How to beat a random walk with a clock?
Locating a Target With an Agent Guided by Unreliable Local Advice

Nicolas Hanusse\textsuperscript{1} \quad David Ilcinkas\textsuperscript{1} \quad Adrian Kosowski\textsuperscript{1,3} \quad Nicolas Nisse\textsuperscript{2}

\textsuperscript{1}CNRS-LaBRI, Université Bordeaux, Projet INRIA Cépage
\textsuperscript{2}INRIA Sophia-Antipolis, Projet Mascotte
\textsuperscript{3}Gdańsk University of Technology, Poland

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Many thanks to Nicolas Hanusse for his slides
How to find Dorian in MASCOTTE's floor?

Take advantage of your knowledge
Map, sense of direction (left/right, north/south...), topology...

Hanusse, Ilcinkas, Kosowski, Nisse

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Algorithm A (Advice)

Keep on following the advice until the target $t$ is found.
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Algorithm A (Advice)

Keep on following the advice until the target $t$ is found (LOOP).
How to find a cash machine in NY?

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Searching for information in networks with liars

Numerous adversaries

- the network map and the target location \textit{unknown}
- dynamicity $\rightarrow$ local information unreliable
Searching for information in networks with liars

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source: where to go?

ADVICE

IS IT A GOOD ADVICE?

shortest path

LIAR

shortest path

destination

source: where to go?
Searching for information in networks with liars

Numerous adversaries
- the network map and the target location unknown
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Some models of searching with errors
For every target $t$, each node gives an advice. If this advice is bad, the node is a liar.
- *Searching with uncertainty*, SIROCCO’1999, Kranakis-Krizanc

How to beat a random walk with a clock?
A model of searching with errors

**Local information**

- **advice**: For each target \( t \), every node \( u \) points to an incident link \( e \);
- If \( e \) is on a shortest path from \( u \) to \( t \), \( u \) is a **truth-teller**, otherwise a **liar**.

**Hypothesis**

- Advice and topology are **unchanged** during the search;
- **worst-case analysis**: The adversary knows the algorithm and chooses the worst configuration of advice.
A model of searching with errors

Local information
- advice: For each target $t$, every node $u$ points to an incident link $e$;
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- Advice and topology are unchanged during the search;
- worst-case analysis: The adversary knows the algorithm and chooses the worst configuration of advice.
### Performance Measures

- **$n$: # nodes, $k$: # liars, $d$: distance to the target**
- **Time $\mathcal{T}$**: # hops to reach the target
- **Memory $\mathcal{M}$**: of the mobile agent

![Diagram](https://via.placeholder.com/150)

With sense of direction:
- $\mathcal{T} \leq 2n$ et $\mathcal{M} = \Theta(1)$ - whole exploration
- $\mathcal{T} = \Theta(d)$ et $\mathcal{M} = \Theta(\log n)$ - zigzag
- $\mathcal{T} \leq d + 4k + 2$ et $\mathcal{M} = \Theta(\log k)$ - using advice, know $k$
Performance Measures

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State of the art

2 types of results

- specialized deterministic algorithms: ring, hypercube, complete graph, ...
- universal randomized algorithms with $M = 0$: random walk (R), biasest random walk (BR)

Examples

- $T = \Omega(d + 2^k)$ for binary trees for any algorithms;
- $T = O(d + 2^{\Theta(k)})$ for bounded degree graphs (BR);
- $T = \Omega(d + 2^k)$ for the path (BR)
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State of the art

Idea of the lower bound in trees

\[ T = \Omega(d + 2^k) \] for binary trees for any algorithms;

Everything is symmetric: \( \Omega(2^k) \) leaves look the same.
Contribution

Goal

Designing *universal* algorithms using the least amount of requirements:

- no hypothesis on ports/nodes labeling ...
- light mobile agents \( (M = O(\log n)) \)
- no knowledge on \( k \).

New results

New algorithm: using the moody walk or R/A and 
\[ M = O(\log k) \]

- \( E(T) = 2d + O(k^5) \) for the path
- \( E(T) = O(k^3 \log^3 n) \) for some expanders (random regular)
**Goal**

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New algorithm: using the moody walk or R/A and $M = O(\log k)$
- $\mathbb{E}(T) = 2d + O(k^5)$ for the path
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Algorithm BR

- With probability $p$, follow the advice (A);
- Otherwise, the agent chooses a neighbor at random (R).

Algorithm R/A $[L_R, L_A]$

Keep on alternating Algorithm R for $L_R$ hops and Algorithm A for $L_A$ hops.
Ring or path

Gain: r.v $X$ - distance reduction to $t$ in one iteration.

Phase R during L steps

- With probability $1 - 2/k^c$, $|X_R| < \sqrt{L \log k}$;
- With probability $1 - 2/e$, $|X_R| < \sqrt{L}$.

Algorithm R/A[$L, L$]

- Safe area: whp. $X_A > L - \sqrt{L \log k}$.
- Dangerous area: connected component of nodes at distance at most $\sqrt{L \log k}$ from liars.
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Path

Phase A (dangerous area)

1. \(-k \leq X_A < 0\) with prob. \(O(k/\sqrt{L})\);
2. \(X_A \geq \sqrt{2L}/k - 1\) with prob. \(\Theta(1)\);
3. \(\mathbb{E}(X) = \mathbb{E}(X_A)\) and \(\mathbb{E}(X_A) \geq \frac{\sqrt{L}}{40k} - 1\).
Performance of Moody walk on Path

With knowledge of $k$

For $L > 32k^3$, R/A[L,L] finds the target in

$T = 2d + O(Lk^2 \log k) + o(d)$ with probability $1 - \Theta(1/k^{c-3})$,  
\[ M = O(\log k) \]

Without knowledge of $k$

Iterating R/A[$2^{3i}$, $2^{3i}$], increasing $i$ every $i2^{2i}$ phases:

$T = 2d + O(k^5 \log k) + o(d)$ with probability $1 - \Theta(1/k^{c-3})$

Remainder: BR: $T = \Theta(d + 2^k)$ for the path, $M = 0$
Graphs of high expansion are useful

- For any set $S$ of nodes, the neighborhood of $S$ is of size $\Omega(|S|)$
- P2P networks, error-correcting codes, small-world

A family of expanders: random $\Delta$-regular graphs

- Diameter $= \log_{\Delta-1} n + \log_{\Delta-1} \log n + D$ (Bollobás, De la Vega 1982)
- Fast mixing: Every node is reached with probability $\Theta(1/n)$ following a random walk of length $8 \frac{\log n}{\log(\Delta/4)}$ (Friedman 2003, Cooper 2005)
- Local Tree-like: There is a constant $c = 1/2$ such that the neighborhood at distance $c \log_{\Delta-1} n$ is a tree (Cooper-Frieze-Radzik 2008)
Expanders

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R/A/E with knowledge

Beat the adversary?
Starting from a random source

- With prob. $\Theta(1)$, only truth-tellers are encountered during phase A.
- With prob. $\Theta(1)$, the target is "close" to the current position (phase E: BFS exploration).
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**R/A/E with knowledge**

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How to beat a random walk with a clock?
Toward R/A without knowledge of $n$ and $k$

**R/A/E WITH knowledge of $n$, $k$ and diameter**

- **Phase R**: $L_R = 8 \frac{\log n}{\log(\Delta/4)}$ steps
- **Phase A**: $L_A = \log_{\Delta-1} n - 1 - \log_{\Delta-1} k$ steps
- **Phase E**: radius $r_E = \log_{\Delta-1} k + \log_{\Delta-1} \log n + D + 1$

**Theorem**: In a random $\Delta$-regular graph, the target is found in $O(k \log n)$ steps with probability $\Theta(1)$.

**R/A[LR + rE, LA]** (Simulation of phase E)

In phase R, a target at distance $r_E$ is reached with probability $\Omega(\frac{1}{k \log n})$.

**Theorem**: In a random $\Delta$-regular graph, the target is reached in $O(k^2 \log^2 n)$ steps with probability $\Theta(1)$. 

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How to beat a random walk with a clock?
### Toward R/A without knowledge of $n$ and $k$

**R/A/E WITH knowledge of $n$, $k$ and diameter**

- **Phase R:** $L_R = 8 \frac{\log n}{\log(\Delta/4)}$ steps
- **Phase A:** $L_A = \log_{\Delta-1} n - 1 - \log_{\Delta-1} k$ steps
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### R/A[$L_R + r_E$, $L_A$] (Simulation of phase E)

In phase R, a target at distance $r_E$ is reached with probability $\Omega\left(\frac{1}{k \log n}\right)$.

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How to beat a random walk with a clock?
R/A/E without knowledge of $n$, $k$ and diameter

R/A without knowledge

- $j \leftarrow 1$;
- $i = 1..j$, Run R/A[100i, i]
- $j++$

**Theorem:** In a random $\Delta$-regular graph, the target is found within $O(c^3k^3\log^3 n)$ steps with probability $1 - 1/2^{\Omega(c)}$. 
To sum up (without knowledge of $n$ and $k$)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Path</th>
<th>Mean search time</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR $[p &lt; 1/2]$</td>
<td>$2^{\Omega(d)}$</td>
<td>$\Theta\left(\min{(\Delta - 1)^k, n^{\Theta(1)}}\right)$</td>
</tr>
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<tr>
<td>R/A*</td>
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<td>$O(k^3 \log^3 n)$</td>
</tr>
<tr>
<td>Lower B.*</td>
<td></td>
<td>$\Omega\left(\min{(\Delta - 1)^k, (\Delta - 1)^d, \log_{\Delta-1} n}\right)$</td>
</tr>
</tbody>
</table>

*No hypothesis on the in-ports is assumed for the upper bounds but the lower bound assumes one.
What you need to retain

Do not blindy follow the advice but find the good proportions !!

Challenges

- Other graphs family: $D$-dimensionnal grids ?
- With less memory ?
- R/A[$n^3, 0$] universal: How to parameter ?
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