Cops and robber games

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Capture an intruder in a network

- \( C \) plays with a team of cops
- \( R \) plays with one robber

**Cops’ goal:**
- \( C \): Capture the robber using \( k \) cops (“few”);
- The minimum called cop-number, \( cn(G) \).

**Robber’s goal:**
- \( R \): Perpetually evade \( k \) cops (“many”);
- The maximum equal \( cn(G) - 1 \).
## Taxonomy of graph searching games

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**Table:** Classification of graph searching games

? = No studies (as far as I know)
X = Very few studies
Cops & robber games [Nowakowski and Winkler; Quilliot, 83]

Initialization:
1. $C$ places the cops;
2. $R$ places the robber.

Step-by-step:
- each cop traverses at most 1 edge;
- the robber traverses at most 1 edge.

Robber apprehended:
A cop occupies the same vertex as the robber.
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State of art

- **Characterization** of *cop-win* graphs $\{ G \mid cn(G) = 1 \}$.  
  [Nowakowski & Winkler, 83; Quilliot, 83; Chepoi, 97]

**Theorem:** $cn(G) = 1$ iff $V(G) = \{ v_1, \cdots, v_n \}$ and for any $i < n$, there is $j > i$ s.t. $N[v_i] \subseteq N[v_j]$ in the subgraph induced by $v_i, \cdots, v_n$.

Trees, chordal graphs, bridged graphs (...) are cop-win.
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**State of art**

- **Algorithms:** $O(n^k)$ to decide if $\text{cn}(G) \leq k$.  
  [Hahn & MacGillivray, 06]

  $\text{cn}(G) \leq k$ iff the configurations’ graph with $k$ cops is copwin.

- **Complexity:** Computing the cop-number is EXPTIME-complete. [Goldstein & Reingold, 95]

- **Lower bound:** $\text{cn}(G) \geq d^t$, where  
  $d + 1 = \text{minimum degree}$, $\text{girth} \geq 8t - 3$. [Frankl, 87]  
  ($\Rightarrow$ there are $n$-node graphs $G$ with $\text{cn}(G) \geq \Omega(\sqrt{n})$)

- **Planar graph** $G$: $\text{cn}(G) \leq 3$.  
  [Aigner & Fromme, 84]
One of the main tool used

**Shortest path principle**

1 cop can protect 1 **shortest** path $P$.

Position $c$ (**shadow**: $\text{dist}(r, z) \geq \text{dist}(c, z)$, $\forall z \in V(P)$.)
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\[ G \text{ planar } \Rightarrow cn(G) \leq 3; \ G \text{ grid } \Rightarrow cn(G) \leq 2 \]

2 shortest paths to surround, 3\textsuperscript{rd} one to reduce the zone. In a grid, 1 shortest path is enough.
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Other applications of the shortest path principle

Bounded genus graphs

$G$ with **genus** $g$: $cn(G) \leq \frac{3}{2}g + 3$ [Schröder, 01]

Minor free graphs

$G$ excluding a minor $H$: $cn(G) \leq |E(H \setminus \{x\})|$, where $x$ is any non-isolated vertex of $H$ [Andreae, 86]

General upper bound

For any connected graph $G$, $cn(G) \leq O\left(\frac{n}{\log n}\right)$ [Chiniforooshan, 08]
Fast robber

Different speeds

Speed = maximum number of edges traversed in 1 step.

\[ \text{speed}_R \geq \text{speed}_C = 1 \]

Computational hardness

Computing \( cn \) for any \( \text{speed}_R \geq 1 \) is NP-hard; the parameterized version is \( W[2] \)-hard. For \( \text{speed}_R \geq 2 \), it is true already on split graphs. [Fomin, Golovach, Kratochvil, 2008]

Impact of higher robber’s speed in planar graphs?

How many cops are needed to capture a fast robber in a grid?

Recall: 2 cops are enough if \( \text{speed}_R = \text{speed}_C \).
Our results \((\text{speed}_R > \text{speed}_C)\)

**Theorem:** cop-number is unbounded in planar graphs

\[\exists c > 0 \forall k \geq 1: \text{take a square grid } f(k) \times f(k), f(k) = c^{k^2}, \text{ then } \text{cn}(\text{Square}_f(k)) \geq k.\]

**Corollary:** \(\text{cn}(\text{Square}_n) = \Omega(\sqrt{\log(n)}).\)

**Gap:** the best known upper bound is \(\text{cn}(\text{Square}_n) = O(n).\)

Does a planar graph “containing” a large grid have a high \(\text{cn}\)?

**Theorem:** No...

\[\exists H \text{ subdivision of an arbitrarily large grid: } \text{cn}(H) = 2.\]

**Theorem:** However...

\[\forall H \text{ planar with an induced subgraph } \text{Square}_{2f(k)}, \text{cn}(H) \geq k.\]
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Cops and robber games
Idea of the proof, $cn(Square_n) = \Omega(\sqrt{\log(n)})$

Definition inductive (on $k$) of a strategy for the robber against $k$ cops: $c_1, c_2, \cdots, c_k$

Key idea

$\forall i \leq k$, partition $G$ into disjoint subgrid of size $O(2^i)$

$\Rightarrow$ In a subgrid of level $i$, consider only $c_1, c_2, \cdots, c_i$

In a level-$i$ subgrid, a strategy will be a path of level-$(i - 1)$ subgrids avoiding $c_i$. 

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1. Design of a strategy
2. Constraints on $n$ for the strategy to be valid.
3. if $n = f(k) = c^{k^2} \Rightarrow \text{constraints satisfied.}$
Robber’s strategy: big picture

Grid of size $n$ divided into 4 subgrids.
Robber’s strategy: big picture

Grid of size $n$ divided into 4 subgrids.

Pass from a position in a subgrid to a position in an adjacent subgrid.
Grid of size $n$ divided into 4 subgrids.

Pass from a **safe** position in a subgrid to a **safe** position in an adjacent subgrid.
Robber’s strategy: big picture

Grid of size $n$ divided into 4 subgrids.

Pass from a safe position in a subgrid to a safe position in an adjacent subgrid.
Robber’s strategy: Goal

Starting from any safe position in a subgrid Move towards any side keeping its position safe.
Robber’s strategy: Induction $k = 1$

Strategy to go from a safe position towards the blue side
Robber’s strategy: Induction $k = 1$

Case 1: straight line

Strategy to go from a safe position towards the blue side
Robber’s strategy: Induction $k = 1$

Case 2: detour

Strategy to go from a safe position towards the blue side
Robber’s strategy: Induction $k = 1$

Case 2: detour

\[ \frac{A_1 + B_1}{\text{speed}_0} < C_1 - \frac{\text{margin}_1}{2} \quad \text{and} \quad \text{time}_1 = \frac{A_1 + B_1}{\text{speed}_0}. \]
Robber’s strategy: Induction $k = 2$

$\text{size}_1$  

$\text{margin}_2$  

$\text{size}_2$

$(A_2 + B_2)time_1 < C_2 - \frac{\text{margin}_2}{2} \quad \& \quad time_2 = time_1(A_2 + B_2)$. 
Robber’s strategy: Induction $k = i$

$$(A_i + B_i) \text{time}_{i-1} < C_i - \frac{\text{margin}_i}{2} \quad \& \quad \text{time}_i = \text{time}_{i-1}(A_i + B_i).$$
Constraints imposed by the strategy

3 variables: \( \text{size}_i, A_i + B_i \approx \text{detour}_i \) and \( \text{margin}_i \)

Define \( \text{zoom}_i = \frac{\text{size}_i}{\text{size}_{i-1}}, \text{speed}_i = \frac{\text{size}_i}{\text{time}_i} \), and \( \text{time}_i = (\text{zoom}_i + \text{detour}_i)\text{time}_{i-1} \).

4 inequalities: \( \forall i \in [1..k] \)

\[
\text{margin}_i \geq \left\lceil \frac{4 + \text{speed}_{i-1}}{\text{speed}_{i-1} - 1} \right\rceil \\
\text{detour}_i / 2 \geq \left\lceil \frac{(2 \times \text{margin}_i + 2)\text{speed}_{i-1}}{\text{speed}_{i-1} - 1} \right\rceil \\
\text{detour}_i / 2 + 2 \times \text{margin}_i + 1 < \text{zoom}_i / 2 \\
\text{speed}_i > 1
\]

\( \exists a, b > 0, \) Inequalities satisfied for \( \text{zoom}_i = ab^i \)

\( \Rightarrow f(k) = \text{size}_k = \text{size}_0 \times \prod_{1 \leq i \leq k} \text{zoom}_i = O(a^k \times b^{k(k+1)/2}) \)
Containment relations on graphs

Graph searching: unbounded speeds, Robber moves simultaneously with Cops

“Many” cops needed ⇔ “large” grid minor. [Robertson, Seymour, Thomas, 94]

If a planar $G$ “contains” a grid of size $n$, $\text{cn}(G) \geq g(n)$?

$G$ contains $H$

- vertex deletion (1)
- edge deletion (2)
- edge contraction (3)

Induced subgraph: 1; Subgraph: 1 & 2; Minor: 1, 2 & 3

cop-number is not closed under taking isometric induced subgraphs

$\text{cn}(H) = 2 > \text{cn}(G) = 1$
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Subdivision can diminish the cop-number

Each horizontal edge, except for $L$, is subdivided into $6n$ edges. The cops use $L$ as a shortcut. $\Rightarrow \text{cn}(H) = 2$
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Planar $H$ contains $Square_{2n}$ as an induced subgraph. Robber's strategy restricted to $Square_n$. $\Rightarrow cn(H) = \Omega(\sqrt{\log(n)})$
Perspectives

In case $\text{speed}_R = \text{speed}_C = 1$

- $G$ of genus $g \Rightarrow \text{cn}(G) \leq \frac{3}{2}g + 3$. [Schröder, 01]
  Conjecture: $G$ of genus $g \Rightarrow \text{cn}(G) \leq g + 3$.

- General upper bound for $\text{cn}$?
  for any connected graph $G$, $\text{cn}(G) \leq O\left(\frac{n}{\log n}\right)$.
  [Chiniforooshan, 08]
  Conjecture: $\text{cn}(G) \leq O(\sqrt{n})$.
  Link with $\Delta$ (maximum degree)?

In case $\text{speed}_R > \text{speed}_C$

- $\Omega(\sqrt{\log n}) \leq \text{cn}(\text{Square}_n) \leq O(n)$.
  What is the exact value?

- What about other graphs’classes?

- Link with graphs’decompositions?
Thank you

Any questions?