k-Chordal Graphs: from Cops and Robber to Compact Routing via Treewidth[1]

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[1] presented to ICALP 2012

Outline

1. Distributed and Compact Routing
   - Routing in a distributed way
   - Compact Routing
   - Using structural Properties: chordality
   - Our results

2. Cops and Robber Games
   - Definitions and related works
   - Cops and Robber in k-chordal graphs

3. Structural result and link with compact routing

What is Routing?

Something/someone is at some node (**the source**) of a network

(city, country, Internet, etc.)

e.g., you at home or in a city,

an email that you want to send in your computer,

etc.

and needs to go to another node (**the destination**)

you want to go somewhere,

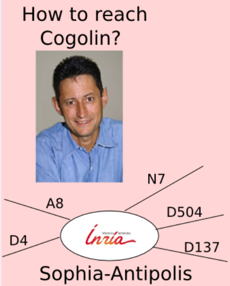
to send the email to someone,

etc.

Goal: to reach the destination quickly.

Why may Routing be difficult?

Jean-Claude wants to reach his destination



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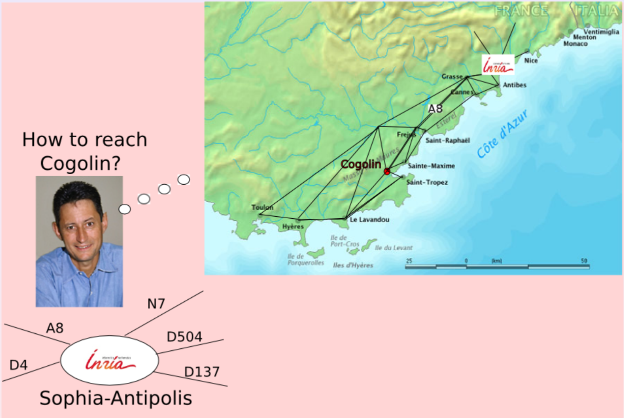
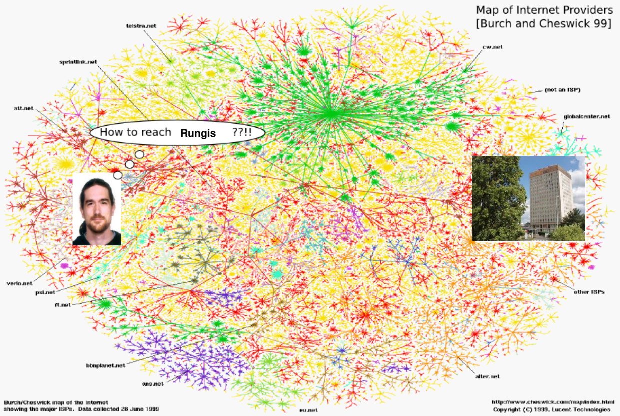


If the network is **small**, **known**, **static**, etc. Easy !!
Apply your favorite shortest path algorithm (e.g., Dijkstra)

Why may Routing be difficult?

What for David?

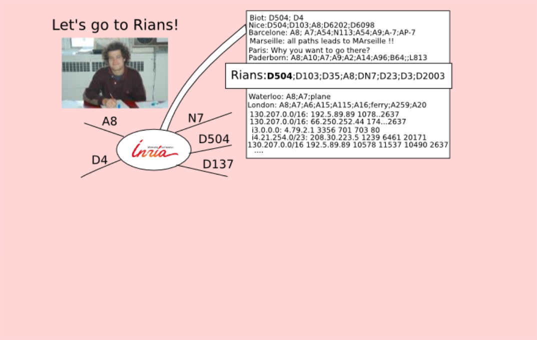


If the network is **Huge**, **only partially known**, **dynamic**, etc.

What to do ??

How Internet works? Border Gateway Protocol

BGP: routing protocol of the Autonomous Systems' (AS) network



- Routing Tables (RT) attached to each AS
- 1 entry/destination: whole path stored (to avoid loops)

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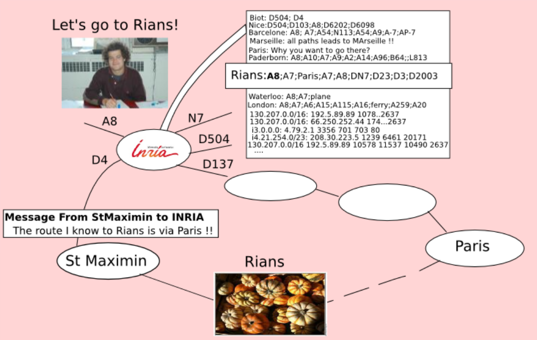


- Routing Tables (RT) attached to each AS
- 1 entry/destination: whole path stored $\Rightarrow$ huge, difficult to read
- to deal with dynamicity: ASs send to each other paths they know

ASs may lie (Policy), paths may be too long

Challenges

	BGP	Ideally
Routing Tables size	$O(n \log n)$ bits	$O(\log n)$ bits
Paths Length	depend on ASs policies	Shortest paths
Update Time	Long ($\approx$ 5 min.)	Fast (using only local information)

(n is the number of ASs)

Large-scale Networks have specific structural properties

- small diameter, high clustering coefficient, power-law degree-distribution
- hyperbolicity, chordality, etc.

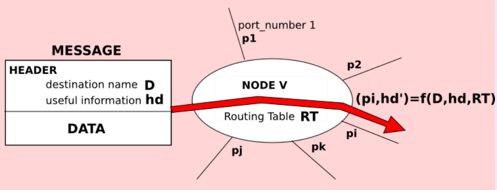
Objectives

- Understand (find new) Properties
- Use it for algorithmic purposes (not only routing)
- Model such networks
- Simulate (static/dynamic behavior)

Compact Routing

Goal To deliver a message in a distributed way

Routing Scheme protocol that directs the traffic in a network



Routing Problem: definitions of:

- Routing Tables **RT**
- Information required in the message headers **hd**
- Routing function **f**: compute next hop / may modify the header

Models

- **labelled/name independent** node Identifiers are part of the design or not
- **design port model** local labeling (port number) are part of the design or not

Performances Measures

Stretch

How far the route actually followed is from a shortest path

- **Multiplicative stretch**: $\max_{x,y \in V(G)} \leq \frac{|route(x,y)|}{dist(x,y)}$.
- **Additive stretch**: $\max_{x,y \in V(G)} \leq |route(x,y)| - dist(x,y)$.

Memory space

- Space necessary to store local routing table (per node)
- Size of the node Identifiers / message header (generally $O(\log n)$)

Time complexity

- Distributed/Centralized protocol
- Time to setup data structures
- Time to update data structures

Example: Interval Routing [Santoro & Khatib, 82]

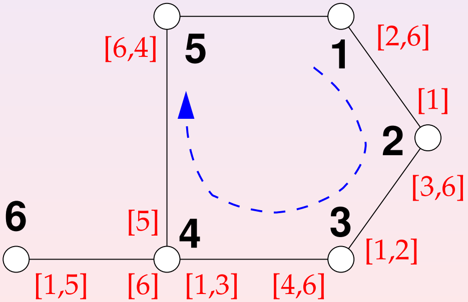
- Nodes labeled using integers
- Outgoing arc labeled with an interval of the name range

Message sent through the arc containing the destination

mult-stretch:
$\frac{route(1,5)}{d(1,5)} = 4$

add-stretch:
$route(1,5)\text{-}d(1,5) = 3$

space per node:
$O(\Delta \log n)$



Related Works

Names and Headers (if any) are of polylogarithmic size

network	mult. stretch	routing table	
		labelled	name-independent
arbitrary ($k \geq 2$)	shortest path	$O(n \log n)$ [folk]	$\Theta(n \log n)$ [Gavoille,Pérennes]
	$O(k)$	$O(n^{1/k})$ [Thorup,Zwick]	$\Theta(n^{1/k})$ [TZ/Abraham et al.]
trees	shortest path	$O(\log n)$ [TZ/Fraigniaud,Gavoille]	$\Omega(\sqrt{n})$ [Laing,Rajaraman]
	$2^k - 1$		$\Theta(n^{1/k})$ [Laing/Abraham et al.]
doubling-α dimension	$O(1) + \epsilon$	$O(\log \Delta)$ [Talwar/Slivkins]	$O(\epsilon^{-\alpha} \log n)$ [Abraham et al.]
		$O(\log n)$ [Chan et al./Abraham et al.]	
planar	$1 + \epsilon$	$O(\log n)$ [Thorup]	
H-minor free	$1 + \epsilon$	$O(\|H\|! \cdot 2^{\|H\|} \log n)$ [Abraham,Gavoille]	

In general graphs, $\Theta(n \log n)$ is optimal.

Can we do better than BGP?

Yes !! (we hope), using structural properties

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Properties of large scale networks — Chordality

Well known properties	graph parameters
small diameter (logarithmic)	($\Rightarrow$ small hyperbolicity)
power law degree distribution	
high clustering coefficient	$\Rightarrow$ **few long induced cycles**

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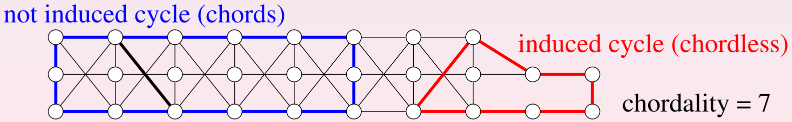
Chordality of a graph G: length of **greatest induced cycle** in G



Brief related work on chordality

Complexity — chordality $\leq k$?

NP-complete	easy reduction from hamiltonian cycle
not FPT [CF'07]	no algorithm $f(k).poly(n)$ (unless $P = NP$)
FPT in planar graphs [KK'09]	Graph Minor Theory

chordality $\leq k \Rightarrow$ treewidth $\leq O(\Delta^k)$ [Bodlaender, Thilikos'97]

Compact routing schemes in graphs with chordality $\leq k$

stretch	RT's size	computation time	
$k+1$	$O(k \log^2 n)$	$poly(n)$	[Dourisboure'05]
header never changes			
$k-1$	$O(\Delta \log n)$	$O(D)$	[NRS'09]
distributed protocol to compute RT's / no header			
$O(k \log \Delta)$	$O(k \log n)$	$O(m^2)$	**[this paper]**
or $O(k)$ and $O(\Delta \log n)$		generalization of [NRS'09]	

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Universal, Labelled scheme, no header

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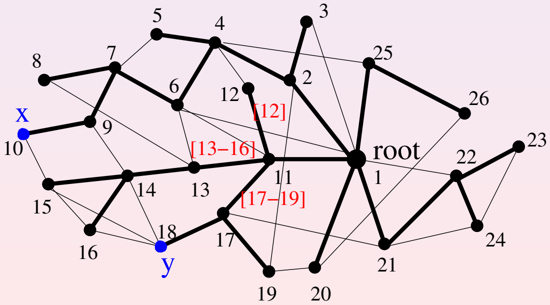
x a source node and y a destination node



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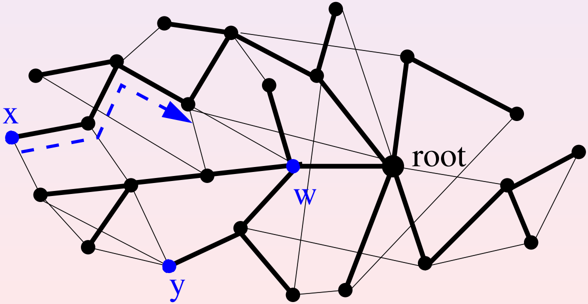

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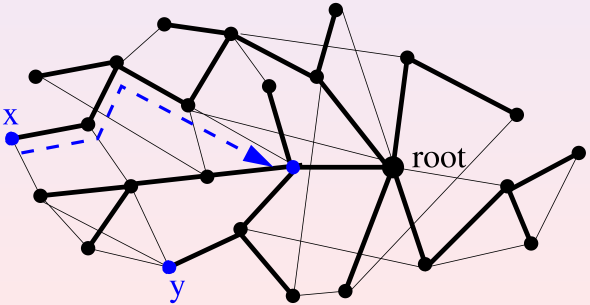

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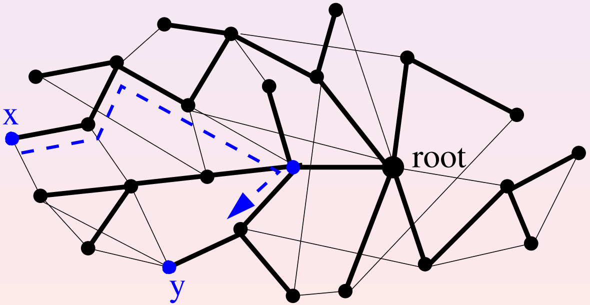

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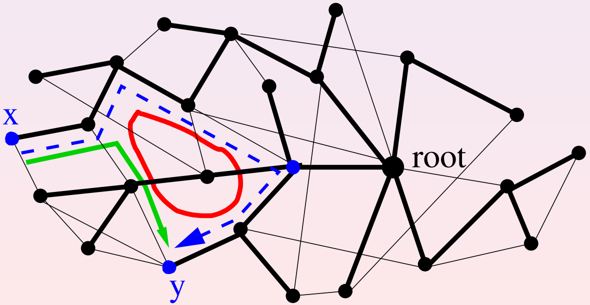

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Once T has been chosen

Space: *labeling of nodes*: any rooted subtree $\Leftrightarrow$ interval
routing table: each node knows the interval of its neighbors
$O(\Delta \log n)$ bits per node

Time: easy to compute in time $O(D)$ in synchronous distributed way

Stretch: if T is a BFS-tree in a k-chordal graph: additive stretch $\leq k - 1$

From Cops and robber to Routing via Treewidth

Compact routing scheme

using structure of k−chordal graphs

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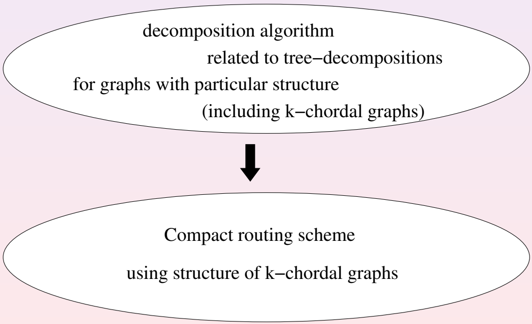

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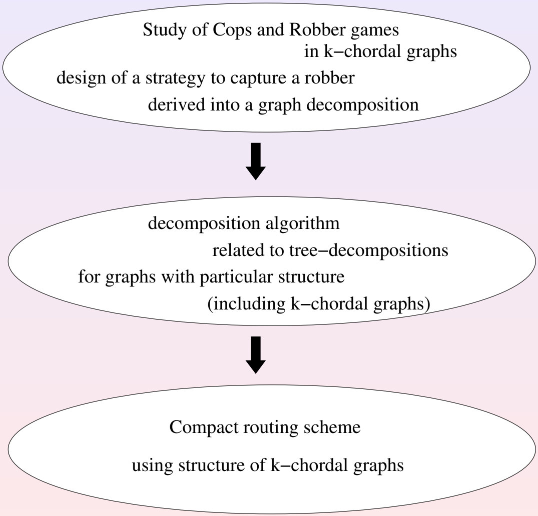

Our results [KLNS12]

Theorem 1: Cops and Robber games

$k-1$ cops are sufficient to capture a robber in k-chordal graphs

Theorem 2: main result

There is a $O(m^2)$-algorithm that, in any m-edge graph G,

- either returns an induced cycle larger than k,
- or compute a tree-decomposition with each bag being the closed neighborhood of an induced path of length $\leq k-1$.

($\Rightarrow$ treewidth $\leq O(\Delta.k)$ and treelength $\leq k$)

Theorem 3: for any graph admitting such a tree-decomposition

there is a compact routing scheme using RT's of size $O(k \log n)$ bits, and achieving additive stretch $O(k \log \Delta)$.

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Cops & robber games [Nowakowski and Winkler; Quilliot, 83]

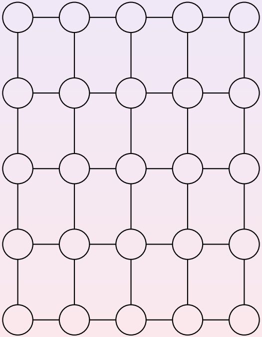
Initialization:

1. $\mathcal{C}$ places the cops;
2. $\mathcal{R}$ places the robber.

Step-by-step:

- each cop traverses at most 1 edge;
- the robber traverses at most 1 edge.

Robber captured:
A cop occupies the same vertex as the robber.



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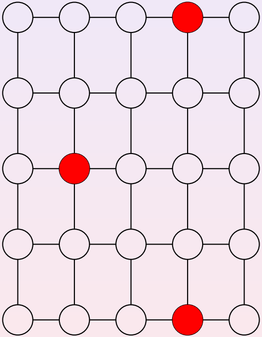

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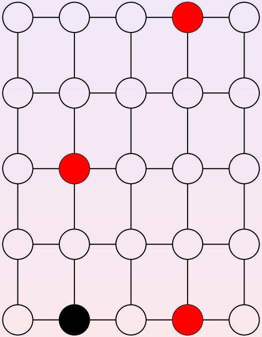

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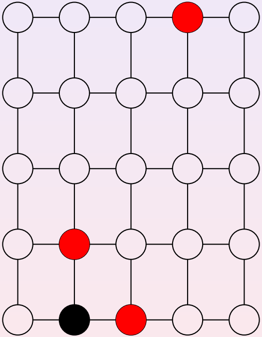

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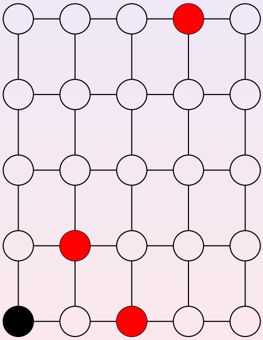

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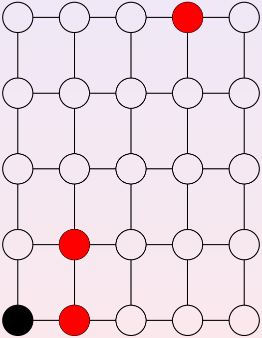

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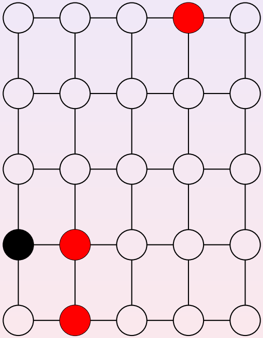

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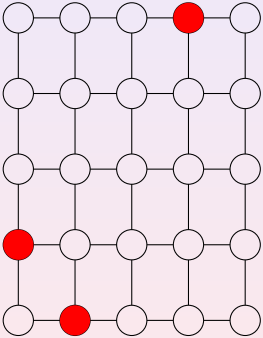
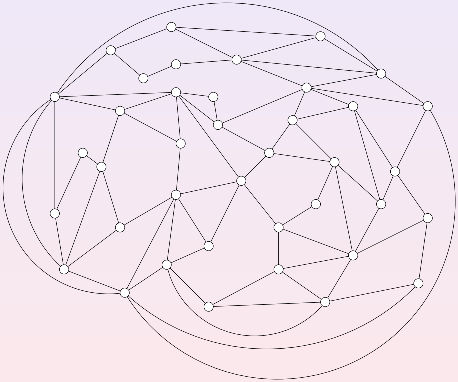


Cop number

$cn(G)$ minimum number of cops to capture any robber

Determine $cn(G)$ for the following graph G?



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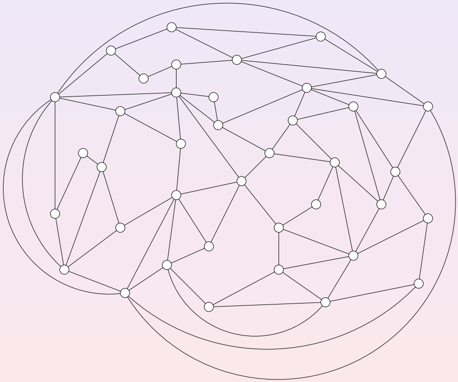

$cn(G) \leq 3$ for any planar graph G [Aigner, Fromme, 84]

Computing $cn(G)$ is NP-hard [FGKNS 10]

Cops & robber games: the graph structure helps!!

- G with **girth** g (min induced cycle) and **min degree** d: $cn(G) \geq d^g$ [Frankl 87]
- $\exists$ n-node graphs G (projective plane): $cn(G) = \Theta(\sqrt{n})$ [Frankl 87]
- G with **dominating set** k: $cn(G) \leq k$ [folklore]
- **Planar graph** G: $cn(G) \leq 3$ [Aigner, Fromme, 84]
- **Minor free graph** G excluding a minor H: $cn(G) \leq |E(H)|$ [Andreae, 86]
- G with **genus** g: $cn(G) \leq 3/2g + 3$ [Schröder, 01]
- G with **treewidth** t: $cn(G) \leq t/2 + 1$ [Joret, Kaminsk,Theis 09]
- G **random graph** (Erdös Reyni): $cn(G) = O(\sqrt{n})$ [Bollobas *et al.* 08]
- **any** n-node graph G: $cn(G) = O(\frac{n}{2^{(1+o(1))\sqrt{\log n}}})$ [Lu,Peng 09, Scott,Sudakov 10]

Conjecture: For any connected n-node graph G, $cn(G) = O(\sqrt{n})$. [Meyniel 87]

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Conjecture: For any connected n-node graph G, $cn(G) = O(\sqrt{n})$. [Meyniel 87]

Theorem 1 [KLNS12]

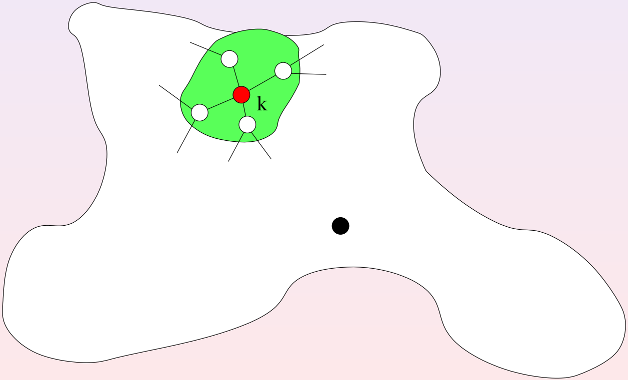
G with chordality k: $cn(G) \leq k - 1$.

Since 25 years, many researchers **study graphs structural properties** and introduce variants in the game to try solving the conjecture.
e.g., [Chiniforooshan 08, Bonato *et al.* 10, FGKNS 10, Alon,Mehrabian11, CCNV11, Clarke,McGillivray11]
see the recent survey book: The Game of Cops and Robbers on Graphs, A.Bonato and R.Nowakovski 2011

Caterpillar strategy — reduce the robber area

initialization: all k cops in one arbitrary node $P = \{v_1\}$

invariant: Cops always occupy an induced path $P = \{v_1, \cdots, v_i\}$



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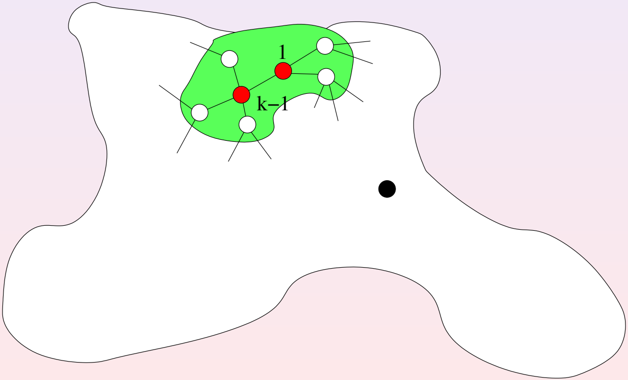

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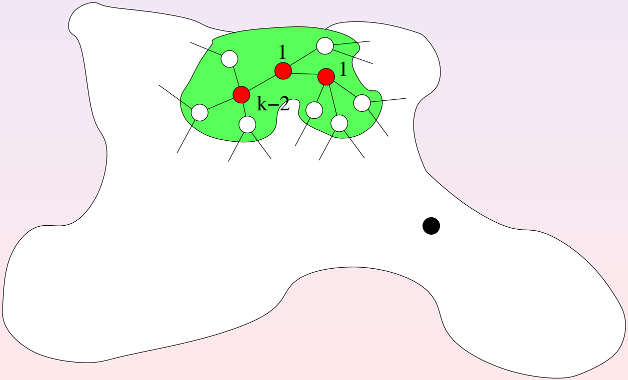

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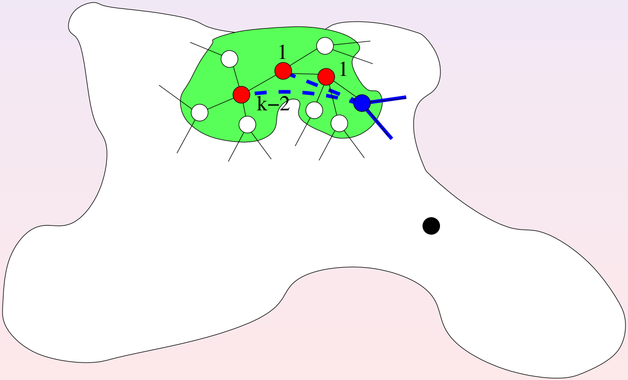

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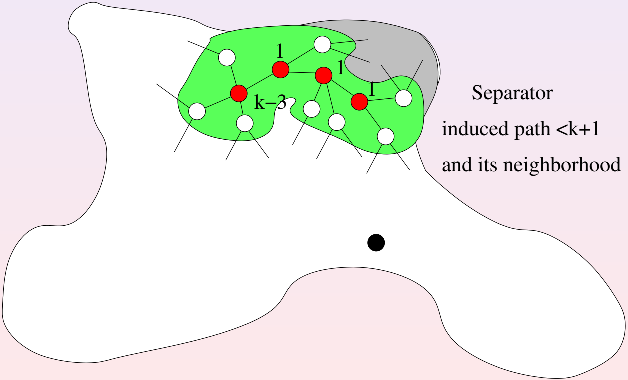

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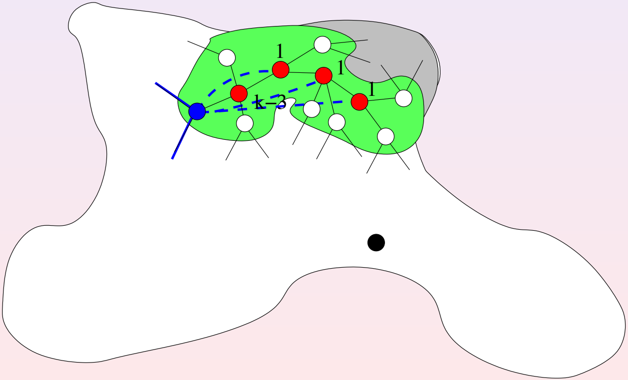

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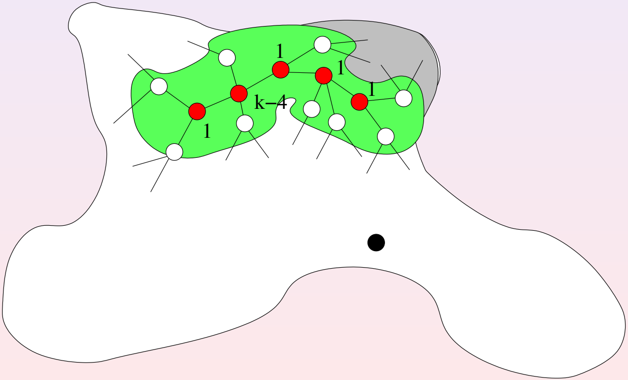

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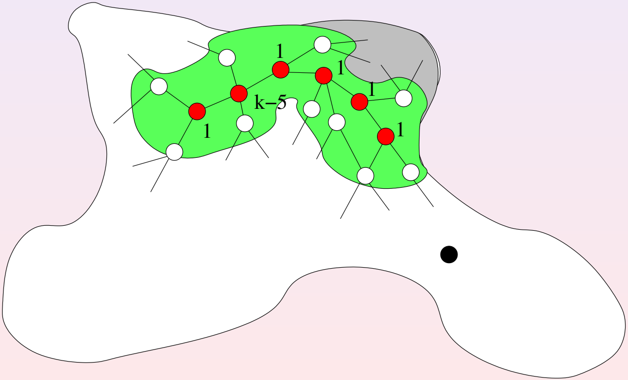

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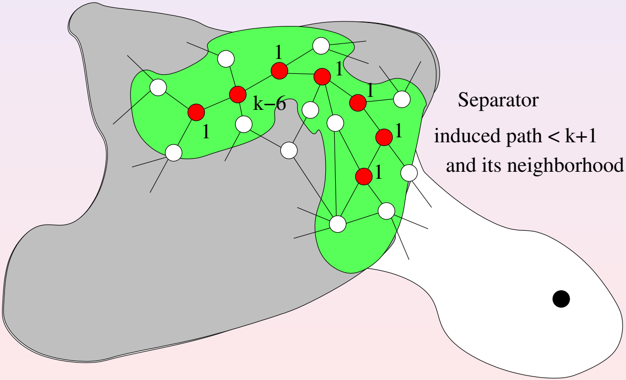

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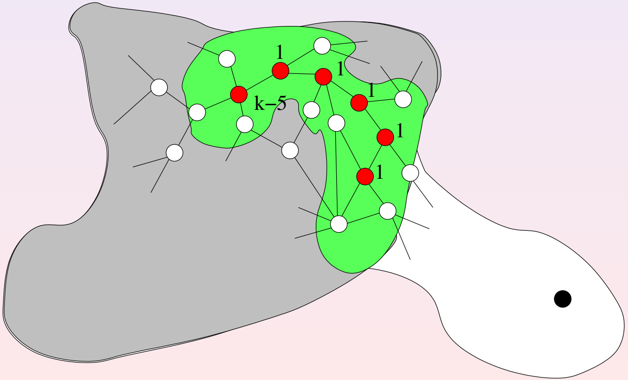

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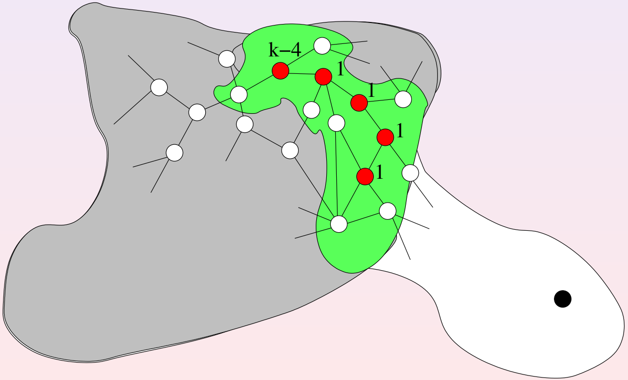

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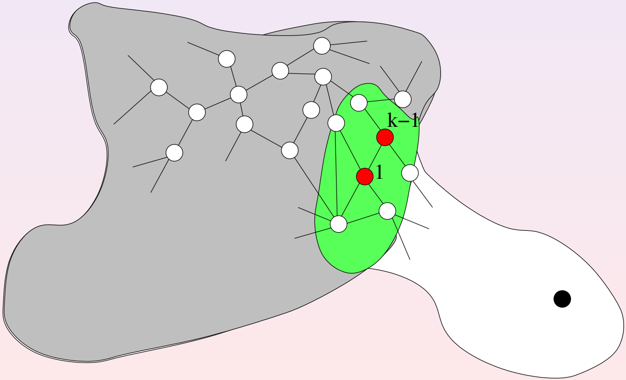

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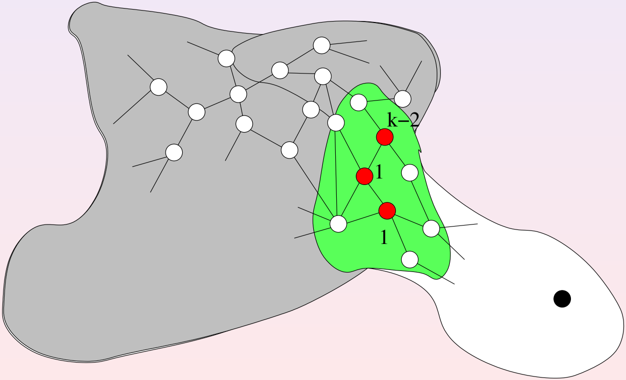
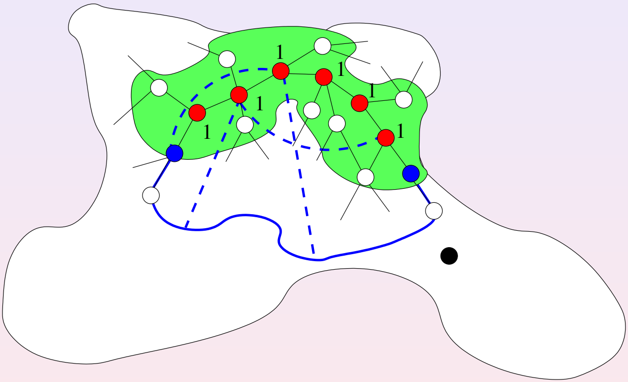


Capture in k-chordal graphs: Caterpillar strategy

$\{v_1, \cdots, v_i\}$ occupied: if no retraction $\Rightarrow$ induced cycle $\geq i+1$



Theorem 1 greedy algorithm

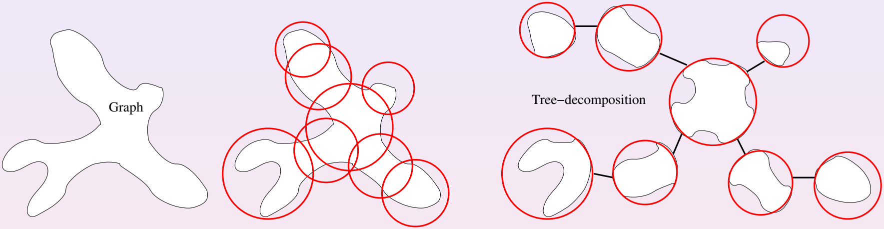
caterpillar strategy uses $\leq k-1$ cops in k-chordal graphs

Outline

1. Distributed and Compact Routing
   - Routing in a distributed way
   - Compact Routing
   - Using structural Properties: chordality
   - Our results
2. Cops and Robber Games
   - Definitions and related works
   - Cops and Robber in k-chordal graphs
3. Structural result and link with compact routing

Tree-decomposition/treewidth (unformal)

Pieces (subgraphs) with tree-like structure (bag=separator)



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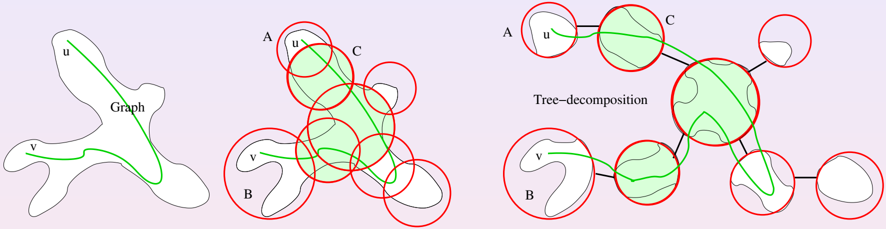

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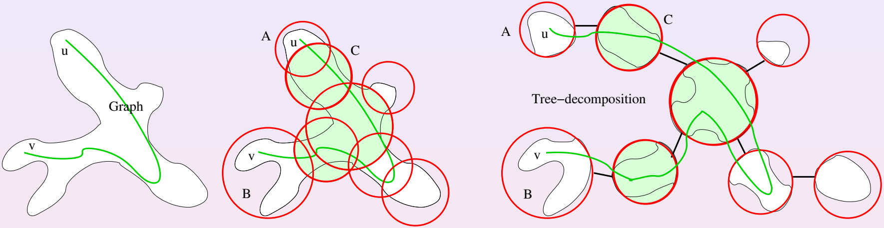
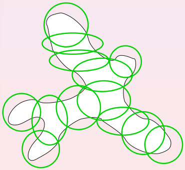


Usually, try to minimize the largest bag (treewidth)



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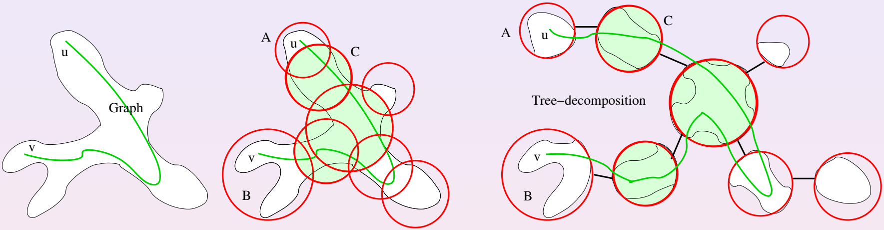
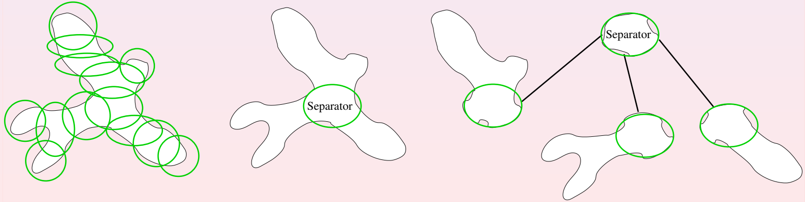
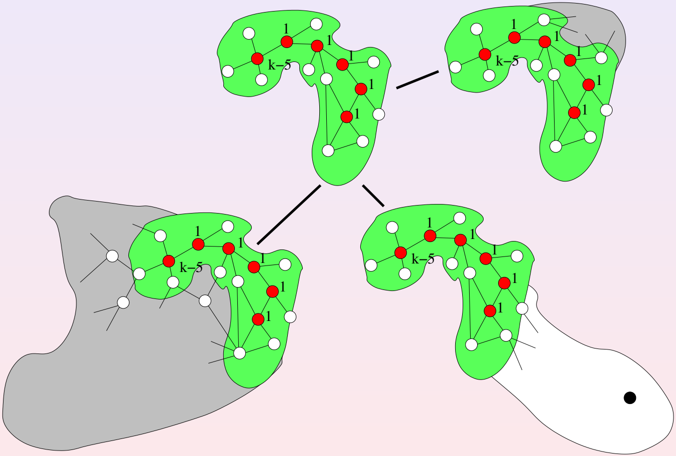


Computation: find a separator with desired properties, then induction



Tree-decomposition with k-induced paths

From k-Caterpillar's strategy



Tree-decomposition with k-induced paths

k-Caterpillar strategy $\Rightarrow$ decomposition with separator$=$ k-caterpillar

Theorem 2: main result [KLNS12]

There is a $O(m^2)$-algorithm that, in any m-edge graph G,

- either returns an induced cycle larger than k,
- or compute a tree-decomposition with each bag being the closed neighborhood of an induced path of length $\leq k-1$.

In case of k-chordal graphs:

$\Rightarrow$ treewidth $\leq O(\Delta.k)$ (improves [Bodlaender,Thilikos'97] result)

$\Rightarrow$ treelength $\leq k$ (already known)

$\Rightarrow$ hyperbolicity $\leq 3k/2$ (better bound is known)

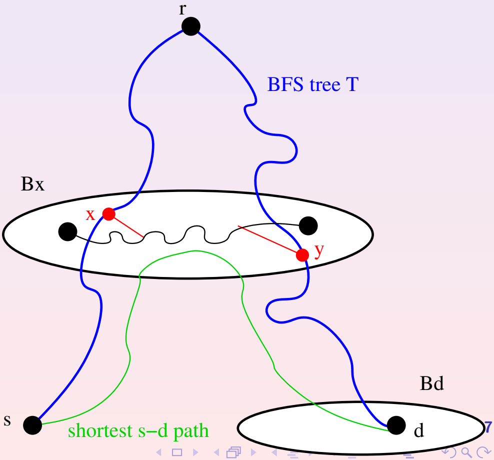
Application to compact routing

stretch $O(k \log \Delta)$ with RT's of size $O(k \log n)$ bits [KLNS12]

BFS-tree T, tree-decomposition D with k-caterpillar separators

From s to d

1. follow the path to r in T
   until find x such that
   B_x is an ancestor of B_d in D
   stretch: $+k$
2. in B_x, find y an ancestor of d in T
   stretch: $+k \log \Delta$
3. follow the path to d in T
   stretch: $+k$



Further Works

short term goals:

Implement the decomposition-algorithm and simulate on large scale models

What if other properties are included?

and then?

Structural Properties of Large-scale Networks

Efficient algorithms for $\geq 10^5$-node graphs

(e.g., for hyperbolicity $O(n^4)$-algorithms cannot work)
what about NP-hard problems? (e.g., chordality)
other "hidden" properties?

Distributed algorithms

How facing dynamicity?

models of dynamicity, localized algorithms

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