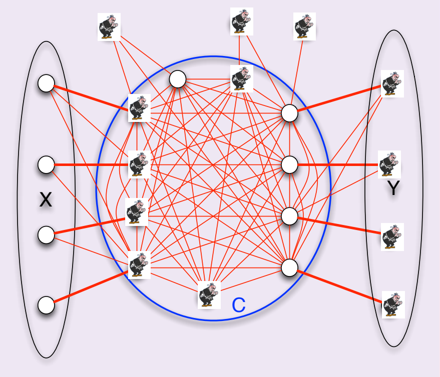

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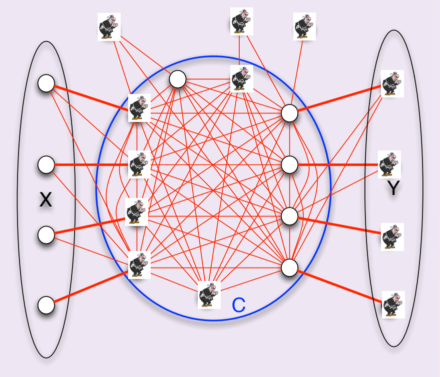
It uses $k = |V| - |X| - |Y| \ (-1)$ searchers



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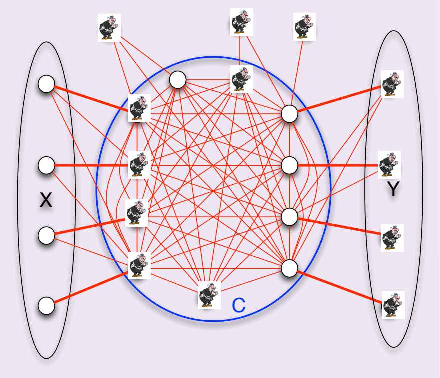
Find particular subsets as large as possible

$$X = \{x_1, \cdots, x_r\} \textit{ and } N(x_i) \setminus \bigcup_{j<i} N(x_j) \neq \emptyset.$$



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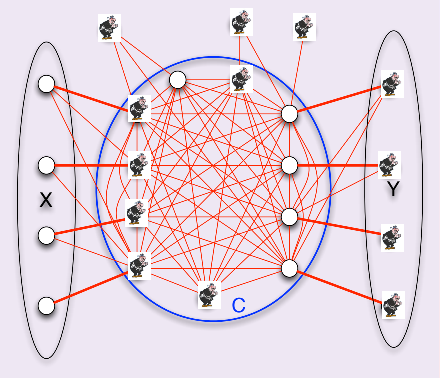

New problem:

input: $\{S_1, \cdots, S_n\}$ subsets of ground set A
output: a sequence $(S_{i_1}, \cdots, S_{i_r})$ such that $(\bigcup_{j \leq k} S_{i_j})_k$ strictly increasing and r is maximum

NP-hard (reduction from MIN-SAT)

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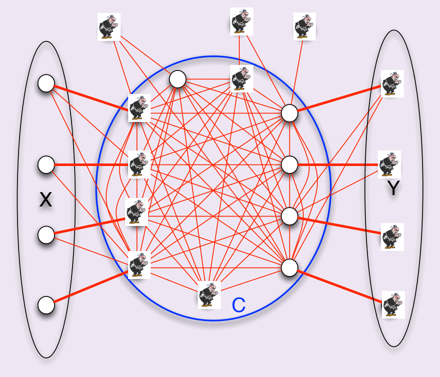
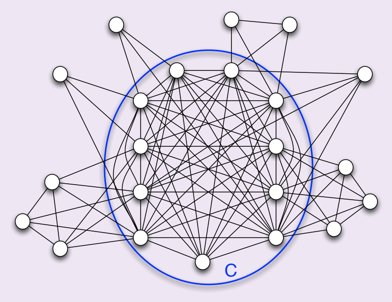
Theorem [Markou,N.,Pérennes]

Computing *mxs* is NP-complete in split graphs (contrary to pathwidth)



Exclusive Graph Searching in Split Graphs

Star-like: One Central clique C_0 and Peripheral cliques intersecting only in C_0



Theorem: Strategies are very constrained [Markou,N.,Pérennes]

G a star-like graph with each peripheral clique has at least two peripheral nodes.

1. Either there is an edge of C_0 that does not belong to any peripheral clique, and $mxs(G) = |V(G)| - r - 1$,
2. or $mxs(G) = |V(G)| - r$.

Perspectives on Exclusive Graph Searching

- Are there graph classes where *pw* is NP-complete
  and *xs* (*mxs*) in *P* and provide good approximation of *pw*?
  (or *vice-versa*)
- Can *xs* (or *mxs*) be approximated?
- *xs* in NP?
- *xs* (or *mxs*) FPT?
- $xs = mxs$ in split graphs?
- ...