Cops and robber games

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Cops & robber/pursuit-evasion/graph searching

Capture an intruder in a network

- $\mathcal{C}$ plays with a team of cops
- $\mathcal{R}$ plays with one robber

Cops' goal:

- $\mathcal{C}$: Capture the robber using k cops ("few");
- The minimum called cop-number, **cn**(G).

Robber's goal:

- $\mathcal{R}$: Perpetually evade k cops ("many");
- The maximum equal $\mathbf{cn}(G) - 1$.

Taxonomy of graph searching games

	robber's characteristics			
	bounded speed		arbitrary fast	
	visible	invisible	visible	invisible
turn by turn	Cops & Robber	X	X	?
simultaneous	?	X	graph searching treewidth	graph searching pathwidth

Table: Classification of graph searching games

? = No studies (as far as I know)
X = Very few studies

Initialization:

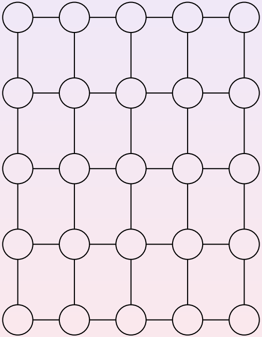
1. $\mathcal{C}$ places the cops;
2. $\mathcal{R}$ places the robber.

Step-by-step:

- each cop traverses at most 1 edge;
- the robber traverses at most 1 edge.

Robber apprehended:

A cop occupies the same vertex as the robber.



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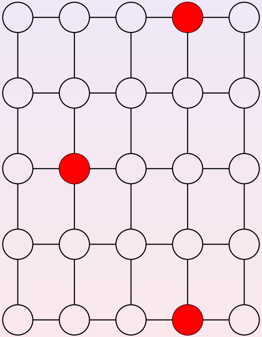

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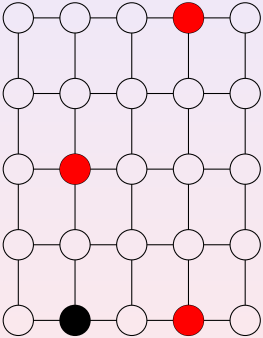

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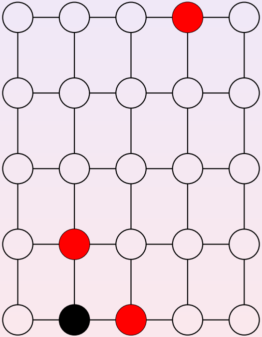

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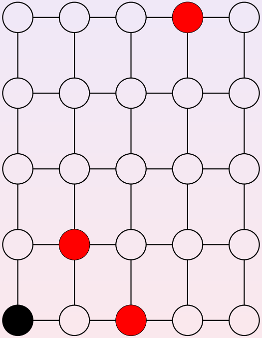

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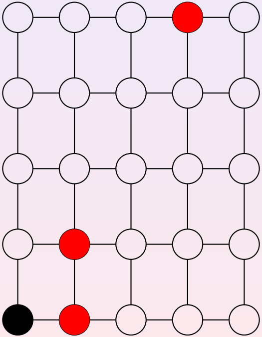

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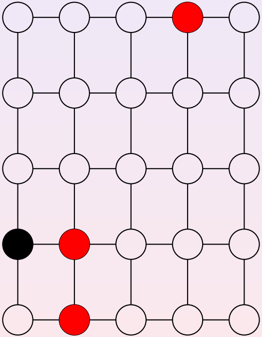

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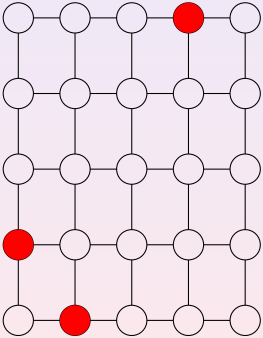

State of art

- **Characterization** of *cop-win* graphs $\{G \mid \mathbf{cn}(G) = 1\}$.
  [Nowakowski & Winkler, 83; Quilliot, 83; Chepoi, 97]

Theorem: $cn(G) = 1$ iff

$V(G) = \{v_1, \cdots, v_n\}$ and for any $i < n$, there is $j > i$ s.t. $N[v_i] \subseteq N[v_j]$ in the subgraph induced by $v_i, \cdots, v_n$

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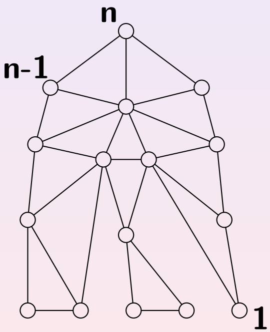

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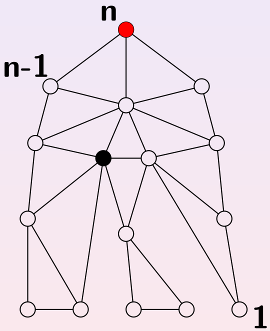

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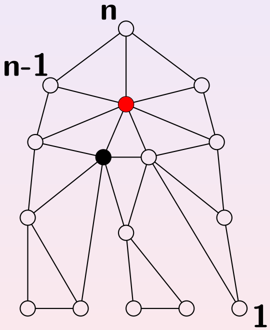

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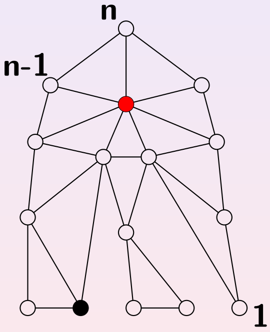

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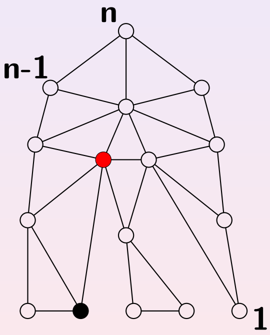

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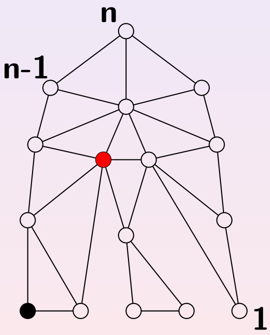

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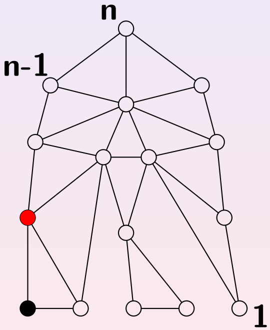

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State of art

- **Algorithms:** $O(n^k)$ to decide if $\mathbf{cn}(G) \leq k$. [Hahn & MacGillivray, 06]

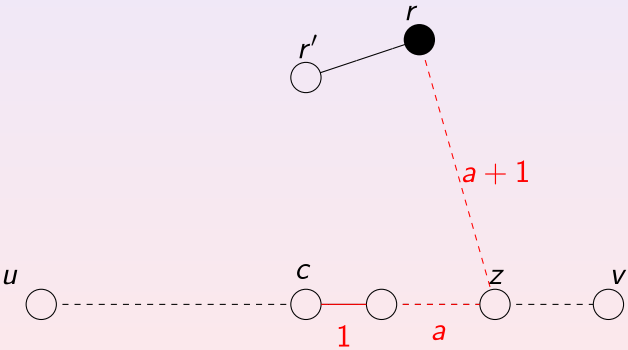
$\mathbf{cn}(G) \leq k$ iff the configurations'graph with k cops is copwin.

- **Complexity:** Computing the cop-number is EXPTIME-complete. [Goldstein & Reingold, 95]
- **Lower bound:** $\mathbf{cn}(G) \geq d^t$, where $d+1 =$ minimum degree, girth $\geq 8t-3$. [Frankl, 87] ($\Rightarrow$ there are n-node graphs G with $\mathbf{cn}(G) \geq \Omega(\sqrt{n})$)
- **Planar graph** G: $\mathbf{cn}(G) \leq 3$. [Aigner & Fromme, 84]

One of the main tool used

Shortest path principle

1 cop can protect 1 **shortest** path P.



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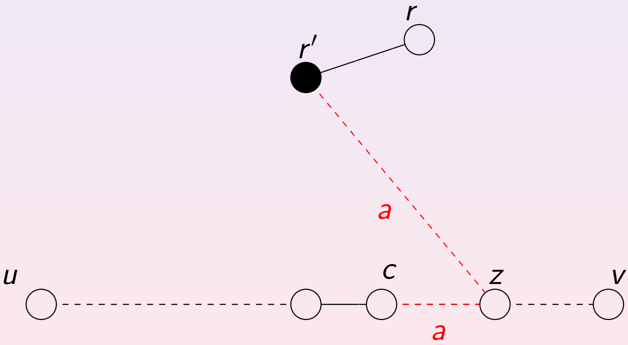

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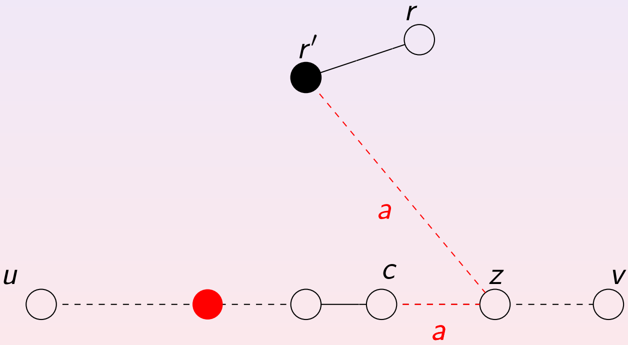

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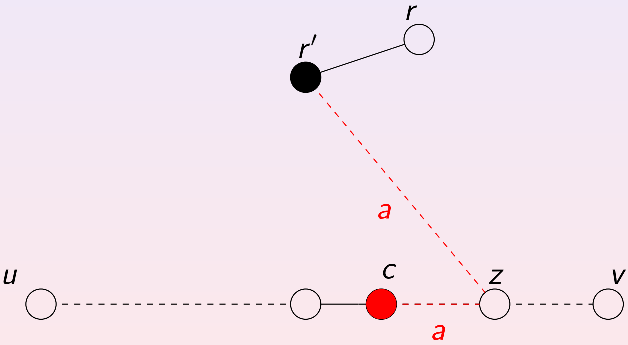

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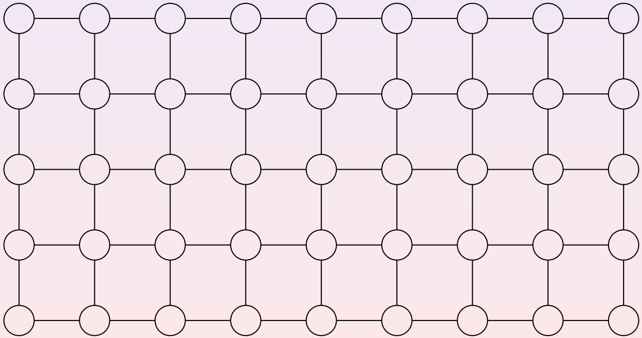
Planar graphs [Aigner & Fromme, 84]

G planar $\Rightarrow cn(G) \leq 3$; G grid $\Rightarrow cn(G) \leq 2$

2 shortest paths to surround, 3^{rd} one to reduce the zone.
In a grid, 1 shortest path is enough.



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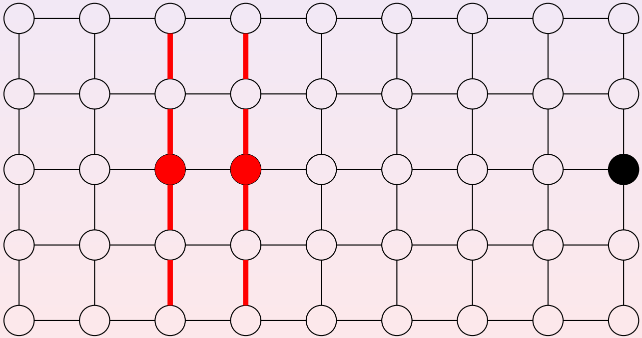

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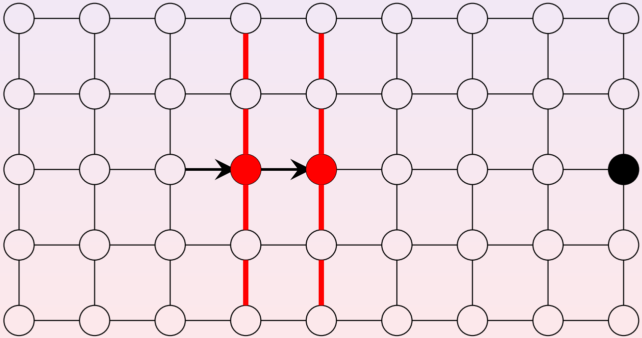

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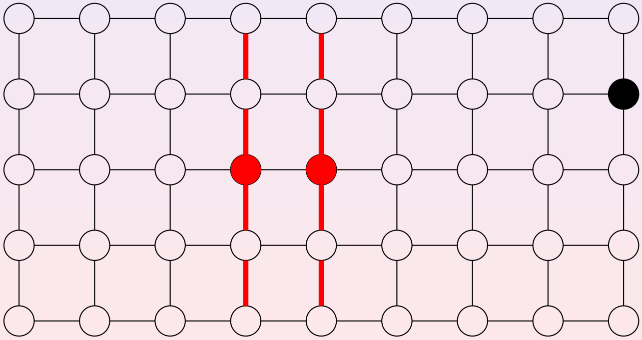

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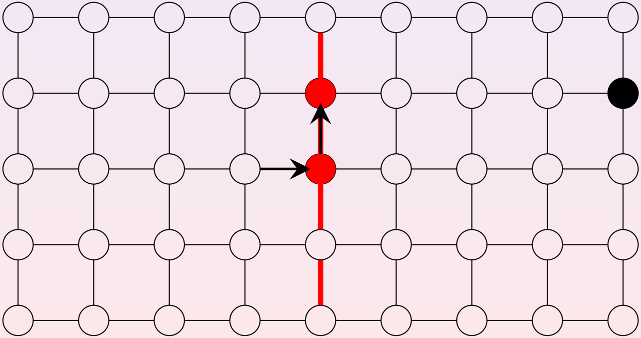

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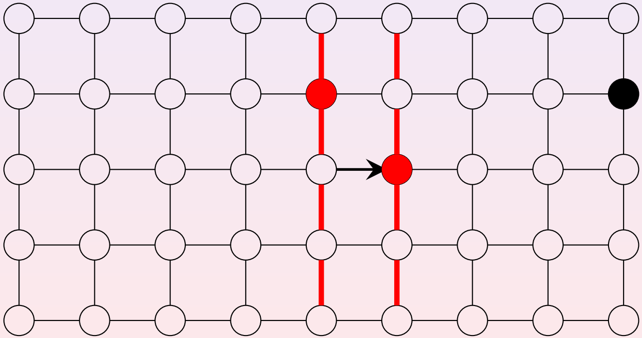

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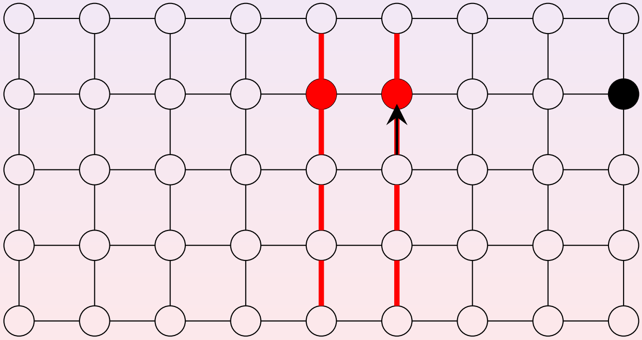

Other applications of the shortest path principle

Bounded genus graphs

G with **genus** g: $cn(G) \leq \frac{3}{2}g + 3$ [Schröder, 01]

Minor free graphs

G **excluding a minor** H: $cn(G) \leq |E(H \setminus \{x\})|$, where x is any non-isolated vertex of H [Andreae, 86]

General upper bound

For any connected graph G, $cn(G) \leq O(\frac{n}{\log n})$ [Chiniforooshan, 08]

Fast robber

Different speeds

Speed $=$ maximum number of edges traversed in 1 step.

$speed_{\mathcal{R}} \geq speed_{\mathcal{C}} = 1$

Computational hardness

Computing **cn** for any $speed_{\mathcal{R}} \geq 1$ is NP-hard; the parameterized version is $W[2]$-hard. For $speed_{\mathcal{R}} \geq 2$, it is true already on split graphs. [Fomin, Golovach, Kratochvil, 2008]

Impact of higher robber's speed in planar graphs?

How many cops are needed to capture a fast robber in a grid?
Recall: 2 cops are enough if $speed_{\mathcal{R}} = speed_{\mathcal{C}}$.

Our results ($speed_{\mathcal{R}} > speed_{\mathcal{C}}$)

Theorem: cop-number is unbounded in planar graphs

$\exists c > 0 \forall k \geq 1$: take a square grid $f(k) \times f(k)$, $f(k) = c^{k^2}$, then $\mathbf{cn}(Square_{f(k)}) \geq k$.

Corollary: $\mathbf{cn}(Square_n) = \Omega(\sqrt{\log(n)})$.
Gap: the best known upper bound is $\mathbf{cn}(Square_n) = O(n)$.
Does a planar graph "containing" a large grid have a high **cn**?

Theorem: No...

$\exists H$ subdivision of an arbitrarily large grid: $\mathbf{cn}(H) = 2$.

Theorem: However...

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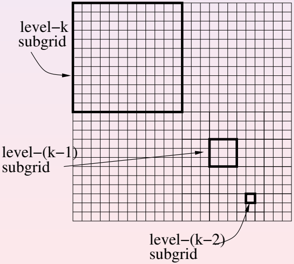
Definition inductive (on k) of a strategy for the robber against k cops: $c_1, c_2, \cdots, c_k$

Key idea

$\forall i \leq k$, partition G into disjoint subgrid of size $O(2^i)$
$\Rightarrow$ In a subgrid of level i, consider only $c_1, c_2, \cdots, c_i$

In a level-i subgrid, a strategy will be a path of level-$(i-1)$ subgrids avoiding c_i.



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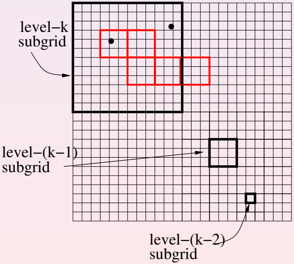

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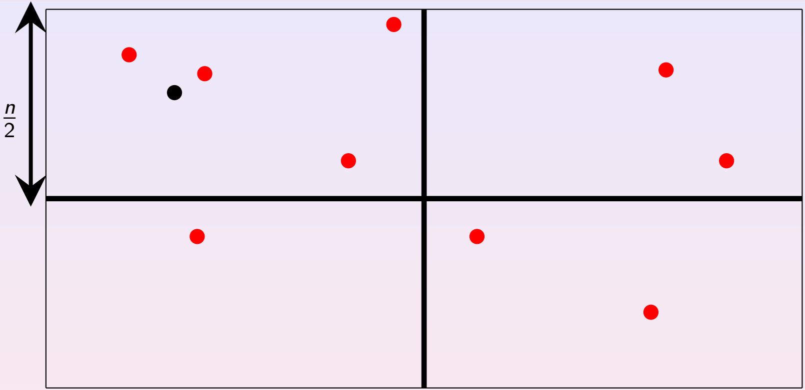
1. Design of a strategy
2. Constraints on n for the strategy to be valid.
3. if $n = f(k) = c^{k^2} \Rightarrow$ constraints satisfied.

Robber's strategy: big picture



Grid of size n divided into 4 subgrids.

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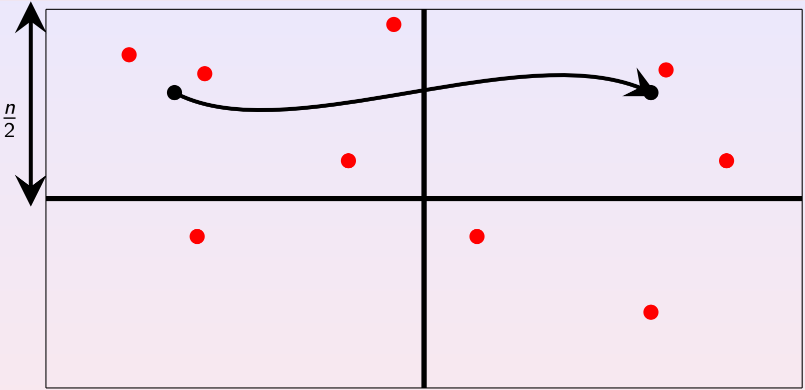

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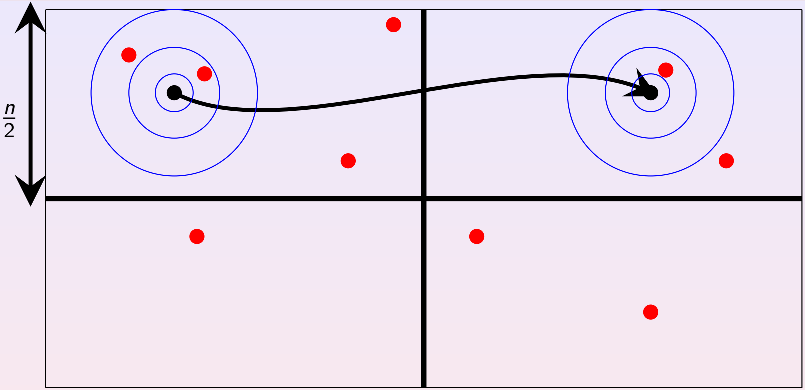
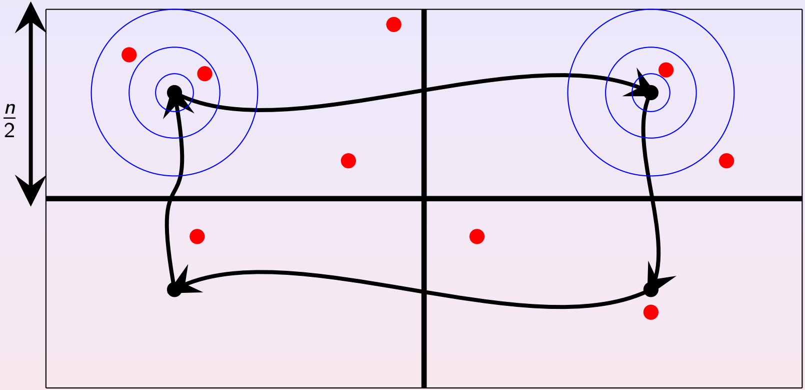


Grid of size n divided into 4 subgrids.

Pass from a safe position in a subgrid
to a safe position in an adjacent subgrid.

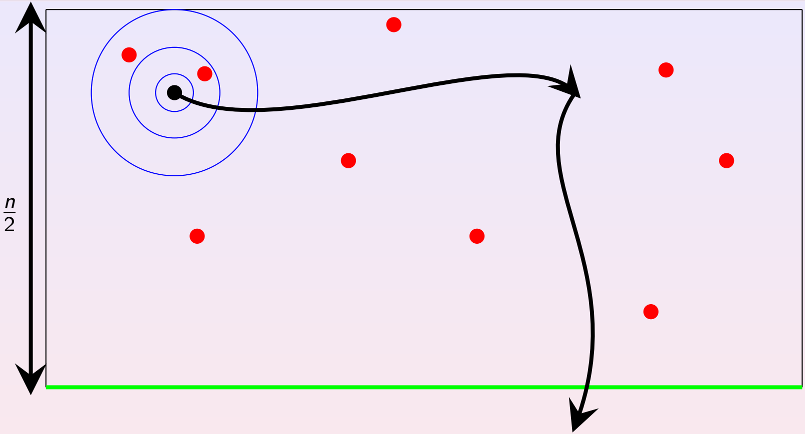
Robber's strategy: big picture



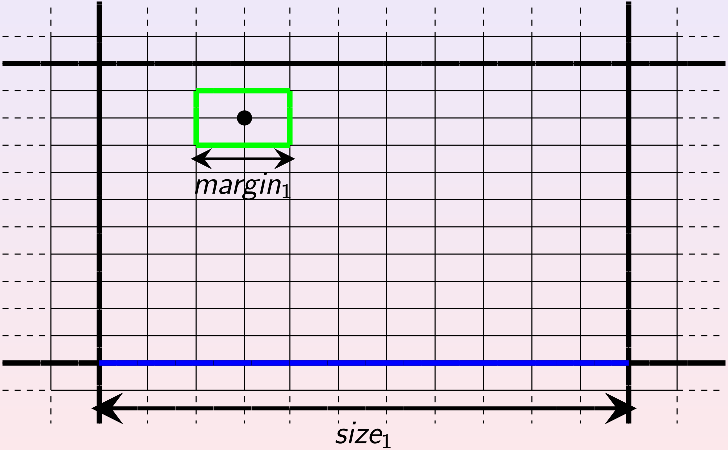
Grid of size n divided into 4 subgrids.

Pass from a safe position in a subgrid
to a safe position in an adjacent subgrid.

Robber's strategy: Goal



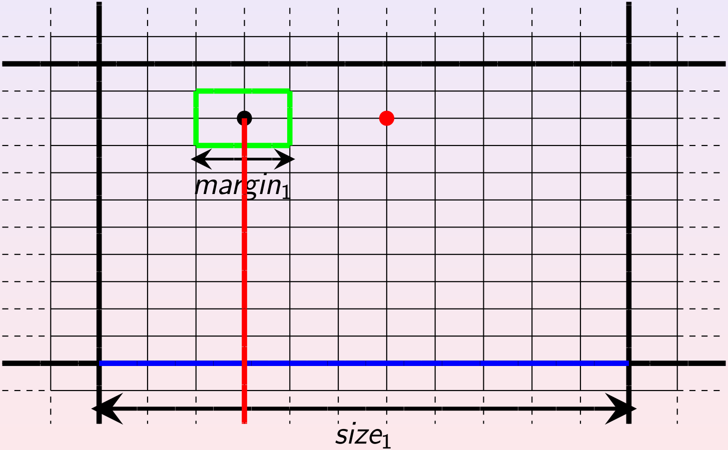
Starting from any *safe* position in a subgrid
Move towards *any side* keeping its position safe.



Strategy to go from a safe position towards the blue side

Robber's strategy: Induction $k = 1$

Case 1: straight line



Strategy to go from a safe position towards the blue side

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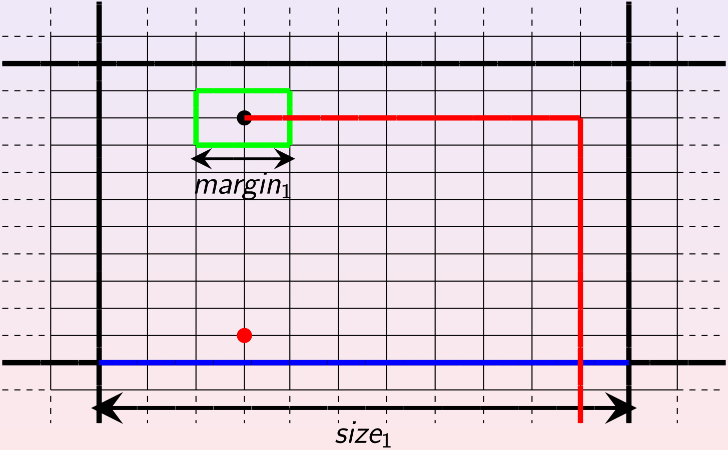
Case 2: detour



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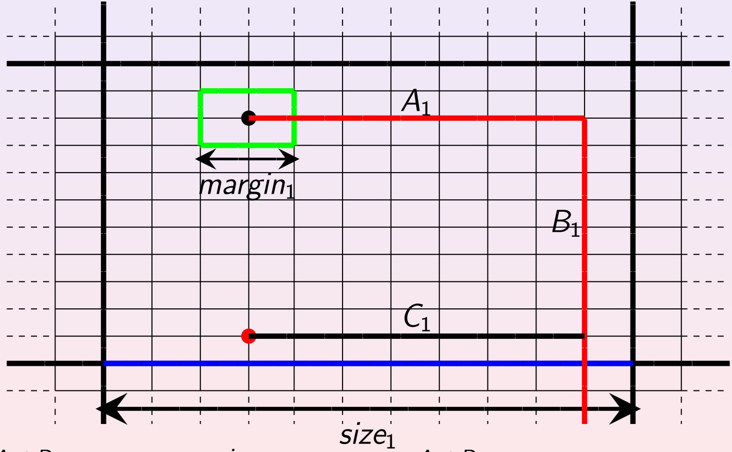
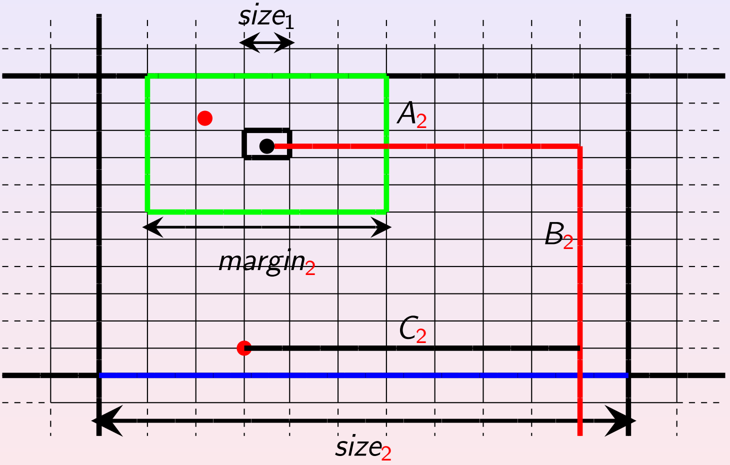


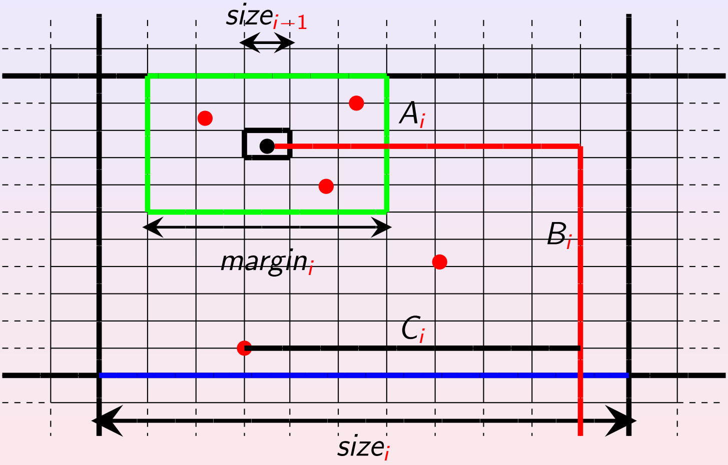
$$\frac{A_1+B_1}{speed_0} < C_1 - \frac{margin_1}{2} \;\&\; time_1 = \frac{A_1+B_1}{speed_0}.$$

Robber's strategy: Induction $k = 2$



$$(A_2 + B_2)time_1 < C_2 - \frac{margin_2}{2} \ \& \ time_2 = time_1(A_2 + B_2).$$

Robber's strategy: Induction $k = i$



$$(A_i + B_i)time_{i-1} < C_i - \frac{margin_i}{2} \;\&\; time_i = time_{i-1}(A_i + B_i).$$

Constraints imposed by the strategy

3 variables: $size_i$, $A_i + B_i \approx detour_i$ and $margin_i$

Define $zoom_i = size_i / size_{i-1}$, $speed_i = size_i / time_i$, and $time_i = (zoom_i + detour_i) time_{i-1}$.

4 inequalities : $\forall i \in [1..k]$

$margin_i \geq \lceil \frac{4 + speed_{i-1}}{speed_{i-1} - 1} \rceil$

$detour_i / 2 \geq \lceil \frac{(2 * margin_i + 2) speed_{i-1}}{speed_{i-1} - 1} \rceil$

$detour_i / 2 + 2 * margin_i + 1 < zoom_i / 2$

$speed_i > 1$

$\exists a, b > 0$, Inequalities satisfied for $zoom_i = ab^i$

$\Rightarrow f(k) = size_k = size_0 * \prod_{1 \leq i \leq k} zoom_i = O(a^k * b^{k(k+1)/2})$

Containment relations on graphs

Graph searching: unbounded speeds, Robber moves simultaneously with Cops

"Many" cops needed $\Leftrightarrow$ "large" grid minor.
[Robertson, Seymour, Thomas, 94]

If a planar G *"contains"* a grid of size n, $\mathbf{cn}(G) \geq g(n)$?

G contains H

vertex deletion (1), edge deletion (2), edge contraction (3)
Induced subgraph: 1; **Subgraph**: 1 & 2; **Minor**: 1, 2 & 3

cop-number is not closed under taking isometric induced subgraphs
$\mathbf{cn}(H) = 2 > \mathbf{cn}(G) = 1$

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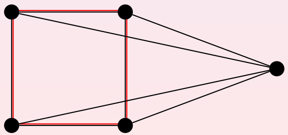

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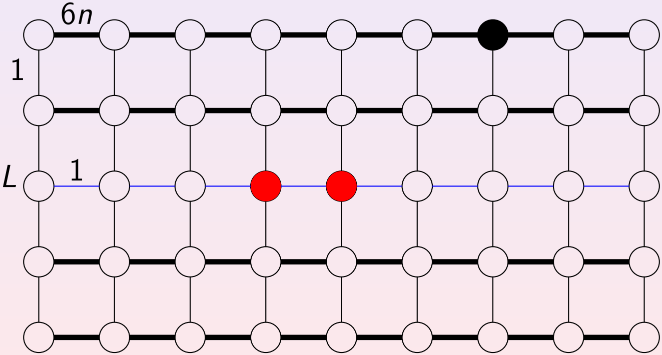
Subdivision can diminish the cop-number

Each horizontal edge, except for L, is subdivided into $6n$ edges. The cops use L as a shortcut. $\Rightarrow$ $\mathbf{cn}(H) = 2$



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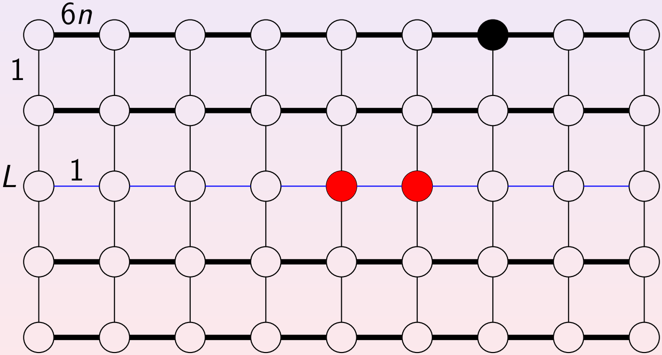

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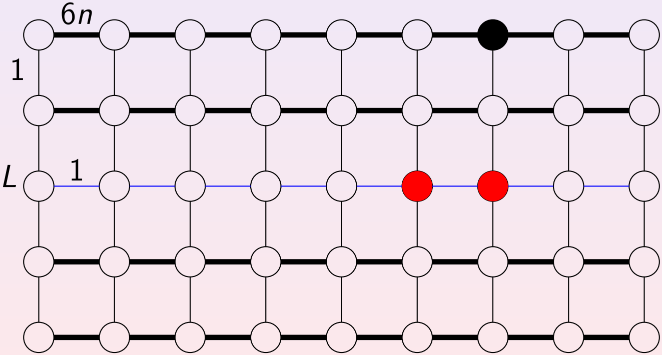

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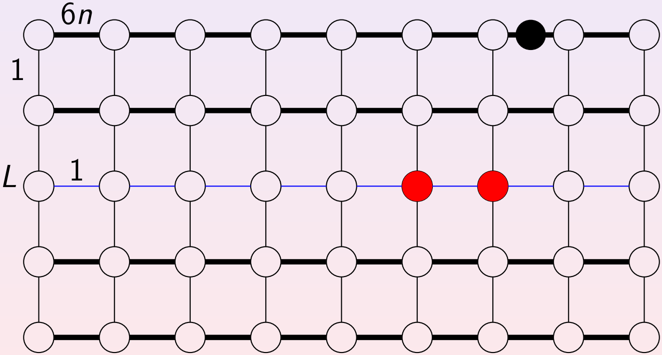

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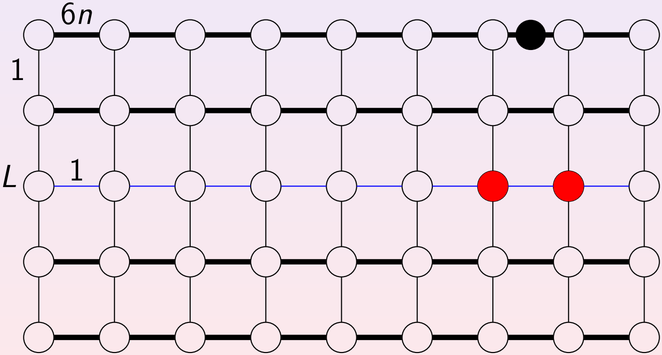

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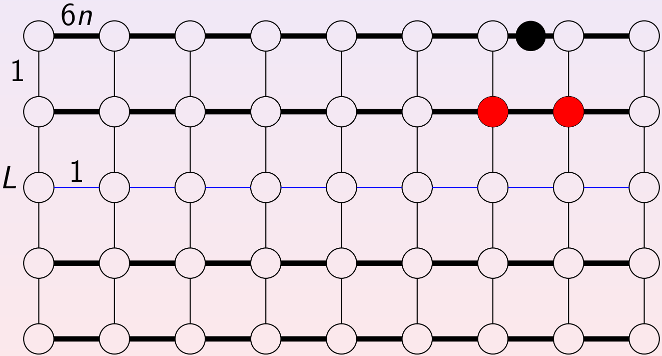

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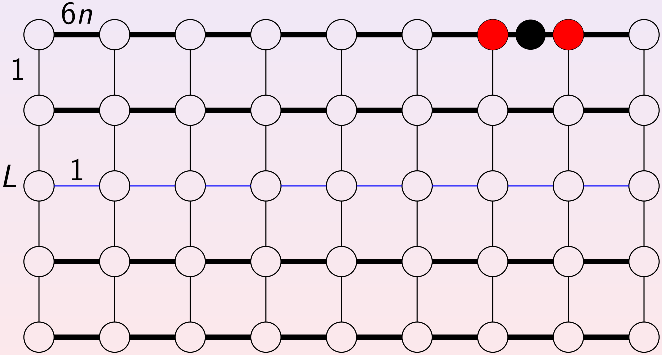
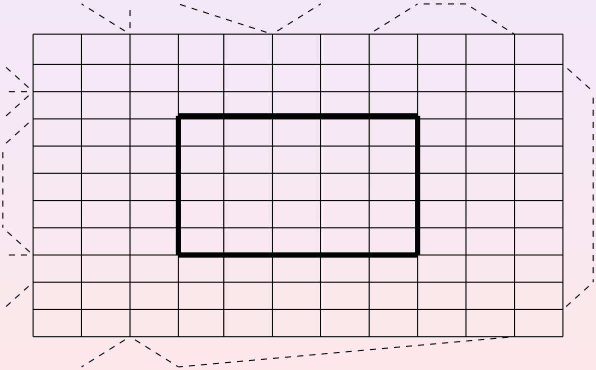


Large square as an induced subgraph

Planar H contains $Square_{2n}$ as an induced subgraph. Robber's strategy restricted to $Square_n$. $\Rightarrow$ $\mathbf{cn}(H) = \Omega(\sqrt{\log(n)})$



Perspectives

In case $speed_{\mathcal{R}} = speed_{\mathcal{C}} = 1$

- G of **genus** $g \Rightarrow \mathbf{cn}(G) \leq \frac{3}{2}g + 3$. [Schröder, 01]
  Conjecture: G of genus $g \Rightarrow \mathbf{cn}(G) \leq g + 3$.
- General upper bound for cn ?
  for any connected graph G, $cn(G) \leq O(\frac{n}{\log n})$.
  [Chiniforooshan, 08]
  Conjecture: $cn(G) \leq O(\sqrt{n})$.
  Link with Δ (maximum degree)?

In case $speed_{\mathcal{R}} > speed_{\mathcal{C}}$

- $\Omega(\sqrt{\log(n)}) \leq \mathbf{cn}(Square_n) \leq O(n)$.
  What is the exact value?
- What about other graphs'classes?
- Link with graphs'decompositions?

Thank you

Any questions?