Graph Searching and related problems.

Graph Minors, connected search strategies, and distributed approach

Nicolas Nisse

DIM, Universidad de Chile, Project Anillo en Redes.

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Outline

General problem

Context

A fugitive is running in a graph.
A team of searchers is aiming at capturing the fugitive.

Goal

To design a strategy that capture **any** fugitive using the **fewest searchers as possible**.

Initially studied by Breish (67) and Parson (76)

To save a speleologist lost in a caves'network.
Equivalently, to clear a contaminated pipelines'network.

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Motivations

Layout Problems

Numerical analysis, VLSI design, etc.
Related to: bandwidth, cutwidth, profile, etc.

Computational Complexity

Relationship between graph searching and pebble games [Kirousis and Papadimitriou 86]
Tradeoff space/time complexity of computation.

Graph Minors Theory

Robertson and Seymour [JCTB, 1983-]
Tree decomposition, Path decomposition

Search Strategy, Parson. [GTC,1978]
Model of Kirousis and Papadimitriou. [TCS,86]

The fugitive

- occupies vertices of the graph;
- is arbitrary fast;
- can move from one vertex to another by following a path in G, as long as it does not cross any vertex occupied by a searcher.

The searchers

- can jump from one vertex to any other vertex of the graph;
- are visible.

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Sequence of two basic operations,. . .

1. Place a searcher at a vertex of the graph;
2. Remove a searcher from a vertex of the graph.

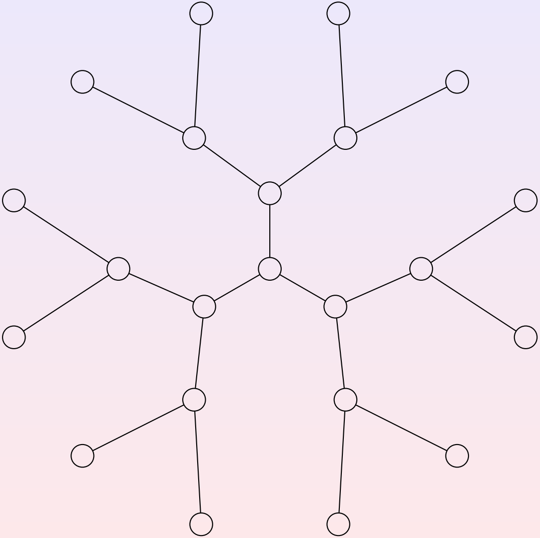
. . . that must result in catching the fugitive

The fugitive is caugth when it occupies the same vertex as a searcher and it cannot move away.

The node-search number

Let $\mathbf{s}(G)$ be the smallest number of searchers needed to catch an invisible fugitive in a graph G.

Simple example: A ternary tree



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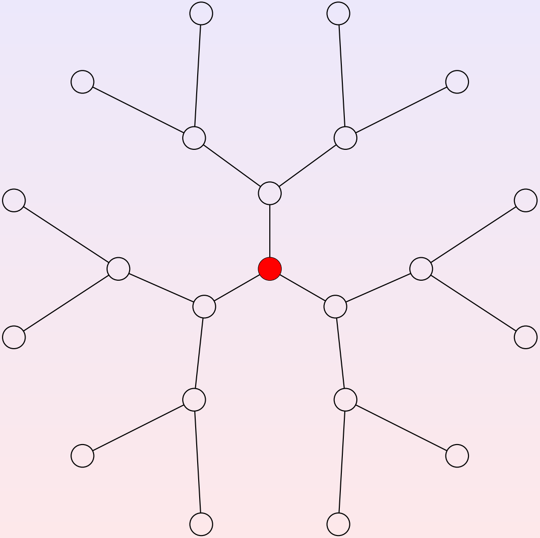

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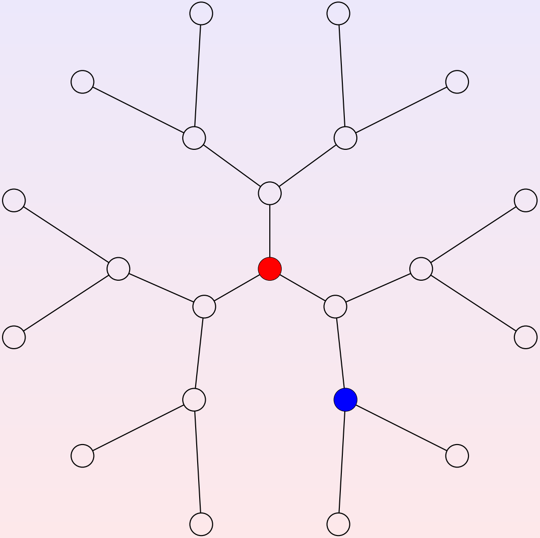

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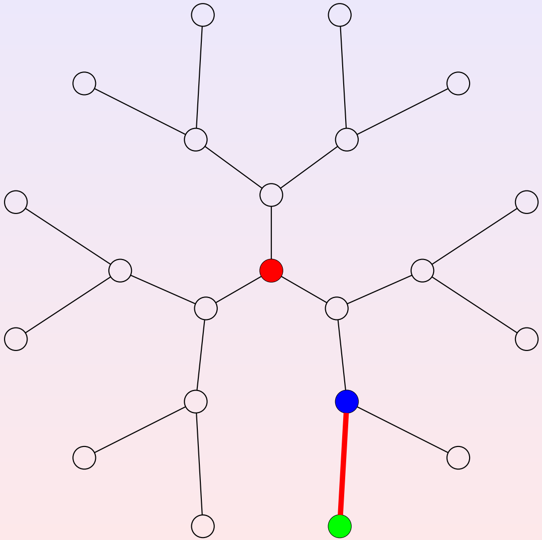

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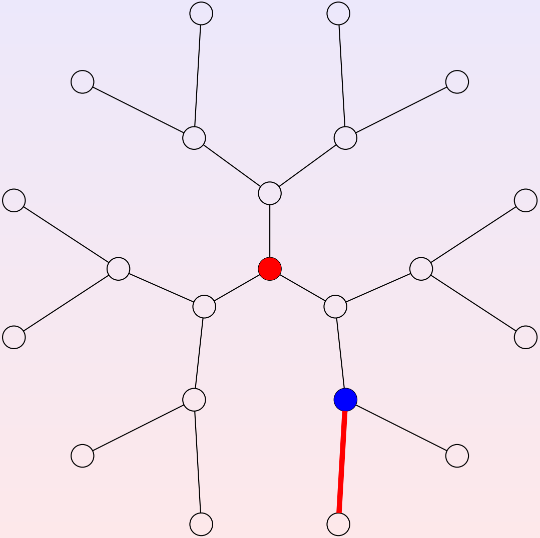

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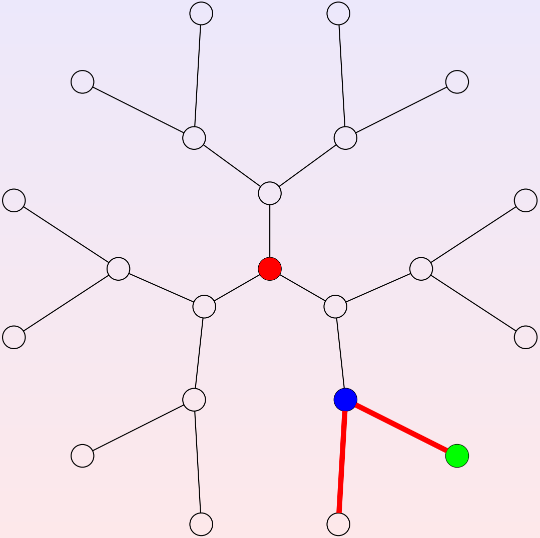

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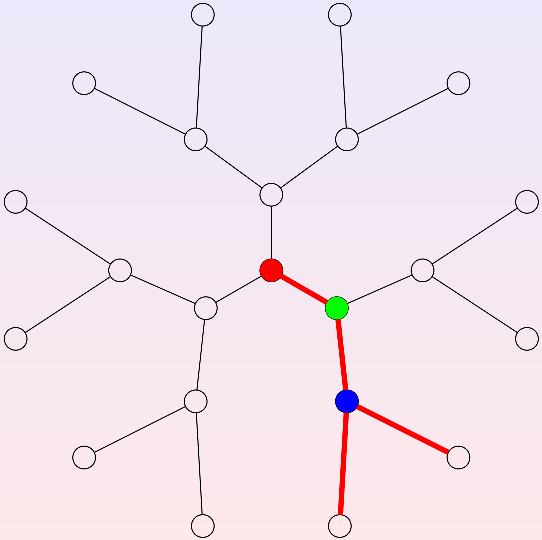

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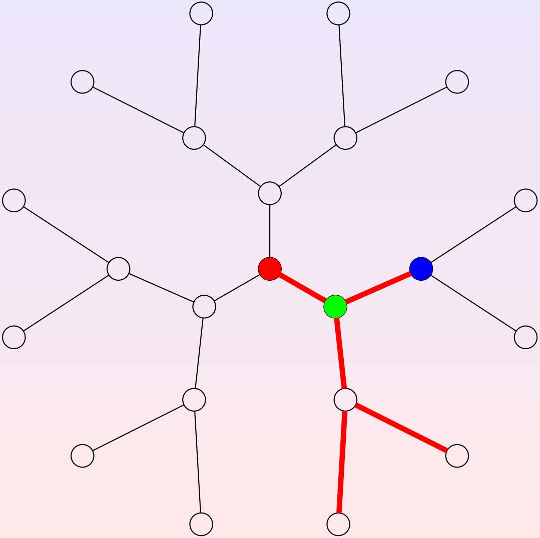

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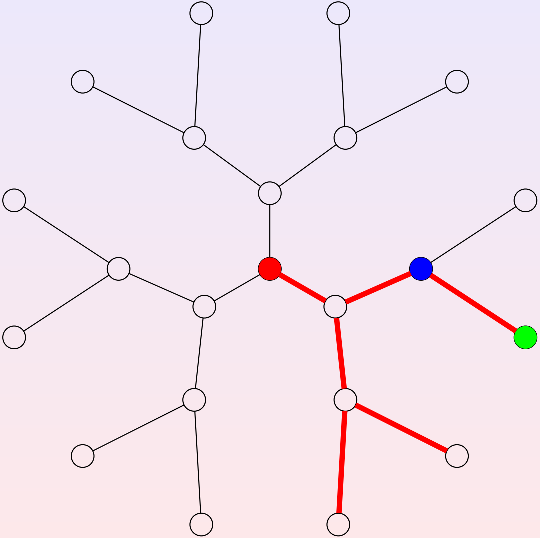
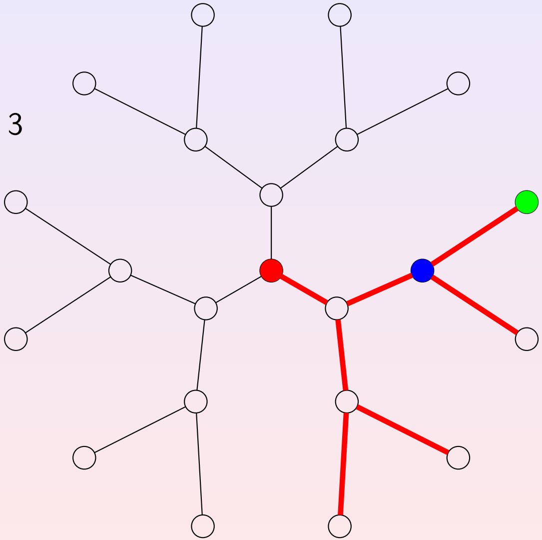


Simple example: A ternary tree

$\mathbf{s}(T) = 3$



Visibility of the fugitive

Visible fugitive

The fugitive is visible if, at every step, searchers know its position.
Let $\mathbf{vs}(G)$ be the visible search number of the graph G.

Obviously, for any graph G, $\mathbf{vs}(G) \leq \mathbf{s}(G)$.

Invisible fugitive vs. visible fugitive in trees

For any n-nodes tree T, $\mathbf{s}(T) \leq 1 + \log_3(n-1)$ (tight) Megiddo *et al.* [JACM 88]

For any tree T (with at least 2 vertices), $\mathbf{vs}(T) = 2$.

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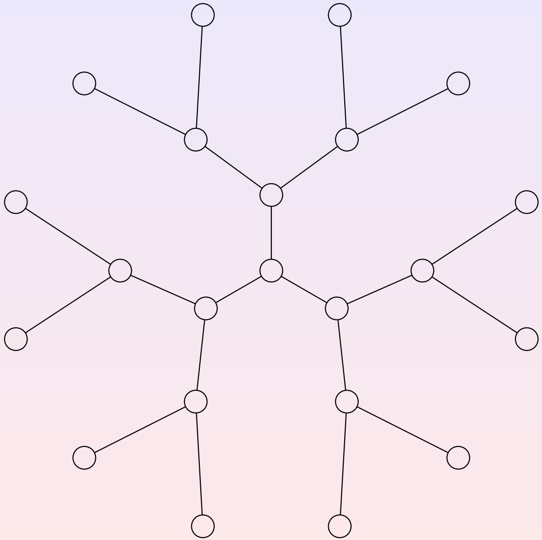

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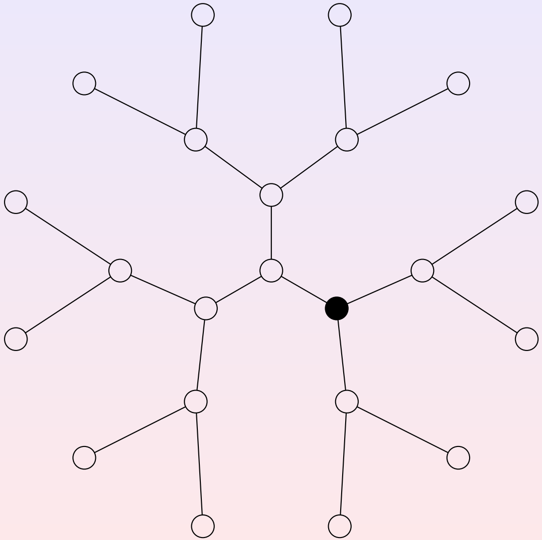

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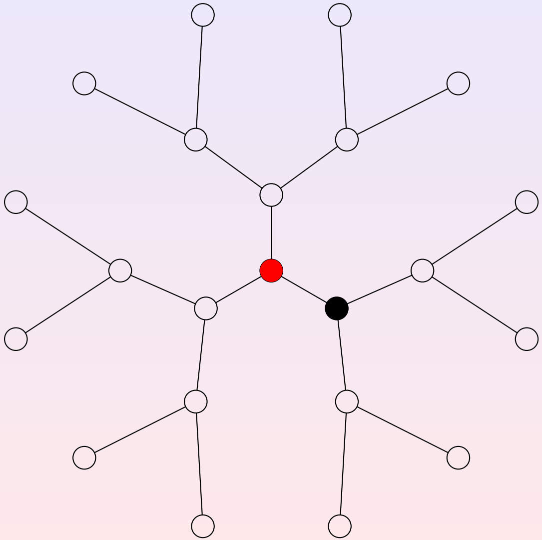

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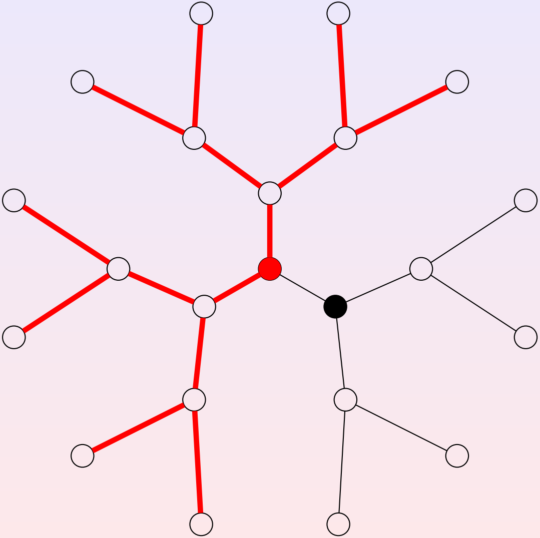

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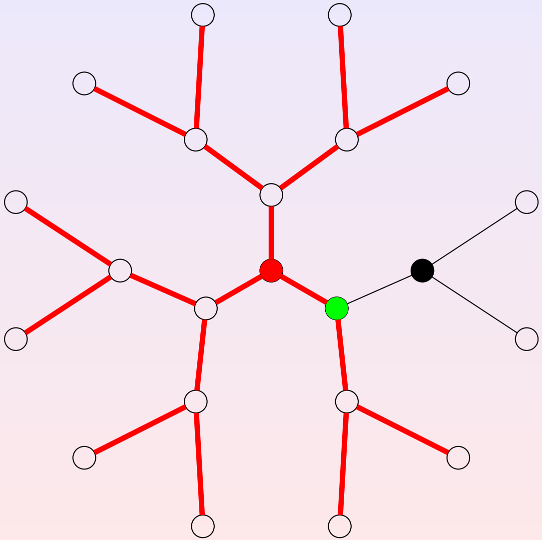

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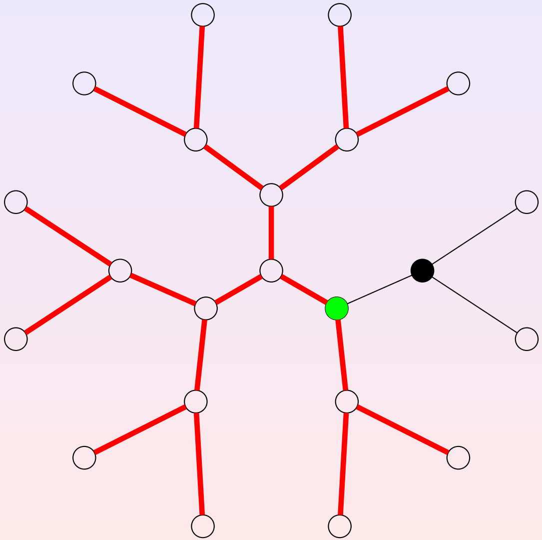
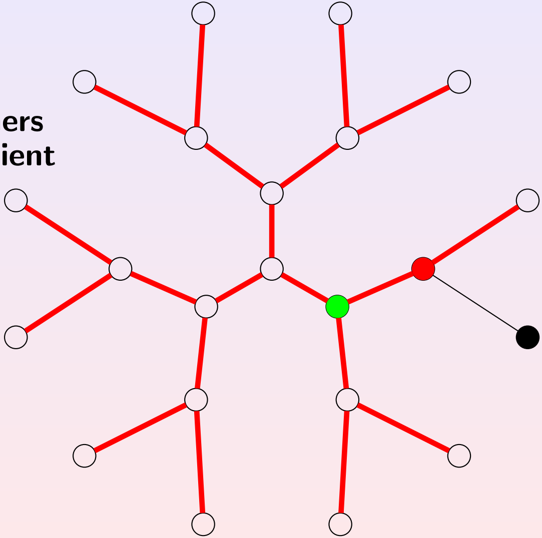


Visible graph searching in a tree

2 searchers are sufficient



NP-hardness

The following problems are NP-hard

Input:	a graph G, an integer $k > 0$,	Megiddo *et al.*,
Output:	$\mathbf{s}(G) \leq k$?	[JACM 88]
Input:	a graph G , an integer $k > 0$,	Seymour and Thomas
Output:	$\mathbf{vs}(G) \leq k$?	[JCTB 93]

Remark: linear in the class of trees, Skodinis [JAlg 03]

NP-membership? Certificate?

A strategy is a certificate, but what is its size ?

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Monotonicity

Monotone strategies

A vertex v is recontaminated if the fugitive can move to v after v has been occupied by a searcher.
A search strategy is monotone if no recontamination ever occurs. That is, a vertex is occupied by a searcher only once.

A monotone strategy consists of a polynomial number of steps

Question: Does recontamination help?

In other words, for any graph, does there always exist a monotone strategy using the smallest number of searchers?

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Recontamination does not help

Case of an invisible fugitive

- **Bienstock and Seymour**, J.of Alg., 1991
  Monotonicity in graph searching.
- **LaPaugh**, J.of ACM, 1993
  Recontamination does not help to search a graph.

Constructive proofs: Any strategy using k searchers can be turned into a *monotone* strategy using $\leq k$ searchers.

Case of a visible fugitive

- **Seymour and Thomas**, J. of Comb. Th., 1993.
  Graph searching and a min-max theorem for tree-width

Non constructive proof.

Corollary: To compute $\mathbf{s}(G)$ (resp., $\mathbf{vs}(G)$) is NP-complete.

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Search numbers and graphs'decompositions

Thanks to the monotonicity, we get:

Search number and Pathwidth (**pw**)

For any graph G, $\mathbf{s}(G) = \mathbf{pw}(G) + 1$,
Kinnersley [IPL 92],
Ellis, Sudborough, and Turner [Inf.Comp.94]

Visible search number and Treewidth (**tw**)

For any graph G, $\mathbf{vs}(G) = \mathbf{tw}(G) + 1$,
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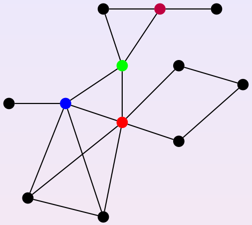
Tree and Path Decompositions



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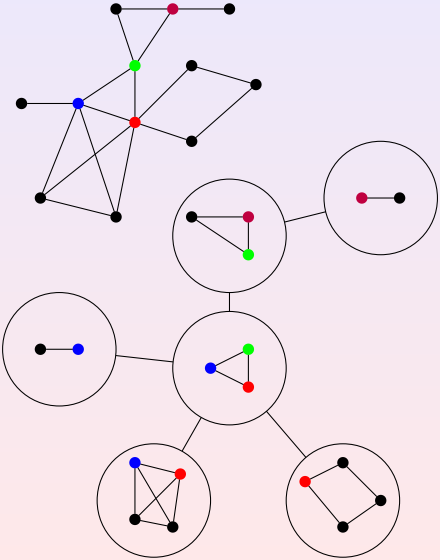

a tree T and bags $(X_t)_{t \in V(T)}$

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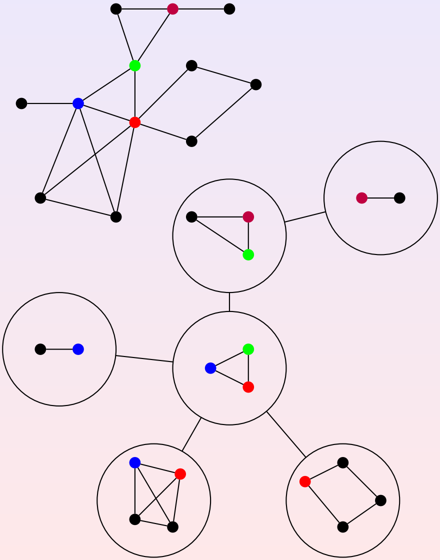

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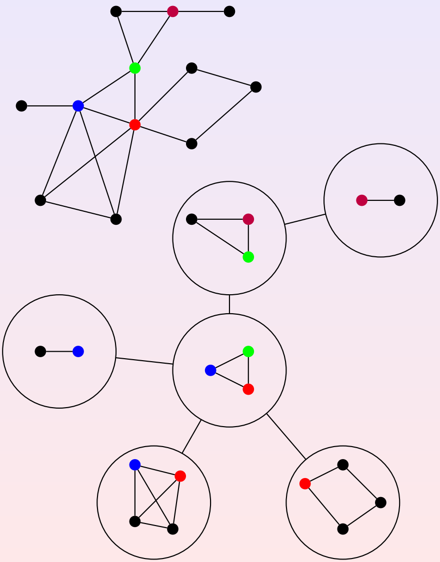

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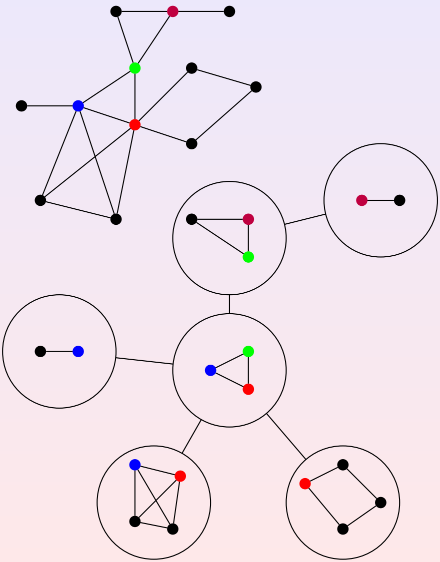

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- For any vertex of G, all bags that contain it, form a subtree.

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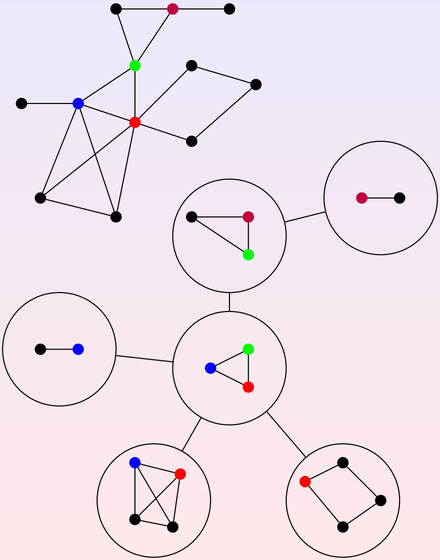

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Width = Size of largest Bag -1

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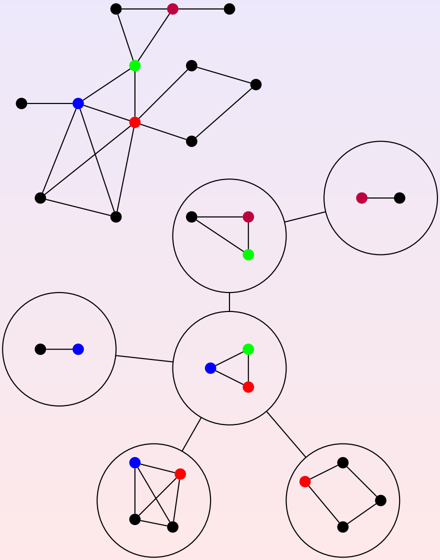

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treewidth of G

tw(G), minimum width among any tree-decomposition

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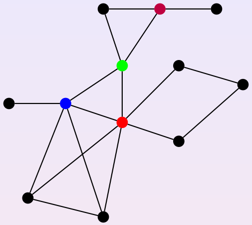

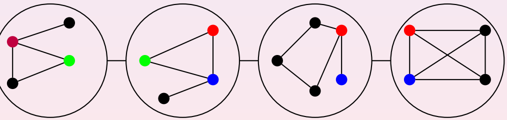

a path P and bags $(X_t)_{t \in V(P)}$

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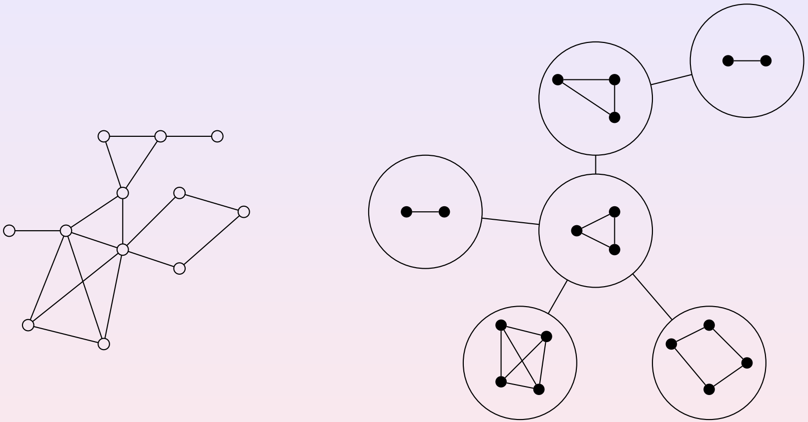
- For any vertex of G, all bags that contain it, form a **subpath**.

Width = Size of largest Bag -1

pathwidth of G

pw(G), minimum width among any path-decomposition

Monotone Visible Search Number $=$ Treewidth $+1$



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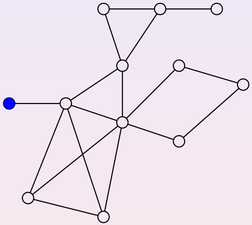

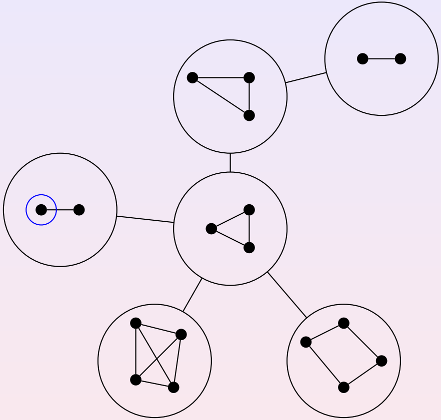

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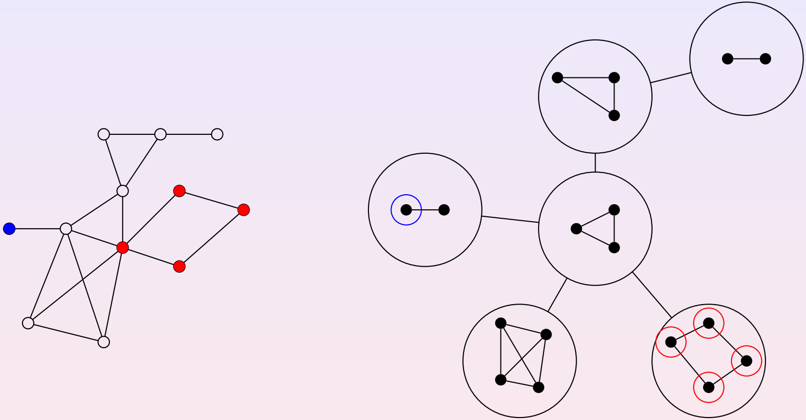

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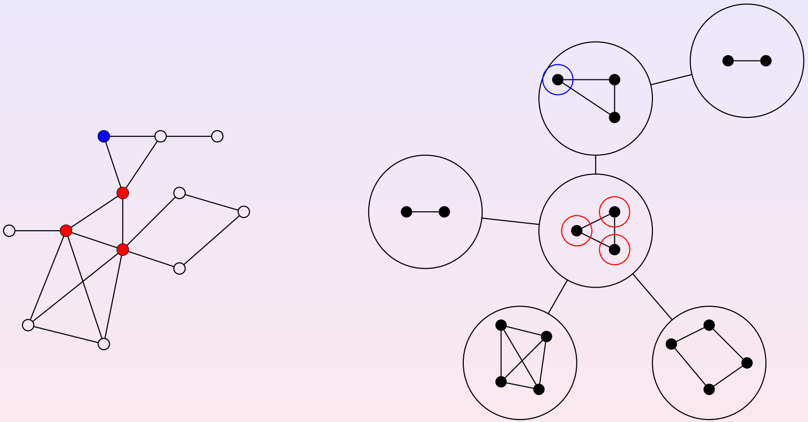

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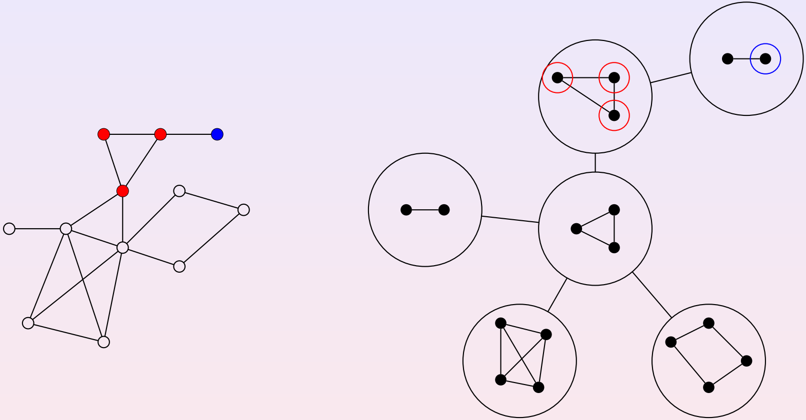

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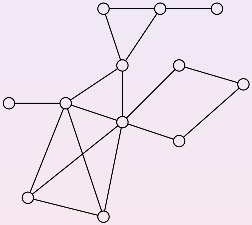

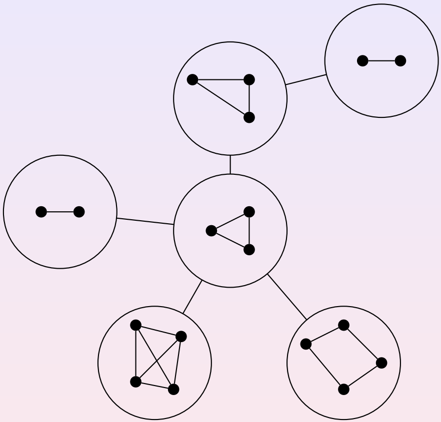

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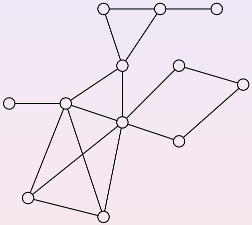

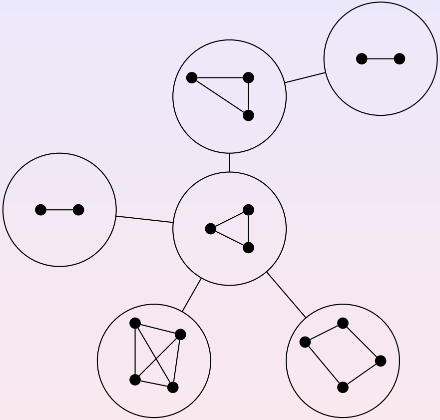

Thus, $\mathbf{vs(G)} \leq \mathbf{tw(G)} + 1$

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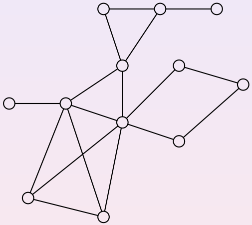

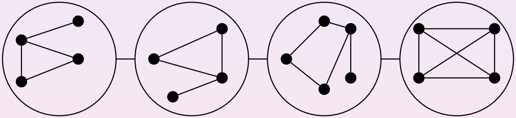

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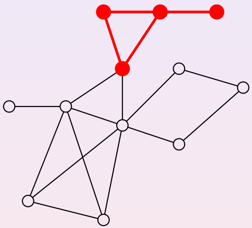

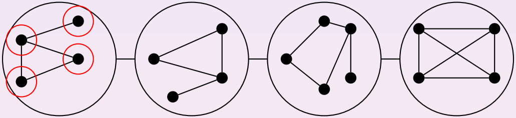

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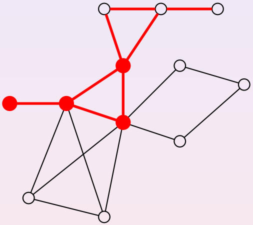

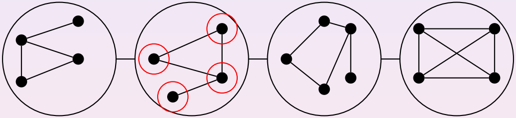

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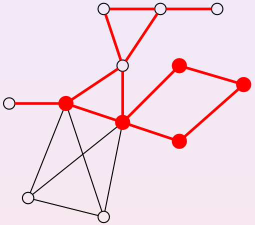

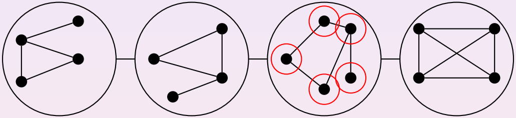

Thus, $\mathbf{s(G)} \leq \mathbf{pw(G)} + \mathbf{1}$

Outline

Non-deterministic Graph Searching

Invisible fugitive

An Oracle permanently knows the position of the fugitive

One extra operation is allowed

Searchers can perform a query to the oracle:
"What is the current position of the fugitive?"

Sequence of three basic operations

1. Place a searcher at a vertex of the graph;
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Tradeoff number of searchers / number of queries

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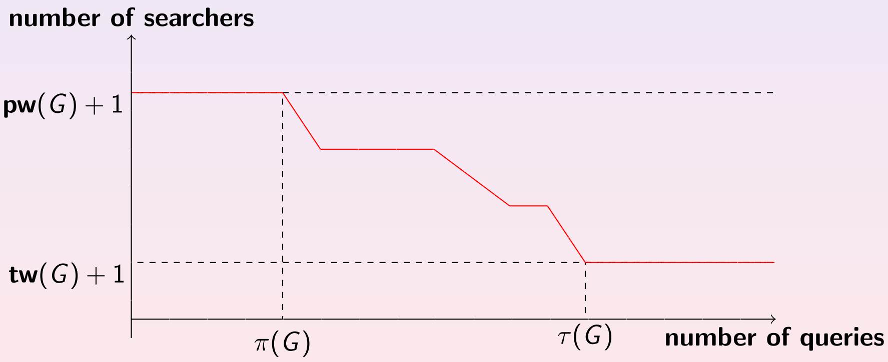
Tradeoff number of searchers / number of queries

Controlled Amount of Nondeterminism

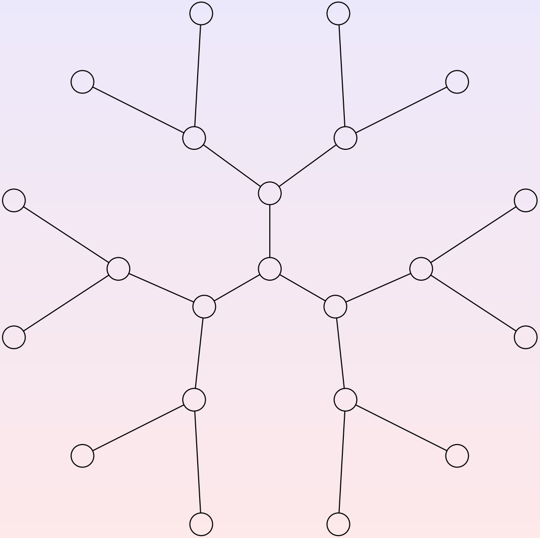
q-limited (non-deterministic) search number, $s_q(G)$

- $\mathbf{s}_0(G) = \mathbf{pw}(G) + 1$, invisible search number of G;
- $\mathbf{s}_\infty(G) = \mathbf{tw}(G) + 1$, visible search number of G.



Still the same ternary tree

$s_0(T)=3$



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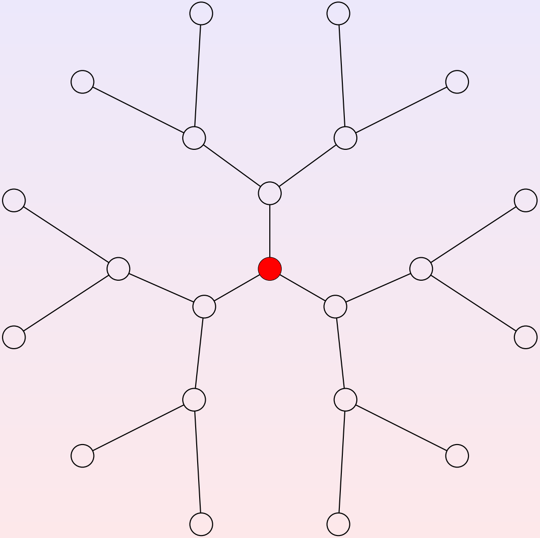
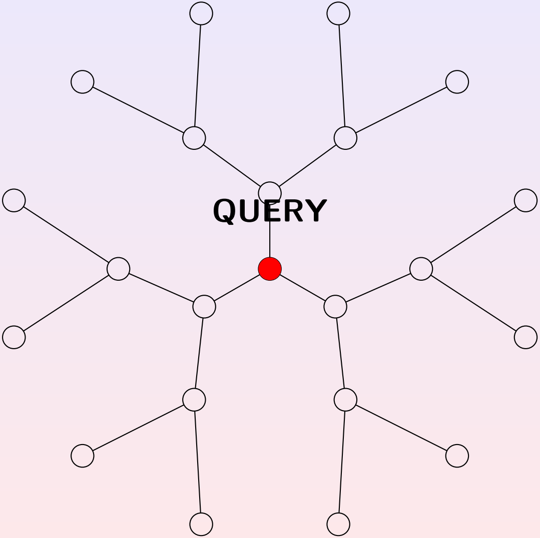


Still the same ternary tree

1 query



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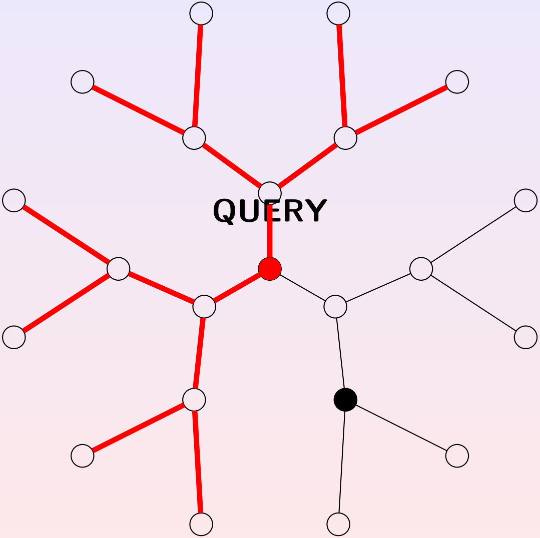

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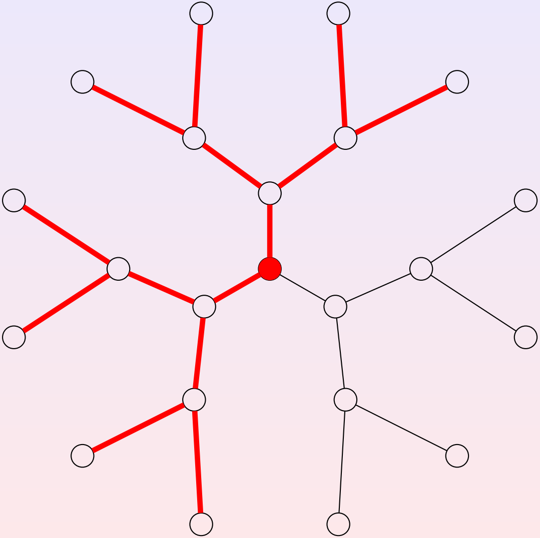

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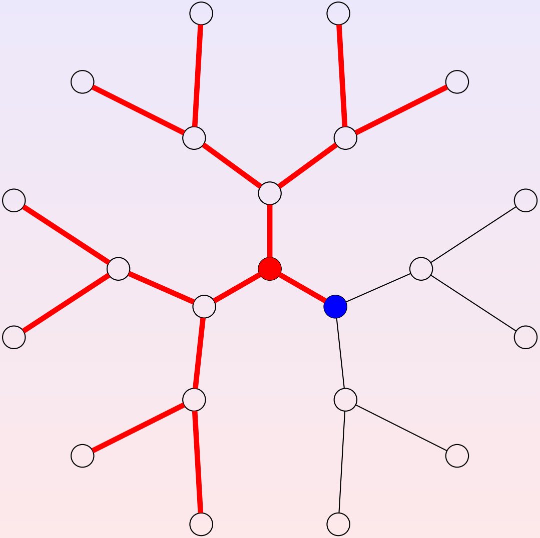
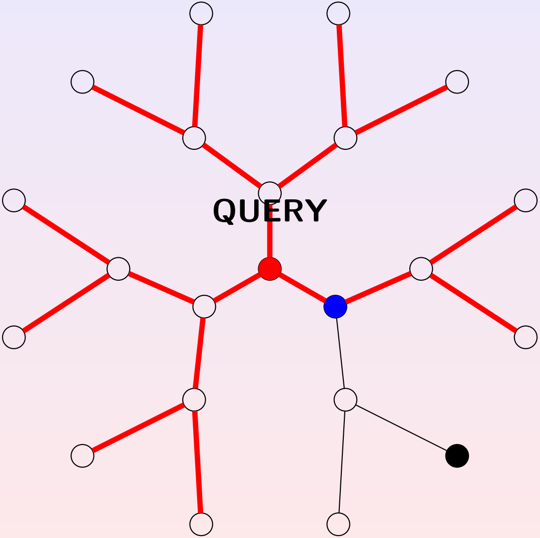


Still the same ternary tree

2 queries



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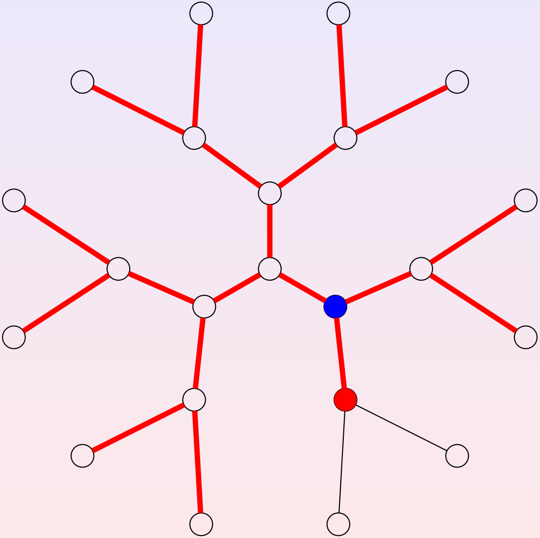

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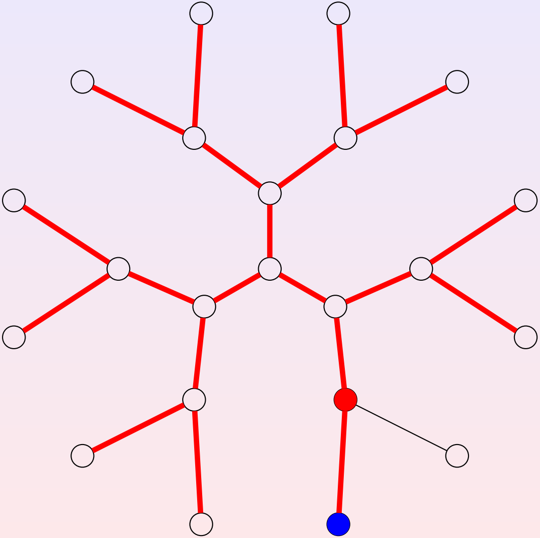
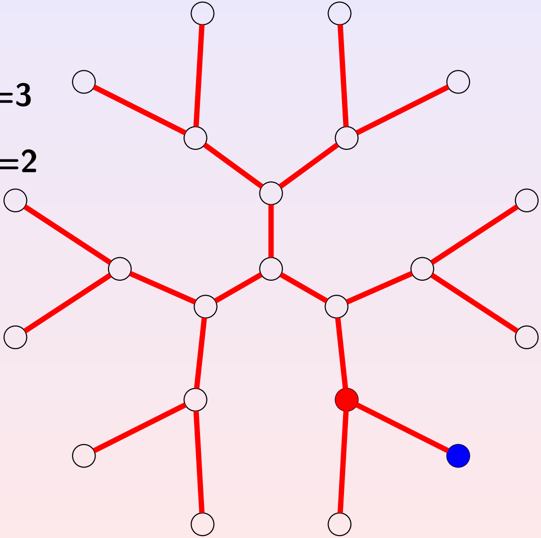


Still the same ternary tree

$s_0(T)=s_1(T)=3$

$s_2(T)=s_\infty(T)=2$



Results

Monotonicity [Mazoit and N.]

For any $q \geq 0$, recontamination does not help to catch a fugitive in G performing at most q queries.

- Constructive proof;
- Generalize the existing proofs ($q = 0$ and $q = \infty$).

Graph's Decompositions [Fomin, Fraigniaud and N.]

- Equivalence between non-deterministic graph searching and branched tree-decomposition;
- Exponential exact algorithm computing $\mathbf{s}_q(G)$ in time $O^*(2^n)$;
- $\mathbf{s}_q(G) \leq 2\, \mathbf{s}_{q+1}(G)$ (almost tight).

Monotonicity: Search-tree

Auxiliary structure inspired by the tree-labelling [Robertson and Seymour, Graph Minor X]:

Search-tree = A rooted tree T labelled with subsets of $E(G)$

For any vertex $v \in V(T)$ incident to $e_1, \ldots, e_p$:

- label of v: $\ell(v) \subseteq E(G)$
- label of e_i: $\ell_v(e_i) \subseteq E(G)$

Any edge has two labels: one for each extremity.

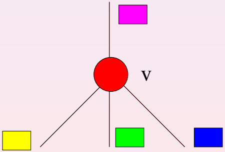
Two Properties

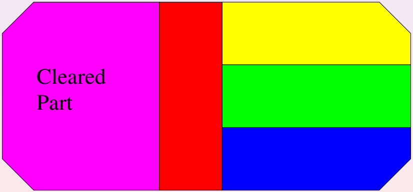
1. $\{\ell(v), \ell_v(e_1), \ell_v(e_2), \ldots, \ell_v(e_p)\}$ partition of $E(G)$;
2. $\forall e = \{u, v\} \in E(T)$, $\ell_v(e)$ and $\ell_u(e)$ are disjoint.

Monotonicity: Search-tree

Non-deterministic search strategy $\Rightarrow$ Search-tree

- placement of searchers $\Rightarrow$ vertex of T
- query $\Rightarrow$ fork (vertex of T with more than one child)
- removal of searchers $\Rightarrow$ edge of T





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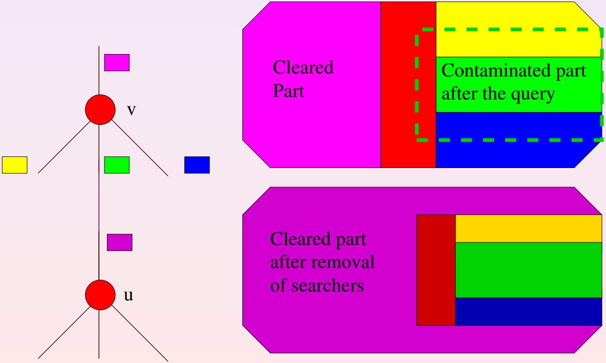

Monotonicity

Search-tree = a rooted tree T labelled with subsets of $E(G)$.

Remark: a search tree $\approx$ relaxed tree decomposition

Sketch of the proof

- (possibly non monotone) strategy $\Rightarrow$ Search-tree
- weight function over the search-trees
- minimal search-tree $\Rightarrow$ monotone strategy
- local optimization without increasing neither the number of searcher, nor the number of queries.

Recent Results and Open Problems

Class of trees

- polynomial time algorithm to compute $\mathbf{s}_1(T)$ and polynomial time 2-approximation to compute $\mathbf{s}_q(T)$, for any $q \geq 0$. [Amini, Coudert, and N.]
- $\exists$? polynomial (linear?) time algorithm to compute $\mathbf{s}_q(T)$, for any tree T, and $q \geq 0$.

Search-tree

- Generalization of the min-max theorem for treewidth [Amini, Mazoit, N. and Thomassé]
- (FPT) Algorithms? Excluding Minor Theorem?
- Application to matroids.

Outline

Connected Graph Searching

Limits of the Parson's model

- Searchers cannot move at will in a real network;
- Secured communications.

Connected Search Strategy, Barrière *et al.*, [SPAA 02]

At any step, the cleared part of the graph must induce a connected subgraph.
Let $\mathbf{cs}(G)$ be the connected search number of the graph G.

Two main questions

What is the cost of connectivity? ratio $\mathbf{cs}/\mathbf{s}$?
Monotonicity property of connected graph searching?

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The cost of connectedness

In terms of number of searchers

For any tree T, $\mathbf{s}(T) \leq \mathbf{cs}(T) \leq 2\ \mathbf{s}(T) - 2$. (tight)
Barrière, Flocchini, Fraigniaud, and Thilikos [WG 03]

For any connected graph G, $\mathbf{cs}(G) \leq \mathbf{s}(G)\ (2 + \log |E(G)|)$.
Fomin, Fraigniaud, and Thilikos [Tech. Rep. 04]

About monotonicity

Recontamination does not help in trees.
Barrière, Flocchini, Fraigniaud, and Santoro [SPAA 02]

Recontamination helps in general.
Alspach, Dyer, and Yang [ISAAC 04]

Results: Case of a invisible fugitive

Using the concept of *connected* tree-decomposition.

Cost of connectivity [Fraigniaud and N.]

For any n-node connected graph G, $\mathbf{cs}(G)/\mathbf{s}(G) \leq \log n$.

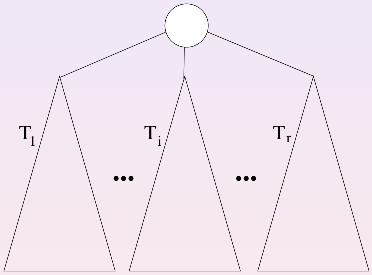
Graphs with bounded chordality k [N.]

(T, X) an optimal tree-decomposition of G
$\mathbf{cs}(G) \leq (\mathbf{tw}(G)\lfloor k/2 \rfloor + 1)\mathbf{cs}(T)$.

$\Rightarrow \mathbf{cs}(G)/\mathbf{s}(G) \leq 2\ (\mathbf{tw}(G) + 1)$ if G chordal

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

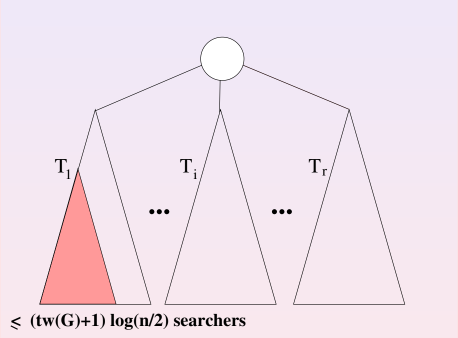
Proof by induction on n: $\mathbf{cs}(G) \leq (\mathbf{tw}(G)+1)\ \log n$



For any $1 \leq i \leq r$, $G[T_i]$ is a connected subgraph with at most $n/2$ vertices.

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

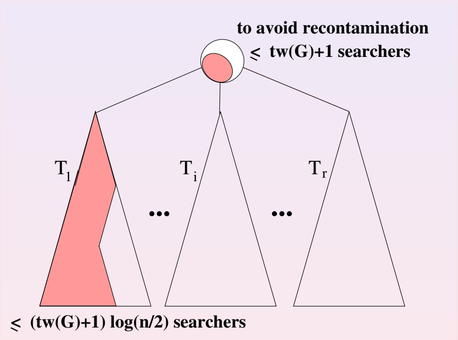
Proof by induction on n: $\mathbf{cs}(G) \leq (\mathbf{tw}(G)+1)\ \log n$



There is a connected search strategy for $G[T_1]$, using at most $(\mathbf{tw}(G)+1)\log(n/2)$ searchers.

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

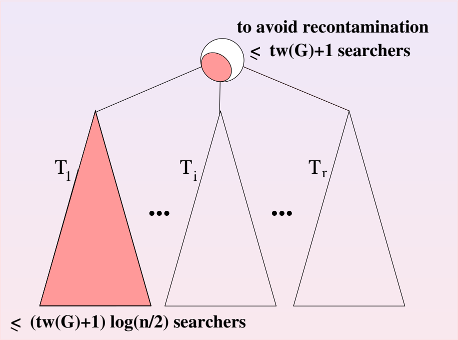
Proof by induction on n: $\mathbf{cs}(G) \leq (\mathbf{tw}(G)+1)\ \log n$



At most $\mathbf{tw}(G)+1$ searchers are required to protect $G[T_1]$ from recontamination from the remaining part of G.

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

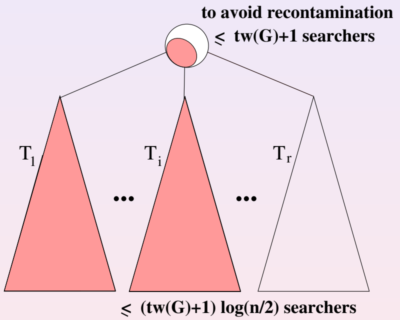
Proof by induction on n: $\mathbf{cs}(G) \leq (\mathbf{tw}(G)+1)\ \log n$



Then, we can terminate the clearing of $G[T_1]$.

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

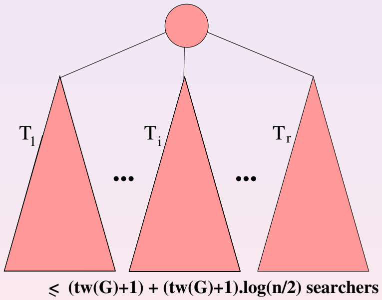
Proof by induction on n: $\mathbf{cs}(G) \leq (\mathbf{tw}(G)+1)\ \log n$



The $(\mathbf{tw}(G)+1)\log(n/2)$ searchers can be used to clear another subgraph $G[T_i]$, and so on...

Sketch of proof: $\mathbf{cs}(G) \leq \mathbf{s}(G)\ \log n$

Proof by induction on n: $\mathbf{cs}(G) \leq (\mathbf{tw}(G)+1)\ \log n$



Connected search strategy using at most $(\mathbf{tw}(G)+1)\log n$ searchers. Thus, $\mathbf{cs}(G) \leq \mathbf{s}(G)\log n$

Results: Case of a visible fugitive

Cost of connectivity [Fraigniaud and N.]

For any n-node graph G, $\mathbf{cvs}(G)/\mathbf{vs}(G) \leq \log n$
tight for monotone strategies: $\mathbf{mcvs}(G)/\mathbf{vs}(G) \geq \Omega(\log n)$.

Non Monotonicity [Fraigniaud and N.]

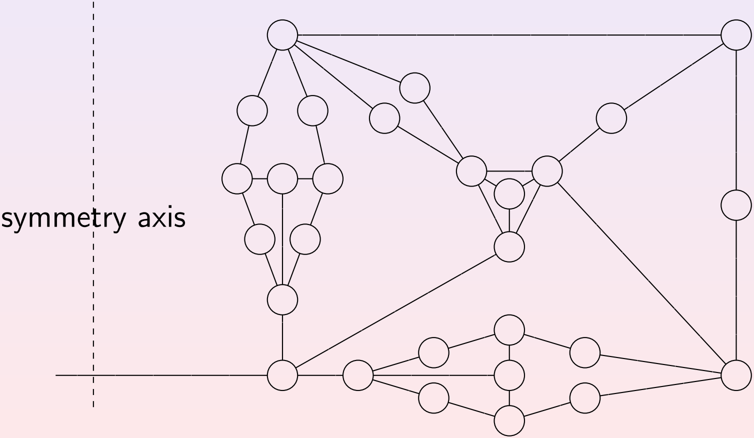
In visible connected graph searching, recontamination helps

For any $k \geq 4$, there exists a graph G such that $\mathbf{cvs}(G) = 4k+1$ and any monotone connected visible search strategy uses at least $4k+2$ searchers.

Non-monotonicity

Recontamination helps in visible connected graph searching

Let G be the graph below: **mcvs**$(G) >$ **cvs**$(G) = 4$.



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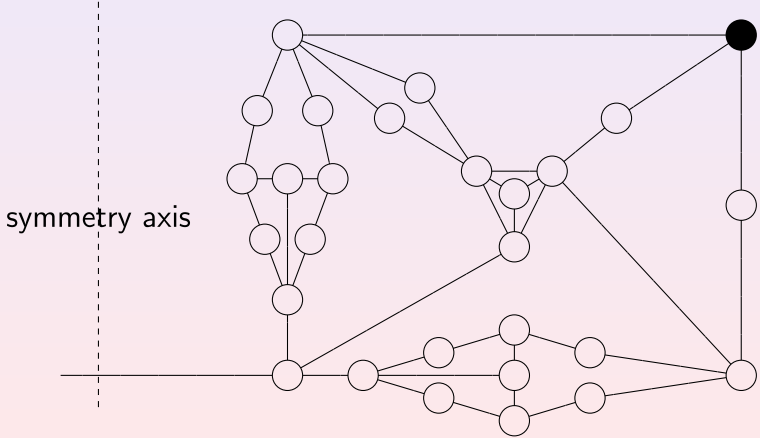

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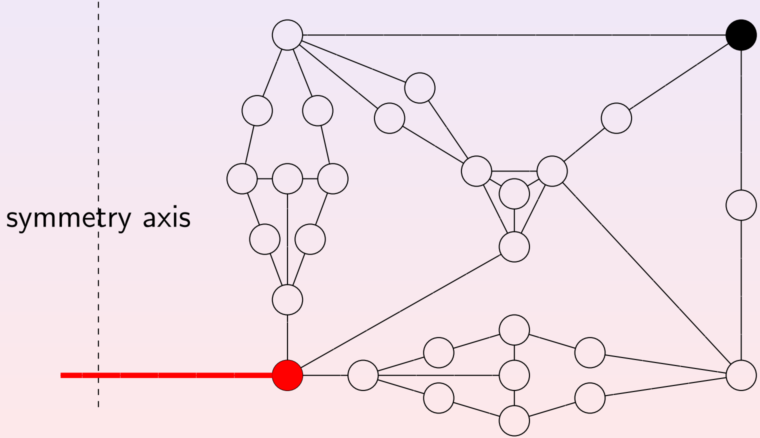

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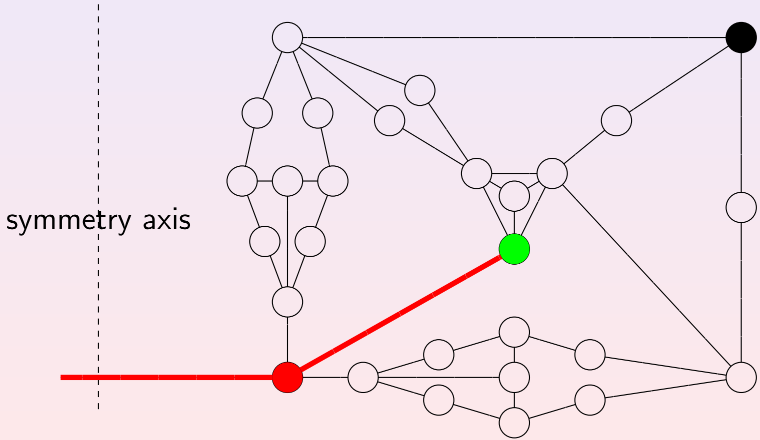

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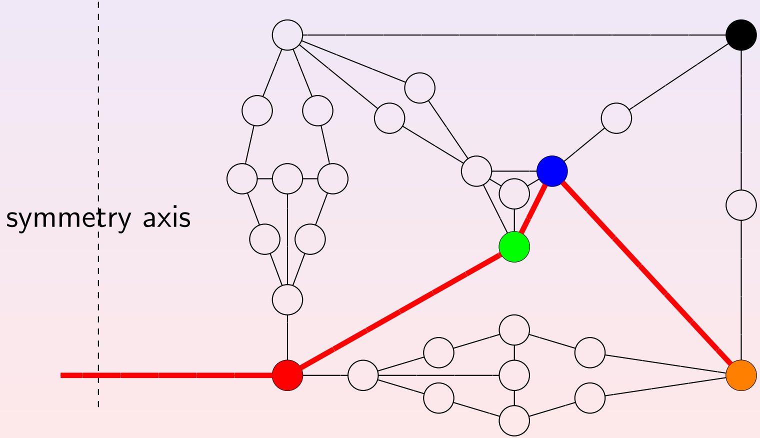

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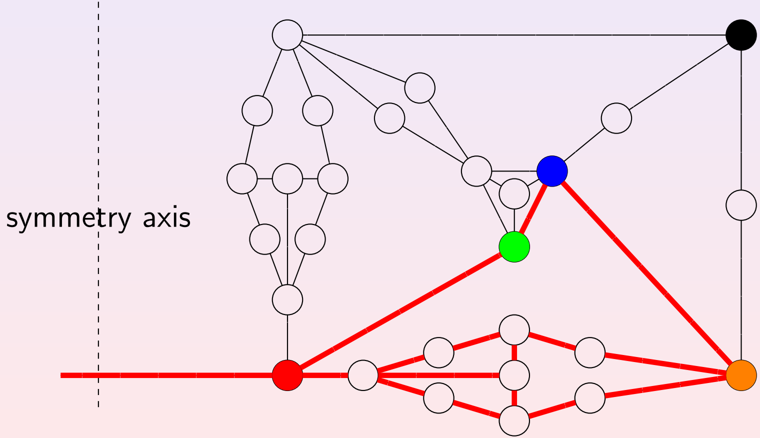

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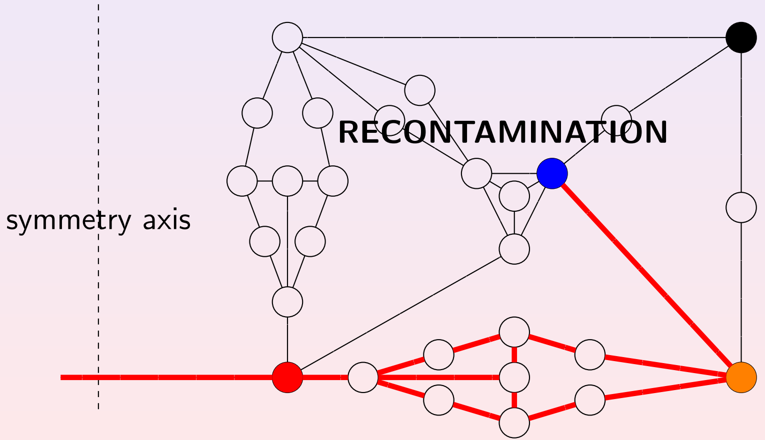

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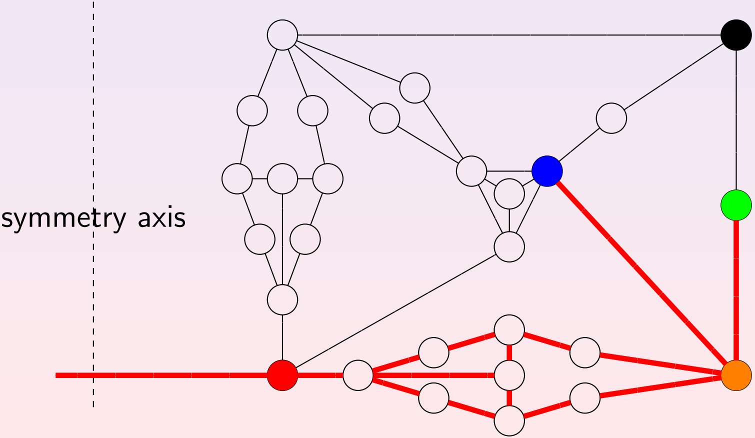

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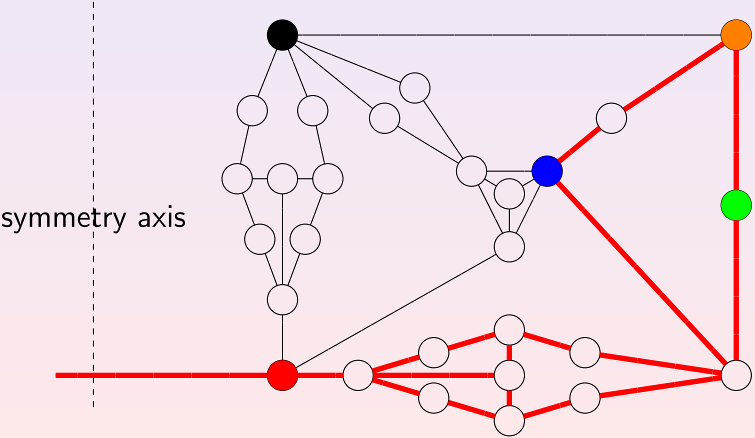

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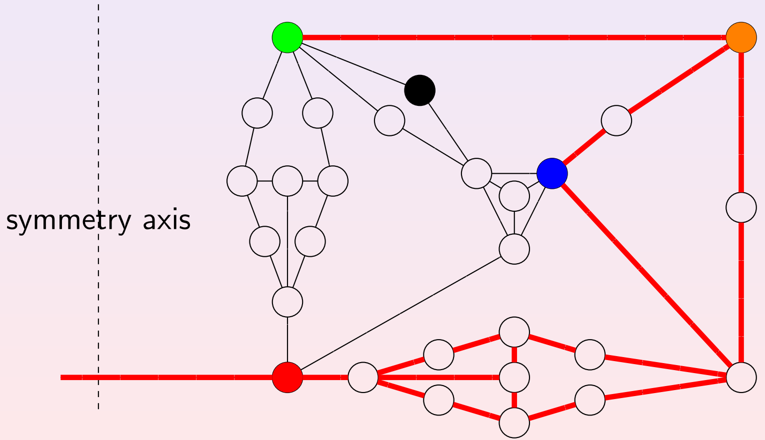

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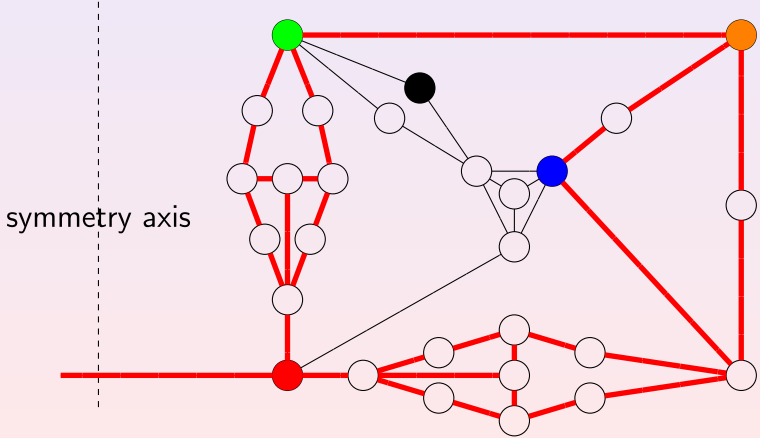

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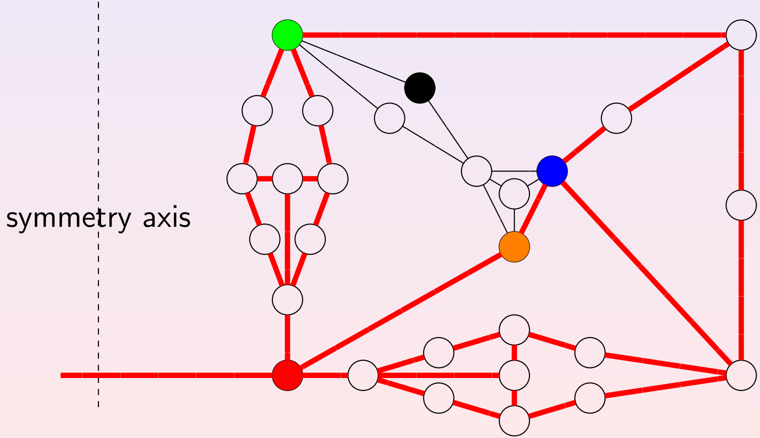

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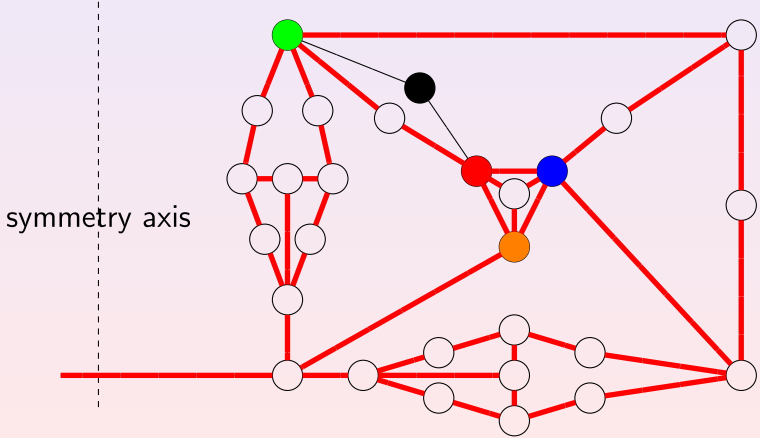

Connected Graph Searching: Open Problems

Cost of connectivity

Conjecture: For any graph G, $\mathbf{cs}(G)/\mathbf{s}(G) \leq 2$ [Barrière *et al.*]

FPT Algorithm

FPT algorithm to compute $\mathbf{cs}(G)$?

NP membership

Bound on the number of recontamination-steps?

Outline

Graph searching in a distributed way

Distributed search problem

To design a *distributed protocol* that enables the *minimum number* of searchers to clear the network.
The searchers must compute themselves a strategy.

We consider connected search strategies.

mcs refers to the smallest number of searchers required to catch an invisible fugitive in a monotone connected way.

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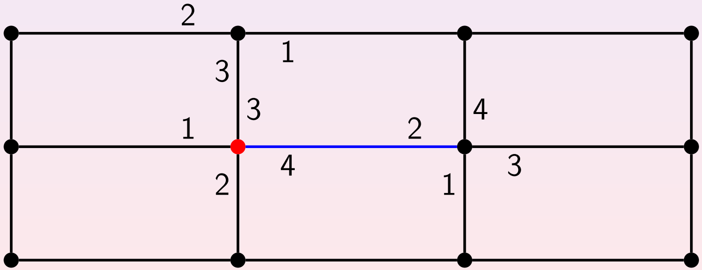
Distributed graph searching: the network

- undirected connected graph;
- local orientation of the edges;
- whiteboards on vertices;
- asynchronous environment.



Distributed graph searching: the searchers

- autonomous mobile computing entities with distinct IDs;
- automata with $O(\log n)$ bits of memory.

Decision is computed locally and depends on:

- its current state;
- the states of the other searchers present at the vertex;
- the content of the local whiteboard;
- if appropriate the incoming port number.

A searcher can decide to:

- leave a vertex via a specific port number;
- switch its state.
- write/erase content of the local whiteboard.

Distributed graph searching: related work

The searchers have a priori knowledge of the topology.

Protocols to clear specific topologies

- **Tree**. Barrière *et al.*, [SPAA 02]
- **Mesh**. Flocchini, Luccio, and Song. [CIC 05]
- **Hypercube**. Flocchini, Huang, and Luccio. [IPDPS 05]
- **Tori**. Flocchini, Luccio, and Song. [IPDPS 06]
- **Sierpinski's graph**. Luccio. [FUN 07]

A monotone connected strategy is performed using **mcs** $+ 1$ searchers, in polynomial time.

Remark:

The extra searcher is due to the asynchronicity of the network and it is necessary [CIC 05].

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Results

Distributed algorithm [Blin, Fraigniaud, N. and Vial]

Distributed protocol that enable **$\mathbf{mcs}(G) + 1$** searchers to clear an unknown graph G in a connected way

Sketch of the algorithm

For $k \geq 0$ to n **do**

Test all connected search strategies using $\leq k$ searchers.
(Difficulty: to ensure that all strategies will be checked)
If this fails, another searcher is called (k++).

Drawback: the strategy is not monotone and may be performed in exponential time.

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Results

Unless P=NP, there is no hope to clear an unknown graph in a monotone way, using the optimal number of searchers.

Provide knowledge about the graph [N. and Soguet]

$\Theta(n \log n)$ bits of information must be provided to the $\textbf{mcs}(G)$ searchers to clear a unknown graph G in a monotone connected way.

Use more searchers [Ilcinkas, N. and Soguet]

$\Theta(\frac{n}{\log n})\textbf{mcs}(G)$ searchers are necessary and sufficient to clear any unknown graph G in a connected *monotone* way.

Distributed Graph Searching: Open Problems

Tradeoff between the number of searchers and the amount of information about the graph.

Better algorithm to clear a graph, possibly computing non-monotone search strategies?

Outline

Further Works

Directed graph searching

Several recent works [Obdrzalek, Hunter *et al.*, Adler, etc.] using "directed" tree decompositions.
Duality therorem?
(FPT) Algorithms?

Cost of connectivity

Conjecture: For any graph G, $\mathbf{cs}(G) \leq 2\mathbf{s}(G)$ [Barrière *et al.*]

Search-tree

- Generalization of the min-max theorem for treewidth [Amini, Mazoit, N. and Thomassé]
- (FPT) Algorithms? Excluding Minor Theorem?
- Application to matroids.